Chapter 6

Filter Characteristics of LTI Systems

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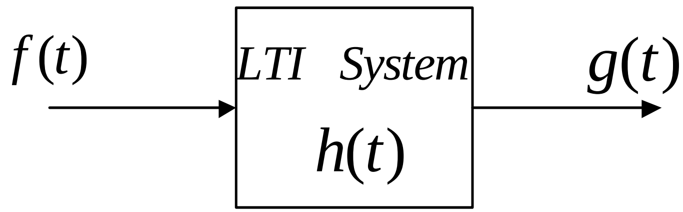
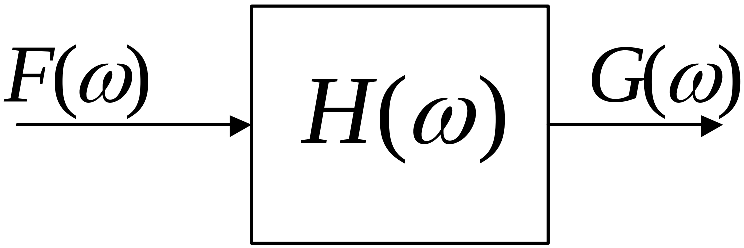
- **We also give some examples**

In time domain



- In time domain: The output g(t)=h(t)*h(t)
- In time domain, the system modifies the shape of the input signal

In frequency domain



- In frequency domain, $G(\omega)=H(\omega)X(\omega)$
- $H(\omega)$ is the frequency transfer function of the system which is the Fourier transform of the impulse response $h(t)$.
- In frequency domain, the system modifies the spectrum of the input signal

System Function

- An LTI system acts as a filter on the various frequency components applied to the system. Some frequency components may be amplified, some may be attenuated and some remain un-attenuated. We may also have a phase shift.

$$G(\omega) = |G(\omega)|.e^{j\theta_G(\omega)}$$

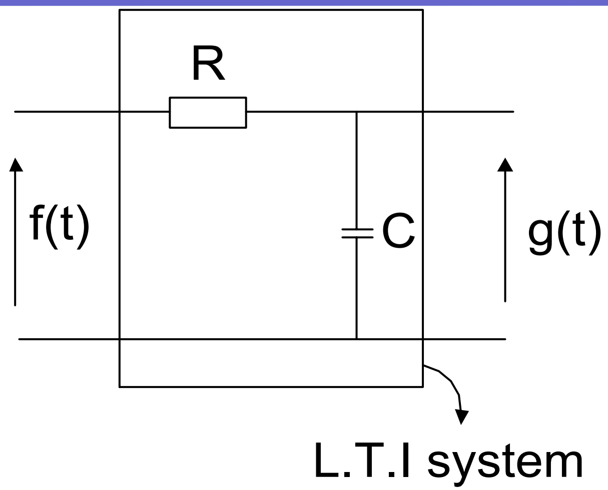
$$H(\omega) = |H(\omega)|.e^{j\theta_H(\omega)}$$

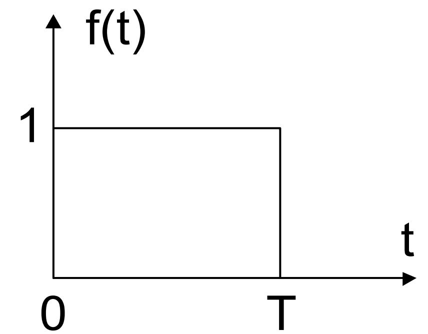
$$F(\omega) = |F(\omega)|.e^{j\theta_F(\omega)}$$

$$|G(\omega)|.e^{j\theta_G(\omega)} = |H(\omega)|.e^{j\theta_H(\omega)}.|F(\omega)|.e^{j\theta_F(\omega)}$$

