AMERICAN UNIVERSITY OF BEIRUT
Department of Electrical and Computer Engineering
EECE340 Signals and Systems -Summer 2011

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Problem Set 1

Out: Thursday, June 23, 2011 Due: Thursday, June 30, 2011

This is your first homework. It is essential that you practice the material. If you have any questions, drop by during office hours. Work individually and write your complete solutions on clean paper. Make sure you staple all the papers together

Problem 1

Consider a system whose input $x(t)$ and output $y(t)$ satisfy the following first order ordinary differential equation

$$\frac{dy(t)}{dt} + 2y(t) = x(t)$$

Given that the system was initially at rest.

a) Determine the output of the system $y_1(t)$ when the input is $x_1(t) = e^{2t}u(t)$. (Recall that the unit step function is $u(t) = 1, t \geq 0$, $u(t) = 0$ otherwise.)

b) Determine the output of the system $y_2(t)$ for an input $x_2(t) = \left[\gamma e^{2t} + e^{5t}\right] u(t)$

c) Determine the output of the system when the input $x_3(t) = Ke^{2(t-T)}u(t-T)$. Show that $y_3(t) = y_1(t-T)$.

Problem 2

Consider the block diagram shown in figure 1.

(a) Write the dynamic equations of the system in state variable representation.

(b) Transform and solve the obtained equations to obtain an expression for $Y(s)$ in terms of the input $G(s)$, and the initial conditions $x_1(0)$ and $x_2(0)$. Point to the ZSR and ZIR in this solution. (In the figure, the input is indicated as $u(t)$, which is meant to be $g(t)$).

(c) find y(t) for input $g(t) = 5u(t)$, where $u(t)$ is the unit step. Assume that the initial conditions $x_1(0) = 4$ and $x_2(0) = -0.5$.

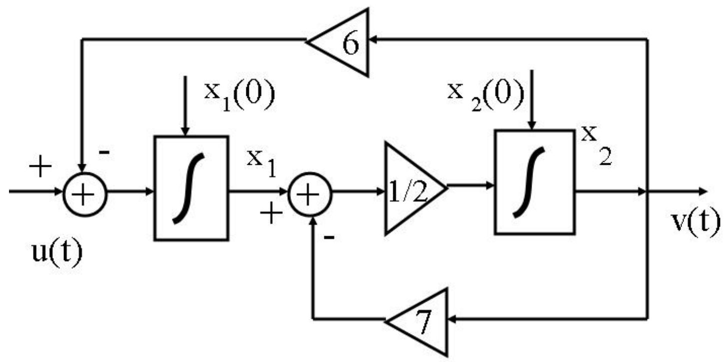

Figure 1: Block diagram of problem 2

Problem 3

Consider the circuit shown in figure 2 (left). The input is a current source i(t), and the output is the voltage $v(t)$ across the resistor.

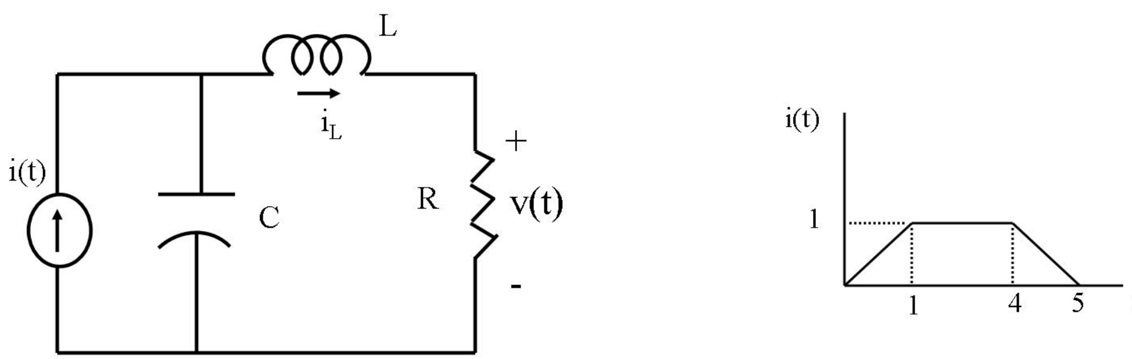

Figure 2: Circuit and input of Problem 3

(a) Determine the system function $H(s) = \frac{V(s)}{I(s)}$

(b) For $R = 4\ \Omega$, $L = 1$ H , $C = 1/4\ F$, $v_c(0) = 4$, and $i_L(0) = 1$, what is the general form of the ZIR.

