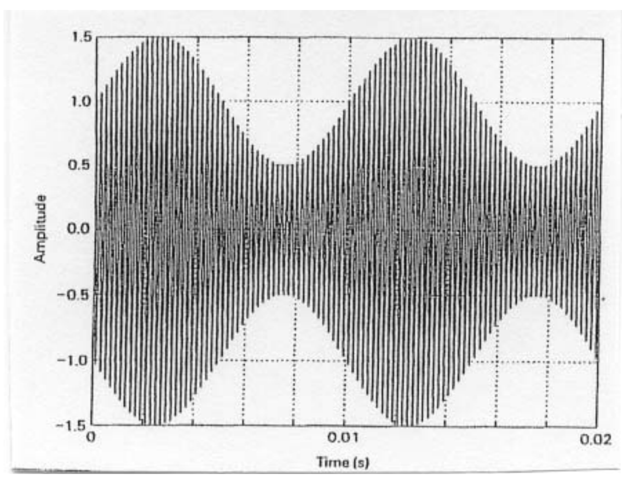
AMERICAN UNIVERSITY OF BEIRUT
FACULTY OF ENGINEERING AND ARCHITECTURE
ELECTRICAL AND COMPUTER ENGINEERING DEPARTMENT

EECE 340 – Signals & Systems
Quiz II-Spring 2010
Solution

Problem 1 (8 pts)

A conventional AM signal is shown below,



Determine the modulation index

$A_{max} = 1.5$

$A_{min} = 0.5$

$$\text{Modulation index}: \mu = \frac{A_{max} - A_{min}}{A_{max} + A_{min}} = \frac{1.5 - 0.5}{1.5 + 0.5} = 0.5$$

Problem 2 (8 pts)

The output signal of an AM modulator is given by:

$$s(t) = 5\cos(1800\pi t) + 20\cos(2000\pi t) + 5\cos(2200\pi t)$$

a. Determine the message signal m(t) (2 pts)

$s(t) = 20[1 + 0.5\cos(200\pi t)]\cos(2000\pi t) = A_c[1 + k_a m(t)]\cos(\omega_c t)$

By comparison

$m(t) = \frac{1}{2k_a}\cos(200\pi t)$ **-1 if coefficient is wrong**

b. Determine the carrier c(t) (2 pts)

$s(t) = 20\cos(2000\pi t)$

c. Determine the modulation index (2 pts)

$\mu = 0.5$

d. Determine the ratio of sideband power to the total power (2 pts)

Sideband power= $P_s = \frac{(5)^2}{2} + \frac{(5)^2}{2} = 25\text{Watts}$ **(1 pt)**

Total power= $P_t = \frac{(5)^2}{2} + \frac{(5)^2}{2} + \frac{(20)^2}{2} = 225\text{Watts}$ **(1 pt)**

Ratio= $\frac{25}{225} = \frac{1}{9}$

<u>Problem 3 (12 pts)</u>

Consider the modulation system shown below

Assume that $m(t) = 2\cos(20\pi t)$ is transmitted using the modulation system with carrier frequency of 500 Hz

a. Assume that A=2
   - What type of modulation does this correspond to? (3 pts)
     $s(t) = 2[1+\cos(20\pi t)]\cos(2000\pi t)$ **2 pts**
     It is DSB-LC=AM **1 pt**

   - What is the modulation index? (3 pts)
     $\mu = 1$ **3 pts**

b. Assume that A=0
   - What type of modulation does this corresponds to? (3 pts)
     $s(t) = 2\cos(20\pi t)\cos(2000\pi t)$ **2 pts**
     DSB-SC **1 pt**

   - What is the power of the output signal (3 pts)
     $s(t) = \cos(2020\pi t) + \cos(1980\pi t)$

$$P = \frac{(1)^2}{2} + \frac{(1)^2}{2} = 1\text{Watt}$$

Problem 4 (10 pts)

Determine the range of permissible cut-off frequencies (Bandwidth) for the ideal low-pass filter used to reconstruct the signal

$$x(t) = 10\cos(600\pi t)\cos^2(1600\pi t)$$

which is sampled at 4000 samples per second.

$$x(t) = 10\cos(600\pi t)\left[\frac{1}{2} + \frac{1}{2}\cos(3200\pi t)\right]$$
$$x(t) = 5\cos(600\pi t) + 2.5\cos(2600\pi t) + 2.5\cos(3800\pi t)$$