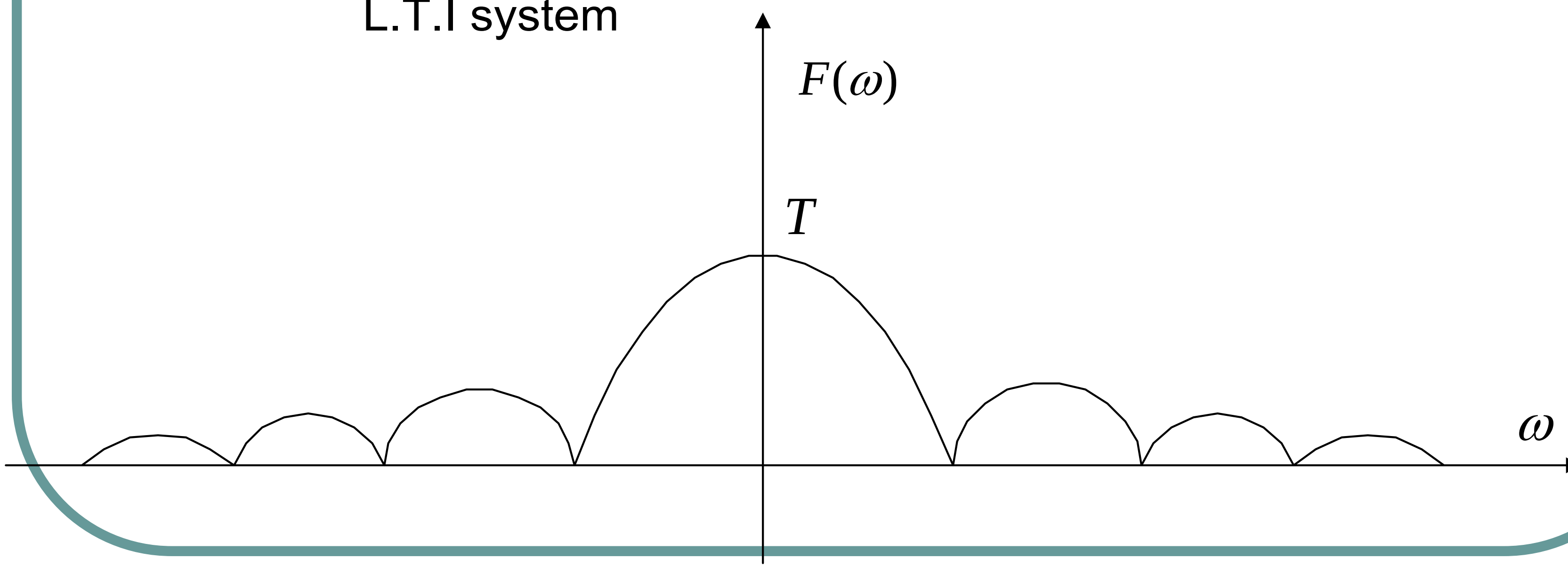
$$|G(\omega)| = |H(\omega)|.|F(\omega)| \qquad \text{and} \qquad \theta_G(\omega) = \theta_H(\omega) + \theta_F(\omega)$$

Example: Input





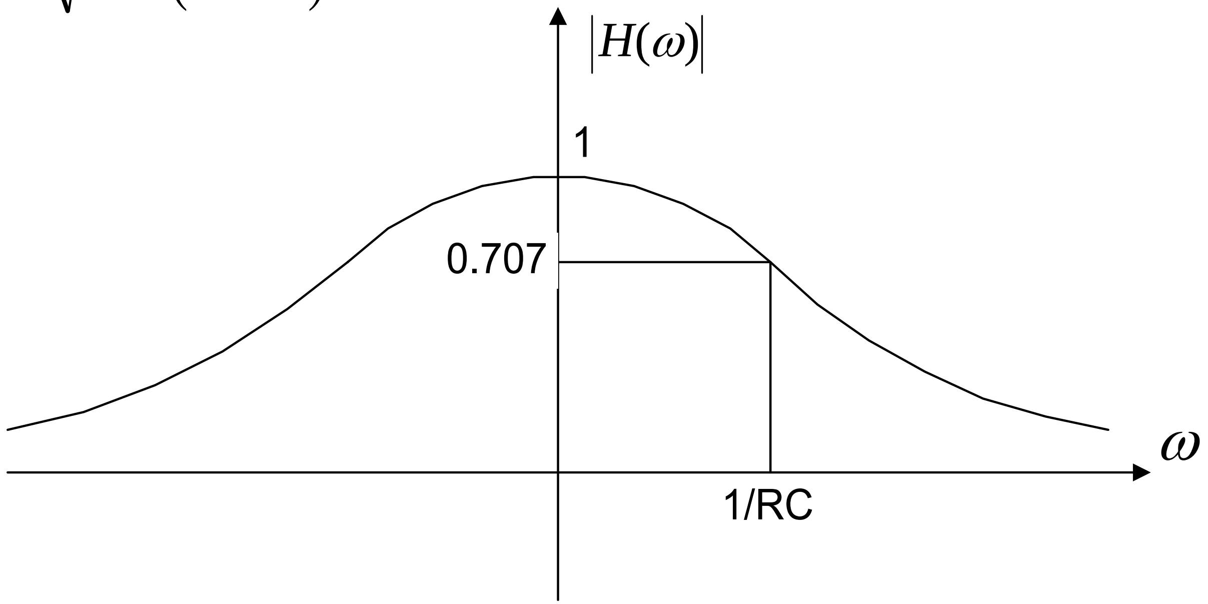
$$H(\omega) = \frac{1}{1 + j\omega RC}$$



