AMERICAN UNIVERSITY OF BEIRUT
ELECTRICAL AND COMPUTER ENGINEERING DEPARTMENT

EECE 340 – Signals and Systems
QUIZ 1 - Solution

Problem 1: (9 pts)

Categorize each of the following signals as an energy signal or a power signal. Sate the reason for your answer.

(a) The continuous-time signal x(t) , defined by

$$x(t) = \begin{cases} 3e^{-2t}, & t \geq 0, \\ 0, & \text{otherwise.} \end{cases}$$

X(t) is an energy signal. (1 pt)

$$\int_{-\infty}^{+\infty} x^2(t)dt = \int_0^{\infty} 9e^{-4t}dt = \frac{9}{4}$$ (2 pts)

Or

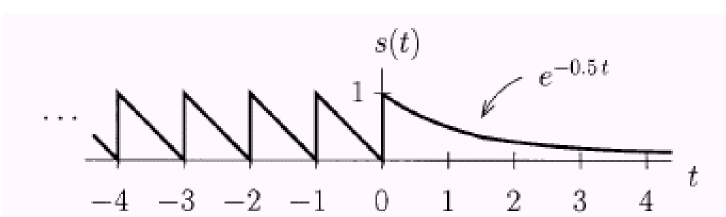
Most of the energy of the signal is within finite period of time (2 pts)

(b) The continuous-time signal z(t) , defined for $-\infty < t < \infty$ by

$z(t) = 3\sin(\pi t) + 2\cos(3\pi t)$

z(t) is a power signal (1 pt)
z(t) is periodic with a period of 2 seconds (2 pts)

(c)



S(t) is not an energy signal nor a power signal (1 pt)
Area under the square of the curve is not finite and the signal is not periodic (2 pts)

Problem 2: (8 pts)

A system is defined by the input-output relationship given by:

$$y(t) = x(t)\sin(2t) + 1$$

a. Is this system linear? Justify your answer

Not a linear system **(1 pt)**

$$x_1(t) \to y_1(t) = x_1(t)\sin(2t) + 1$$

$$x_2(t) \to y_2(t) = x_2(t)\sin(2t) + 1 \qquad \textbf{(1 pt)}$$

$$x_1(t) + x_2(t) \to [x_1(t) + x_2(t)]\sin(2t) + 1 \neq y_1(t) + y_2(t)$$

b. Is this system time-invariant? Justify your answer

Not a time invariant system. **(1 pt)**

$$x(t) \to y(t) = x(t)\sin(2t) + 1$$

$$x(t - t_0) \to x(t - t_0)\sin(2t) + 1 \neq y(t - t_0) = x(t - t_0)\sin(2t - 2t_0) + 1$$

(1 pt)

c. Is this system stable? Justify your answer

Yes, it is a stable system (1 pt)

If x(t) is bounded $|x(t)| \leq M$, then $|y(t)| \leq |x(t)| + 1 \leq M + 1 < M$

(bounded) **(1 pt)**

d. Is this system causal? Justify your answer.

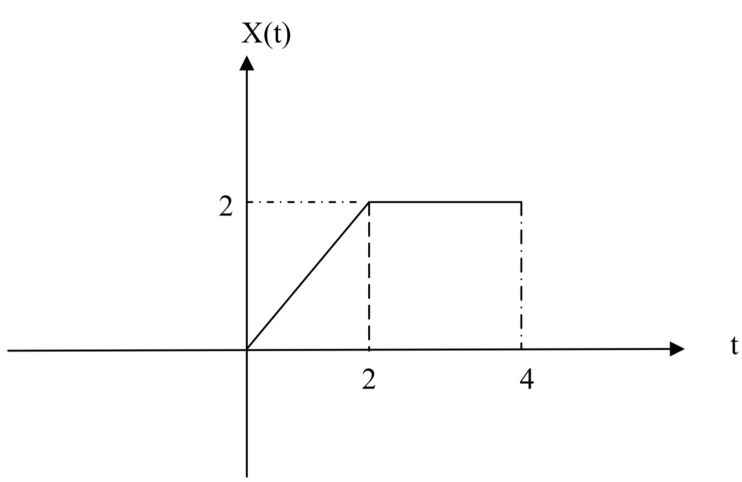
Yes it is a causal system (1 pt)