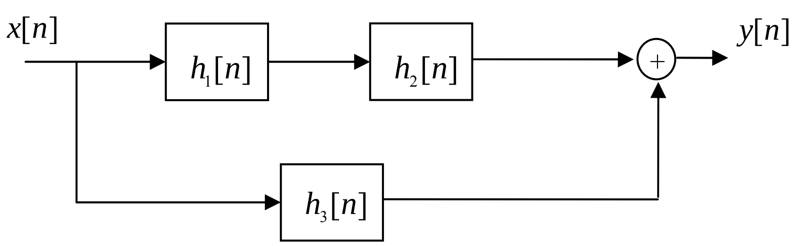
The bandwidth of x(t)=1900 Hz **5 pts**

The permissible cut-off frequency of the LPF is: (1900 --- 2100 Hz) **5 pts**

Problem 5 (10 pts)

Determine the overall impulse response of the DT system shown in the diagram below. The subsystems involved in the diagram are assumed linear and time-invariant and they are given by:

$$h_1[n] = -3\delta[n+1] + 2\delta[n-2]$$
$$h_2[n] = 2\delta[n+2] + \delta[n-1]$$
$$h_3[n] = 3\delta[n+1] - \delta[n] + 2\delta[n-1] + 7\delta[n-3] + 5\delta[n-5]$$



$$h[n] = h_1[n] * h_2[n] + h_3[n]$$
$$H(z) = H_1(z)\cdot H_2(z) + H_3(z) = \left[-3z + 2z^{-2}\right]\left[2z^2 + z^{-1}\right] + \left[3z - 1 + 2z^{-1} + 7z^{-3} + 5z^{-5}\right]$$
$$H(z) = -6z^3 - 3 + 4 + 2z^{-3} + \left[3z - 1 + 2z^{-1} + 7z^{-3} + 5z^{-5}\right]$$
$$H(z) = -6z^3 + 3z + 2z^{-1} + 9z^{-3} + 5z^{-5}$$
$$h[n] = -6\delta[n+3] + 3\delta[n+1] + 2\delta[n-1] + 9\delta[n-3] + 5\delta[n-5]$$

-2 for every term

Problem 6 (8 pts)

The output y[n] of a discrete-time LTI system is related to its input x[n] by

$$y[n]=2^{-n}\sum_{k=-\infty}^{k=\infty}2^{k}x[k]$$

a. Is the system causal? Please explain (4 pts)
   Not causal as the output depends on future values of the input

b. Is the system stable? Explain (4 pts)
   Not stable as the output is not bounded for a bounded input

Problem 7 (8 pts)

Consider two discrete-time systems with the following unit sample responses:

$$x[n]=\delta[n+1]-\delta[n-1]+2\delta[n-2] \text{ and } y[n]=\delta[n]-\delta[n-2]$$

Compute $x[n]*y[n]$

$$X(z)=z-z^{-1}+2z^{-2}$$

$$Y(z)=1-z^{-2}$$

$$X(z)Y(z)=z-2z^{-1}+2z^{-2}+z^{-3}-2z^{-4}$$

$$x[n]*y[n]=\delta[n+1]-2\delta[n-1]+2\delta[n-2]+\delta[n+3]-2\delta[n-4]$$

-2 for every term

Problem 8 (8 pts)

Consider a discrete-time LTI system with unit sample response, h(n), given by:

$$h(n)=a^{-n}u(-n-1) \text{ with } |a|<1.$$

Determine the unit-step response of the system using discrete-time convolution.

$$y[n]=h[n]*u[n]=\sum_{k=-\infty}^{k=\infty}u[k]a^{-n+k}u[-n+k-1]$$

$$y[n]=a^{-n}\sum_{k=0}^{k=\infty}a^{k}u[-n+k-1]=a^{-n}\sum_{k=n+1}^{\infty}a^{k}$$

