Review Problems 1

Problem 1

Determine the homogeneous solution for the systems described by the following difference equations:

a) $y[n] - \alpha y[n-1] = 2x[n]$

$$\begin{aligned} r - \alpha &= 0 \\ y^{(h)}[n] &= c_1 \alpha^n \end{aligned}$$

b) $y[n] + y[n-1] + \frac{1}{4} y[n-2] = x[n] + 2x[n-1]$

$$\begin{aligned} r^2 + r + \frac{1}{4} &= 0 \\ r &= -\frac{1}{2}, -\frac{1}{2} \\ y^{(h)}[n] &= c_1 \left(-\frac{1}{2}\right)^n + c_2 n \left(-\frac{1}{2}\right)^n \end{aligned}$$

Problem 2

We consider the following system: $y[n] - \frac{2}{5} y[n-1] = 2x[n]$

Determine a particular solution for the given inputs:

a. x[n]=2u[n]

$$
\begin{aligned}
y^{(p)}[n] &= ku[n] \\
k - \frac{2}{5}k &= 4
\end{aligned}
$$

$$
\begin{aligned}
k &= \frac{20}{3} \\
y^{(p)}[n] &= \frac{20}{3}u[n]
\end{aligned}
$$

b. $x[n] = -\left(\frac{1}{2}\right)^n u[n]$

$$
\begin{aligned}
y^{(p)}[n] &= k\left(\frac{1}{2}\right)^n u[n] \\
k\left(\frac{1}{2}\right)^n - \frac{2}{5}\left(\frac{1}{2}\right)^{n-1} k &= -2\left(\frac{1}{2}\right)^n \\
k &= -10 \\
y^{(p)}[n] &= -10\left(\frac{1}{2}\right)^n u[n]
\end{aligned}
$$

Problem 3

Solve the following difference equations:

a) $y[n] - \frac{1}{2}y[n-1] = 2x[n]; \quad y[-1] = 3; x[n] = -\left(\frac{1}{2}\right)^n u[n]$

$$\begin{aligned}
n \geq 0 & \quad \text{natural: characteristic equation} \\
r - \frac{1}{2} &= 0 \\
y^{(n)}[n] &= c\left(\frac{1}{2}\right)^n \\
& \quad \text{particular} \\
y^{(p)}[n] &= k\left(-\frac{1}{2}\right)^n u[n] \\
k\left(-\frac{1}{2}\right)^n - \frac{1}{2}k\left(-\frac{1}{2}\right)^{n-1} &= 2\left(-\frac{1}{2}\right)^n \\
k &= 1 \\
y^{(p)}[n] &= \left(-\frac{1}{2}\right)^n u[n] \\
& \quad \text{Translate initial conditions} \\
y[n] &= \frac{1}{2}y[n-1] + 2x[n] \\
y[0] &= \frac{1}{2}3 + 2 = \frac{7}{2} \\
y[n] &= \left(-\frac{1}{2}\right)^n u[n] + c\left(\frac{1}{2}\right)^n u[n] \\
\frac{7}{2} &= 1 + c \\
c &= \frac{5}{2} \\
y[n] &= \left(-\frac{1}{2}\right)^n u[n] + \frac{5}{2}\left(\frac{1}{2}\right)^n u[n]
\end{aligned}$$

b)

$$y[n] + \frac{1}{4}y[n-1] - \frac{1}{8}y[n-2] = x[n] + x[n-1]; \quad y[-1] = 4; y[-2] = -2; x[n] = -(1)^n u[n]$$

$$
\begin{aligned}
n \geq 0 \quad & \text{natural: characteristic equation} \\
r^2 + \frac{1}{4}r - \frac{1}{8} &= 0 \\
r &= -\frac{1}{4}, \frac{1}{2} \\
y^{(n)}[n] &= c_1\left(\frac{1}{2}\right)^n + c_2\left(-\frac{1}{4}\right)^n \\
& \text{particular} \\
y^{(p)}[n] &= k(-1)^n u[n] \\
\text{for } n \geq 1 & \\
k(-1)^n + k\frac{1}{4}(-1)^{n-1} - k\frac{1}{8}(-1)^{n-2} &= (-1)^n + (-1)^{n-1} = 0 \\
k &= 0 \\
y^{(p)}[n] &= 0 \\
y[n] &= c_1\left(\frac{1}{2}\right)^n + c_2\left(-\frac{1}{4}\right)^n \\
& \text{Translate initial conditions} \\
y[n] &= -\frac{1}{4}y[n-1] + \frac{1}{8}y[n-2] + x[n] + x[n-1] \\
y[0] &= \frac{1}{4}4 + \frac{1}{8}(-2) + 1 + 0 = -\frac{1}{4} \\
y[1] &= -\frac{1}{4}\left(-\frac{1}{4}\right) + \frac{1}{8}4 + -1 + 1 = \frac{9}{16} \\
-\frac{1}{4} &= c_1 + c_2 \\
\frac{9}{16} &= \frac{1}{2}c_1 - \frac{1}{4}c_2 \\
y[n] &= \frac{2}{3}\left(\frac{1}{2}\right)^n - \frac{11}{12}\left(-\frac{1}{4}\right)^n
\end{aligned}
$$

Problem 4

Consider the signal $x[n] = \cos(0.0625\,\pi n)$. Determine whether or not x[n] is periodic. If so, determine the number of samples per fundamental frequency.

x[n] is periodic with a period N=32

Problem 5

The input-output of a discrete-time system is given by:

$y[n]=\sin(x[n+1])$

a. Is this system linear? Justify your answer

No, because $\sin(ax[n+1]) \neq a.\sin(x[n+1])$

b. Is this system time invariant? Justify your answer.

Yes, because $x[n-n_0] \rightarrow y[n-n_0]$

Problem 6

The input-output of a discrete-time system is given by:

$y[n]=0.6\, x[n+1] + 0.2\, x[n] + 0.07\, x[n-1]$

a. Determine the system impulse response.

$$h[n] = 0.6\ \delta[n+1] + 0.2\ \delta[n] + 0.07\ \delta[n-1]$$

b. Is this system causal? Justify your answer.

No, because $h[n] \neq 0$ for $n<0$.

Problem 7

A Discrete-time system is represented by the following input-output relation

$$y[n] = \sum_{k=1}^{n} \left(x^4[k+2] + x^2[k+1] - x[k] \right)$$

Where x and y are the input and output of each system

a. Is this system a causal system? Show your work.

No, y[n] depends on Future value of x[n]

b. Is this system a stable system? Show your work.

No, if x[n]=-1, then $y[n] = \sum_{k=1}^{n} 3 \xrightarrow[n \to \infty]{} \infty$

Problem 8

Assume that the response of an LTI system to the input x[n]=u[n] (discrete-time unit step) is given by:

$y[n] = 2\delta[n] - 3\delta[n-2] + \delta[n-3]$

Derive the impulse response h[n] for this system.

$$
\begin{aligned}
h[n] &= y[n] - y[n-1] \\
&= 2\delta[n] - 3\delta[n-2] + \delta[n-3] - 2\delta[n-1] + 3\delta[n-3] - \delta[n-4] \\
&= 2\delta[n] - 2\delta[n-1] - 3\delta[n-2] + 4\delta[n-3] - \delta[n-4]
\end{aligned}
$$