Output does not depend on future values of either the input or the output

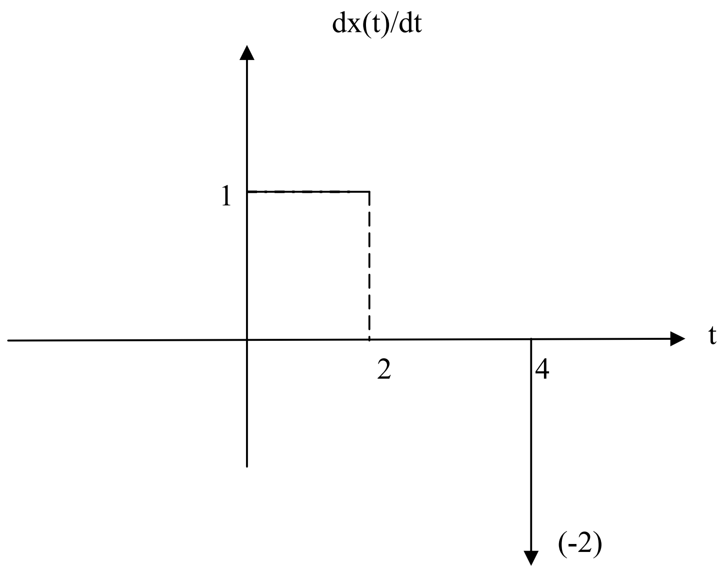
(1 pts)

Problem 3 (6 pts)

Consider the signal shown below.



a. Draw the derivate of x(t).



-2 if the impulse at t=4s is missing

b. Determine the Laplace transform of x(t).

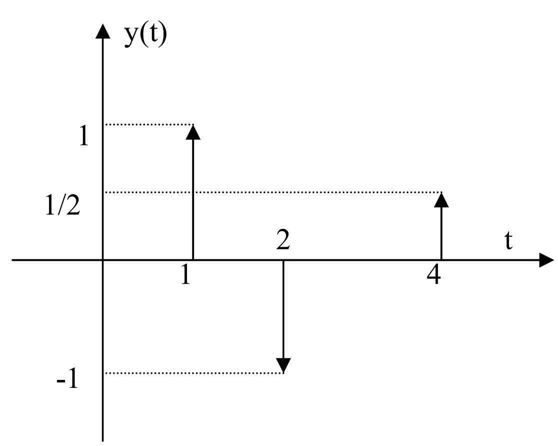
$$LT\left(\frac{dx(t)}{dt}\right) = \frac{1}{s} - \frac{e^{-2s}}{s} - 2e^{-4s} = sX(s) \quad \textbf{(2 pts)}$$

$$X(s) = \frac{1}{s^2} - \frac{e^{-2s}}{s^2} - \frac{2e^{-4s}}{s} \quad \textbf{(1 pt)}$$

<u>Problem 4 (6 pts)</u>

Consider the signal y(t) shown below

a. Write y(t) is time domain



$$y(t)=\delta(t-1)-\delta(t-2)+\frac{1}{2}\delta(t-4)$$ **(3 pts)**

b. Determine the Laplace transform of y(t)

$$Y(s)=e^{-s}-e^{-2s}+\frac{1}{2}e^{-4s}$$ **(3 pts)**

Problem 5 (8 pts)