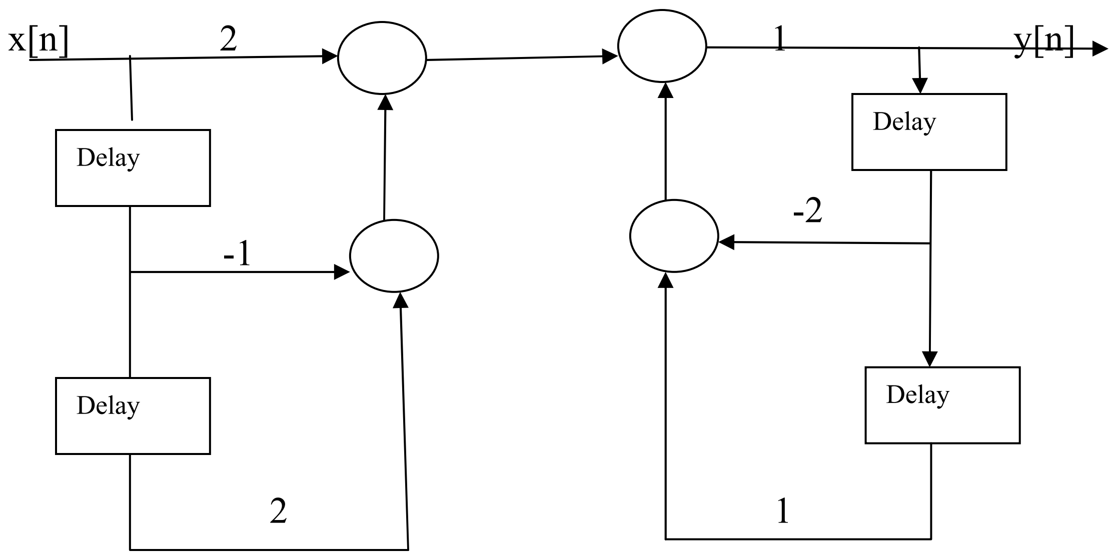
$$y[n]=\frac{a}{1-a}$$

Problem 9 (10 pts)

The input-output relationship of a LTI-DT system is given below

$$y[n] + 2y[n-1] - y[n-2] = 2x[n] - x[n-1] + 2x[n-2]$$

a. Represent this system in Direct Form I.



c. Represent this system in Direct Form II.

Just place system I before system 2

Problem 10 (8 pts)

Assume the impulse response of an LTI system is h[n] given below:

$$h[n] = \delta[n+1] + 2\delta[n-2] - 3\delta[n-4]$$

a. Give the difference equation for this LTI system. (3 pt)

$y[n]=x[n+1]+2x[n-2]-3x[n-4]$

-1 for every wrong term

b. Compute the output y[n] for an input $x[n]=4\delta[n]+2\delta[n-1]+4\delta[n-4]-\delta[n-5]+6\delta[n-6]$ (5 pts)

$y[n]=4\delta[n+1]+2\delta[n]+8\delta[n-2]+8\delta[n-3]-13\delta[n-4]+8\delta[n-6]-2\delta[n-7]$
$+3\delta[n-9]-18\delta[n-10]$

-1 for every wrong term

-2 if not simplified

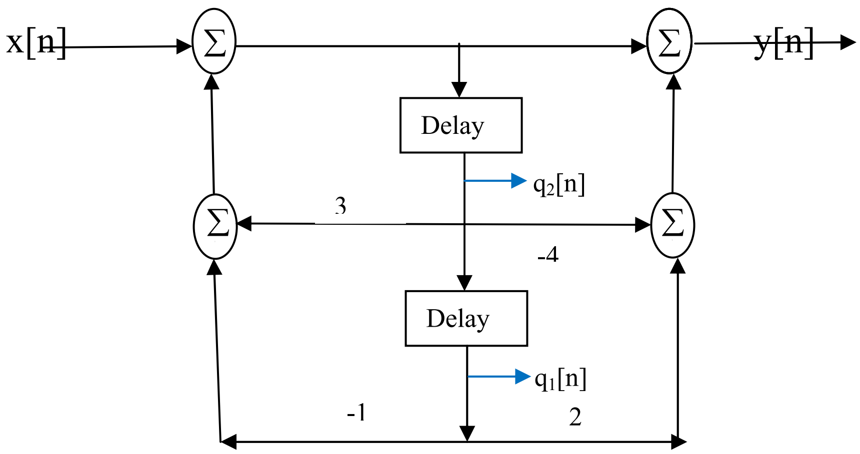
Problem 11 (10 pts)

A LTI-DT system with input x[n] and output y[n] is represented by the following dynamic equations.

$$\begin{bmatrix} q_1[n+1] \\ q_2[n+1] \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -1 & 3 \end{bmatrix} \begin{bmatrix} q_1[n] \\ q_2[n] \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} x[n]$$

$$y[n] = \begin{bmatrix} 1 & -1 \end{bmatrix} \begin{bmatrix} q_1[n] \\ q_2[n] \end{bmatrix} + [1]x[n]$$

Represent this system in Direct Form II.



-2 for every wrong coefficient

-8 for any form different than direct form II