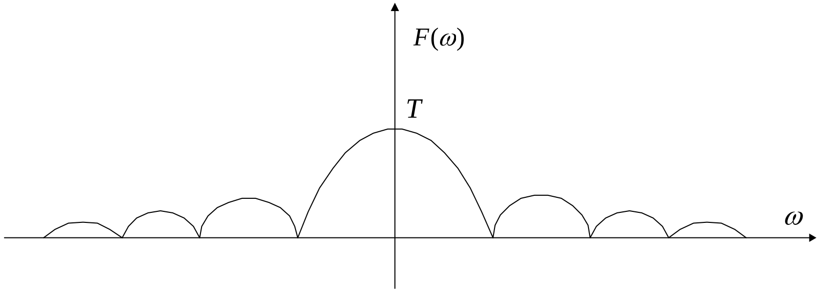
Example: system

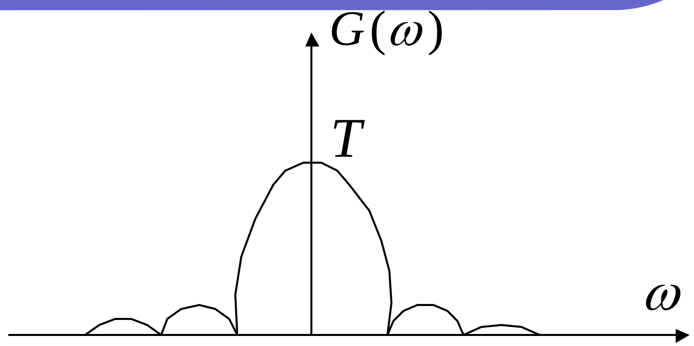
$$H(\omega) = \frac{1}{1 + j\omega RC}$$

$$|H(\omega)| = \frac{1}{\sqrt{1 + (\omega RC)^2}} \qquad \theta_H(\omega) = -\tan^{-1}(\omega RC)$$



Example: Output





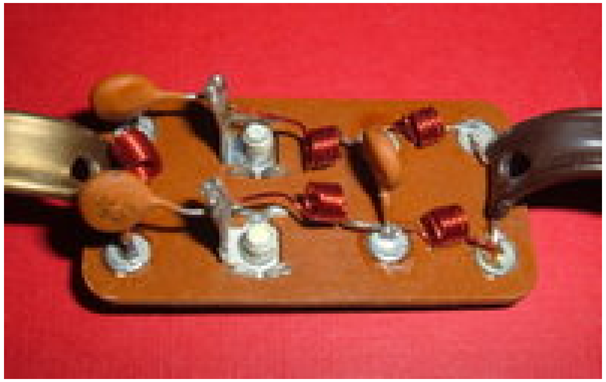
- ***This system acts as a LPF since the high frequency signals are attenuated and the low frequency signals remain almost the same.***

Bandwidth

- Bandwidth is a measure of frequency range in Hertz, of a signal or system.
- Communication signal bandwidth is the (positive) range of frequencies occupied by a modulated carrier wave
  - Only the positive frequency plane is counted. Do not count the symmetric negative frequency plane.
- If considering the frequency range where a signal's FT is non-zero, then many practical systems occupies infinite bandwidth → this is no good

Ideal and Practical Filters

- An electronic or mechanical device or mathematical algorithm to modify the signals
  - Low-pass filter (LPF)
  - High-pass filter (HPF)
  - Bandpass filter (BPF)
  - Bandstop filter (BSF)