For problems 3 and 4, determine the convolution signal z(t)=x(t)*y(t)

$$Z(s)=X(s)Y(s)=\left[\frac{1}{s^2}-\frac{e^{-2s}}{s^2}-\frac{2e^{-4s}}{s}\right]\left[e^{-s}-e^{-2s}+0.5e^{-4s}\right]$$

$$Z(s)=\left[\frac{e^{-s}}{s^2}-\frac{e^{-2s}}{s^2}+0.5\frac{e^{-4s}}{s^2}-\frac{e^{-3s}}{s^2}+\frac{e^{-4s}}{s^2}-0.5\frac{e^{-6s}}{s^2}-2\frac{e^{-5s}}{s}+2\frac{e^{-6s}}{s}-\frac{e^{-8s}}{s}\right]$$

$$Z(s)=\left[\frac{e^{-s}}{s^2}-\frac{e^{-2s}}{s^2}-\frac{e^{-3s}}{s^2}+1.5\frac{e^{-4s}}{s^2}-2\frac{e^{-5s}}{s}-0.5\frac{e^{-6s}}{s^2}+2\frac{e^{-6s}}{s}-\frac{e^{-8s}}{s}\right]$$

(4 pts)

$$z(t)=(t-1)u(t-1)-(t-2)u(t-2)-(t-3)u(t-3)+1.5(t-4)u(t-4)$$
$$-2u(t-5)-0.5(t-6)u(t-6)+2u(t-6)-u(t-8)$$

(4 pts)

Problem 6 (8 pts)

Given

$$f(t)=\int_0^t e^{-3\tau}(t-\tau)e^{-2(t-\tau)}d\tau, \qquad t\geq 0$$

a. Find the Laplace transform of f(t)

$$f(t)=e^{-3t}u(t)*te^{-2t}u(t)$$ **(2 pts)**

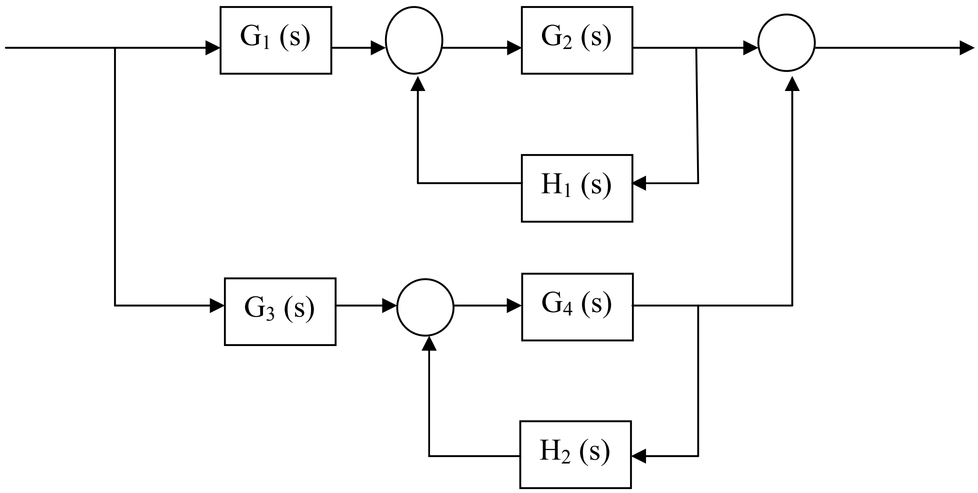
$$F(s)=\frac{1}{(s+3)(s+2)^2}$$ **(2 pts)**

b. Using F(s) and the final value theorem. Can we use the Final value theorem? Justify your answer.

sF(s) has poles at s=-3 and s=-2. Final value theorem can be applied. **(3 pts)**

$$\lim_{t\to\infty} f(t)=\lim_{s\to 0} sF(s)=0$$ **(1 pt)**

Problem 7 (8 pts)
A linear time-invariant control system is rep[present by the block diagram shown below:



Determine the transfer function of the system.

$$\frac{C(s)}{R(s)} = \frac{G_1(s)G_2(s)[1+G_4(s)H_2(s)] + G_3(s)G_4(s)[1+G_2(s)H_1(s)]}{1+G_4(s)H_2(s)+G_2(s)H_1(s)+G_4(s)H_2(s)G_2(s)H_1(s)}$$

(-2 for every missing term)

Problem 8 (12 pts)
The open-loop transfer function of a unity feedback control system is given by:

$$G(s) = \frac{3s+10}{s^3+2s^2+s}$$

1. **1.** Determine the Differential equation representing this system.