(c) Find the ZSR for an input $i(t)$ of the form shown in Figure 2 (right).

Problem 4

Mr Sam Smartguy has just started his new job at the System Hackers Inc. On his second day, he was given a closed electric box to find out as much as he can about it. He was told that the system inside is Linear, Time-Invariant. His experimentation on the system has given him the following:

1. Independent of the state of the system at t=0, an input $x(t) = e^{-4t}$, $t > 0$ yielded an output of the form $y(t) = 5e^{-4t} + k_o e^{-t} + k_1 t e^{-t}$, $t > 0$.

2. Independent of the state of the system at t=0, an input of the form $x(t) = e^{-2t}$, $t > 0$, the output had the form $y(t) = k_2 e^{-t} + k_3 t e^{-t}$, $t > 0$.

Sam is out of ideas. He is asking your help to understand the results of these experiments or he will lose his job. What do you think the system function $H(s)$ must look like? Assume that $H(s)$ is a proper fraction, that is, $H(s) \to 0$ as $s \to \infty$.

(Mini) Problem 5

Show that the response of a CT LTI system with a transfer function $H(jw)$ to a sinusoidal input $x(t) = A\cos(w_o t + \theta)$ is given by

$$y(t) = A|H(jw_o)|\cos(w_o t + \theta_1)$$

where θ_1 is a phase shift to be determined.

Problem 6

This is your first Matlab assignment and is intended to get your feet wet with a very useful tool for engineering. Please make sure you read the Matlab Tutorial found on Moodle and that you do things yourself. Provide both the code and output figures.

Consider the following transfer function:

$$\frac{Y(s)}{G(s)} = H(s) = \frac{100}{s^2 + 15s + 10}$$

This can describe for example, the dynamics of an RLC circuit as you have seen before in EECE210. We will here simulate the response of this system in Matlab to various types of inputs.

a- Create the continuous time system function in Matlab. To do so, use the function **tf**. (*type "help tf" at the command prompt*)

b- Now simulate the unit step response of this transfer function (g(t)=u(t)). Here, the function **step** is all you need.

Note that you can call step(...) at the command prompt in Matlab by itself for an automatic display of the response. Alternatively, you can call it with an output argument:

[OUT,TIME]=step(...)

and then use the **plot** *function to see the output as a function of time. To adjust the scale of the figures look at the command* **axis***, or alternatively edit the figure through its buttons.*

What type of response does this system exhibit?

c- Now we will simulate the output due to any other input. Assume the input is as follows:

$$g(t) = \begin{cases} 1, & \text{t} \leq 40 \text{ sec;} \\ -2, & \text{t} > 40 \text{ sec} \end{cases}$$

1- We first need to create the time vector over which the simulation is to take place. Assume that we are interested in $0 \leq t \leq 100$ sec. Although this is a continuous system, Matlab obviously needs to simulate its behavior in discrete steps. Therefore, time will have to be discretized. To do so, assume that we "sample" time by steps of 0.01 sec. Create the time vector T_{in} accordingly.

2- Now create the input vector G_{in}. Note that this will specify the input at each sampled time point in T_{in}.

3- We are now ready to simulate the response of $H(s)$ to $g(t)$. To do so, look at the function **lsim**.

Plot the output of the system for the given $g(t)$.

d- Practice changing your code so as to change the transfer function $H(s)$ to exhibit oscillatory response.You should be able to do so very easily once you write the code into a *myfirst.m* file and run it in Matlab.