TV signal splitter has a
Hi-pass and a Low-pass filter

Gain of the filter

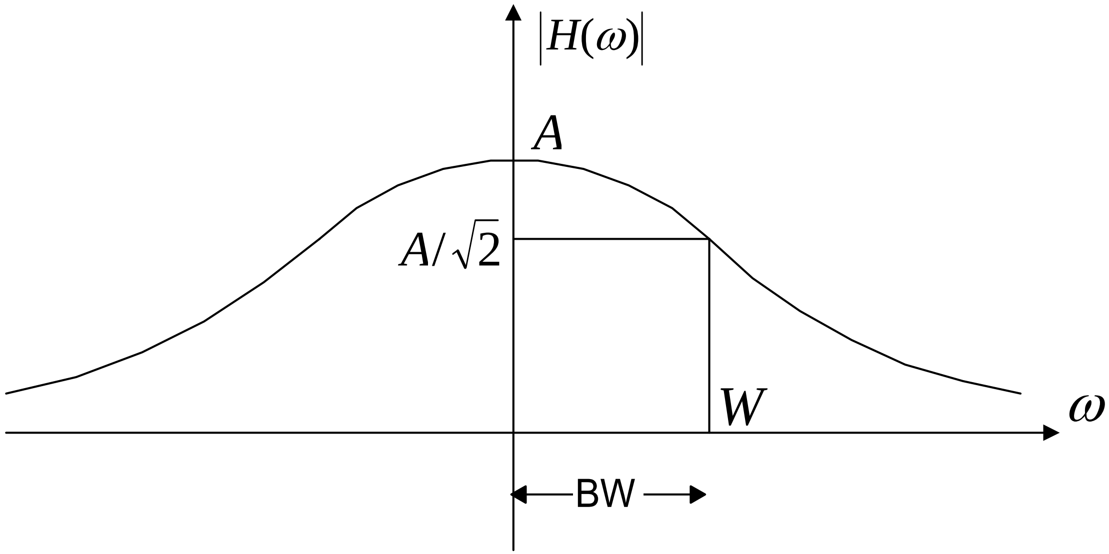
- The gain of the system is denoted by $\alpha(\omega)=|H(\omega)|$. The ***decibels*** unit is usually used for the gain

$$\alpha'(\omega) = 20.\log_{10}\left|H(\omega)\right|$$

Bandwidth of a Low Pass Filter

The bandwidth of a LP system is defined as the positive frequency components at which the amplitude response $|H(\omega)|$ is $1/\sqrt{2}$ times the value at zero frequency or the frequency at which the gain drops by 3 dB below its value at zero frequency.