System transfer function

$$\frac{C(s)}{R(s)} = \frac{3s+10}{s^3+2s^2+4s+10}$$ **(4 pts)**

Differential equation

$$\frac{d^3c(t)}{dt^3} + 2\frac{d^2c(t)}{dt^2} + 4\frac{dc(t)}{dt} + 10c(t) = 3\frac{dr(t)}{dt} + 10r(t)$$ **(2 pts)**

2. Represent this system in a state variable form by determining the state equations and the output equation in matrix format.

Using the same procedures explained in class

$$\begin{bmatrix} X_1'(t) \\ X_2'(t) \\ X_3'(t) \end{bmatrix} = \begin{bmatrix} -2 & 1 & 0 \\ -4 & 0 & 1 \\ -10 & 0 & 0 \end{bmatrix} \begin{bmatrix} X_1(t) \\ X_2(t) \\ X_3(t) \end{bmatrix} + \begin{bmatrix} 0 \\ 3 \\ 10 \end{bmatrix} r(t)$$ **(4 pts)**

$$y(t) = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} X_1(t) \\ X_2(t) \\ X_3(t) \end{bmatrix} + [0]r(t)$$ **(2 pts)**

Problem 9 (7 pts)

The transfer function of a linear time-invariant system, whose input is x(t) and output y(t), is given by

$$\frac{Y(s)}{X(s)} = \frac{3s + 10}{2s^4 + s^3 + 3s^2 + 5s + 10}$$

Check the stability of the system by determining the number of poles of the system in each half of the complex-plane.

S^4	2	3	10
S^3	1	5	
S^2	-7	10	
S^1	45/7		
S^0	10		

2 changes of sign implies 2 roots in the rhp. No complex roots. Root distributions: (2,0,2). The system is unstable.

-2 for every wrong entry in the table
-2 for root locations
-2 for stability conclusion.

<u>Problem 10 (8 pts)</u>

The transfer function of a linear time-invariant system, whose input is x(t) and output y(t), is given by

$$\frac{Y(s)}{X(s)} = \frac{3s+10}{s^4+s^3-s-1}$$

Check the stability of the system by determining the number of poles of the system in each half of the complex-plane.

S^4	1	0	-1	
S^3	1	-1		
S^2	1	-1		$A(S) = S^2 - 1$
~~S^1~~	~~0~~			$A'(S) = 2S$
S^1	2			
S^0	-1			

1 change of sign implies 1 root in the rhp. No complex roots because the roots of A(S) are -1 and +1. Root distributions: (3,0,1). The system is unstable.

-2 for every wrong entry in the table
-4 for wrong table or not using RH table
-2 for A(S) and its solution
-2 for root locations
-2 for stability conclusion.

<u>Problem 11 (5 pts)</u>
What is the Fourier Transform of

$$y(t) = e^{-a|t|}\sin(\omega_0 t)$$

Let $f(t) = e^{-a|t|}$. $F(\omega) = \dfrac{2a}{a^2+\omega^2}$ **(2 pts)**

Shifting property of the FT

$$f(t) \leftrightarrow F(\omega)$$

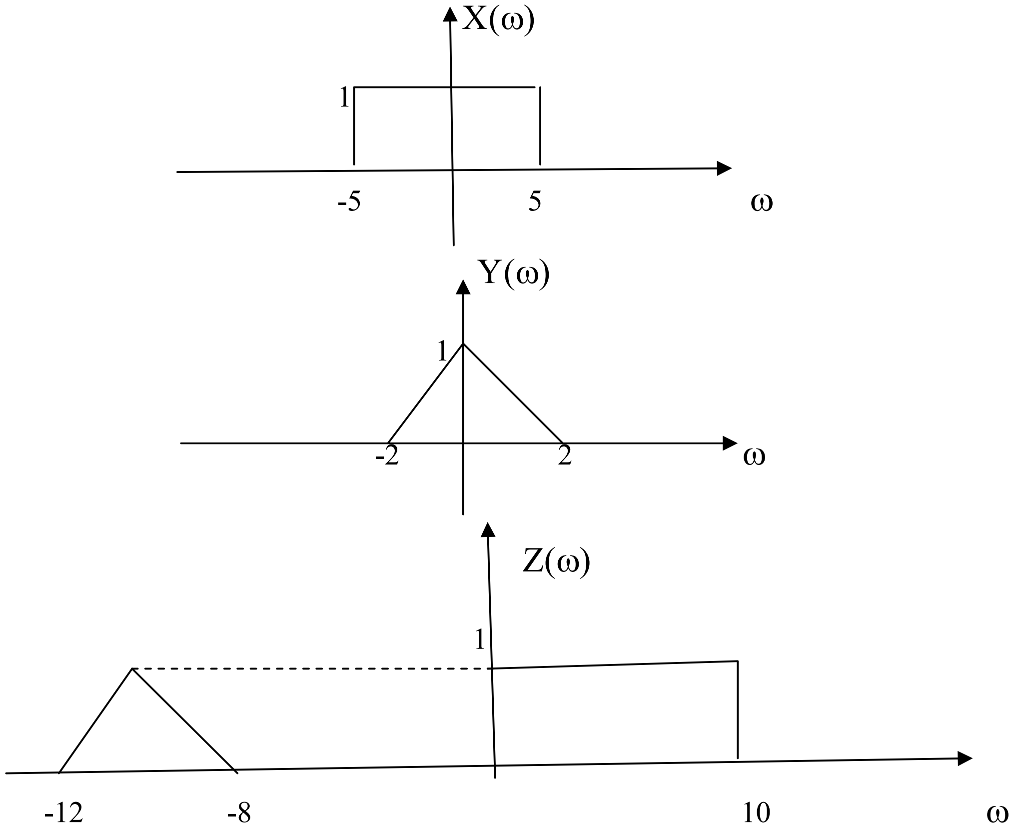
$$f(t)\sin(\omega_0 t) \leftrightarrow \frac{1}{2j}\left[F(\omega-\omega_0) - F(\omega+\omega_0)\right]$$

Accordingly,

$$Y(\omega) = \frac{1}{2j}\left[\frac{2a}{a^2+(\omega-\omega_0)^2} - \frac{2a}{a^2+(\omega+\omega_0)^2}\right]$$ **(3 pts)**

<u>Problem 12 (10 pts)</u>

The Fourier transforms of x(t), y(t), and z(t) are shown below.



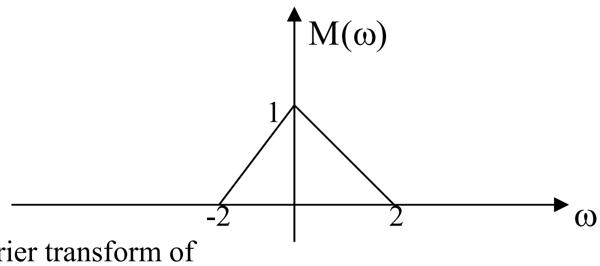
a. What is the bandwidth of x(t)?
   5 rad/s **(2 pts)**

b. What is the bandwidth of y(t)?
   2 rad/s **(2 pts)**

c. Is z(t) a real-valued signal? Explain.
   Not real as there is no symmetry about the origin (even symmetry) **(2 pts)**

d. Write z(t) as a function of x(t) and y(t).
   $z(t) = x(t)e^{i5t} + y(t)e^{-j10t}$ **(4 pts)**

Problem 13 (5 pts)
The Fourier transforms of m(t) is shown below.



Sketch the Fourier transform of

$$s(t) = 10[1 + 0.8m(t)]\cos(20t)$$

$$s(t) = 10\cos 20t + 8m(t)\cos(20t)$$

$$S(\omega) = 10\pi[\delta(\omega - 20) + \delta(\omega + 20)] + 4[M(\omega - 20) + M(\omega + 20)]$$