Example

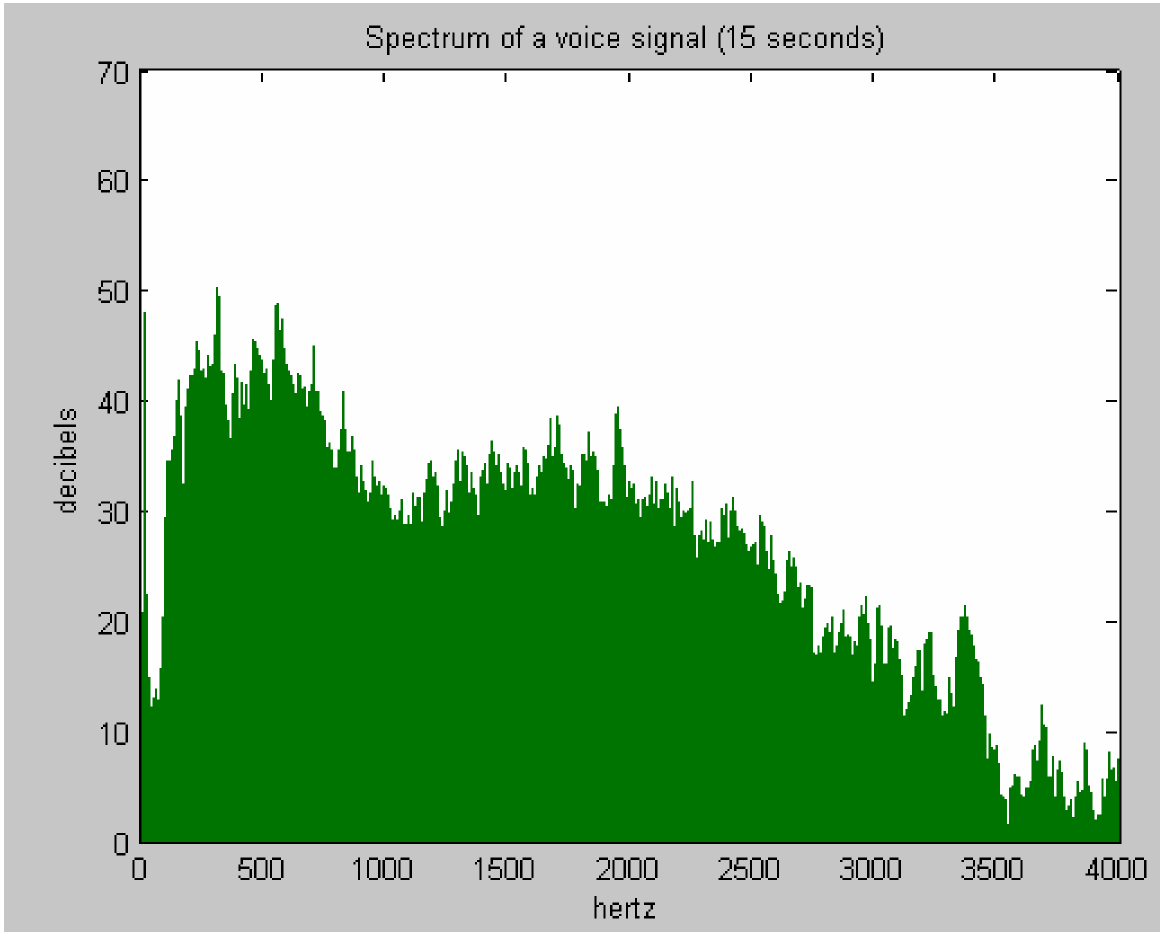


$$20.\log_{10}|H(0)| - 20.\log_{10}\frac{|H(0)|}{\sqrt{2}} = ?db \Rightarrow$$

$$20.\log_{10}|H(0)| - 20.\log_{10}|H(0)| - 20.\log_{10}\sqrt{2} \approx 3db$$

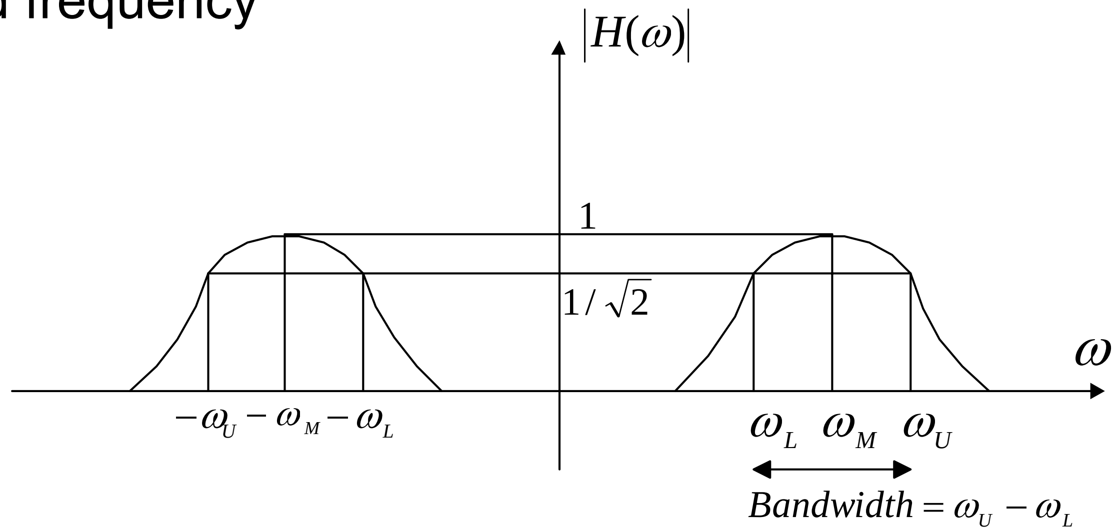
$$where \quad |H(0)| = A$$

Spectrum of a Voice signal



Bandwidth of a band-pass system (filter)

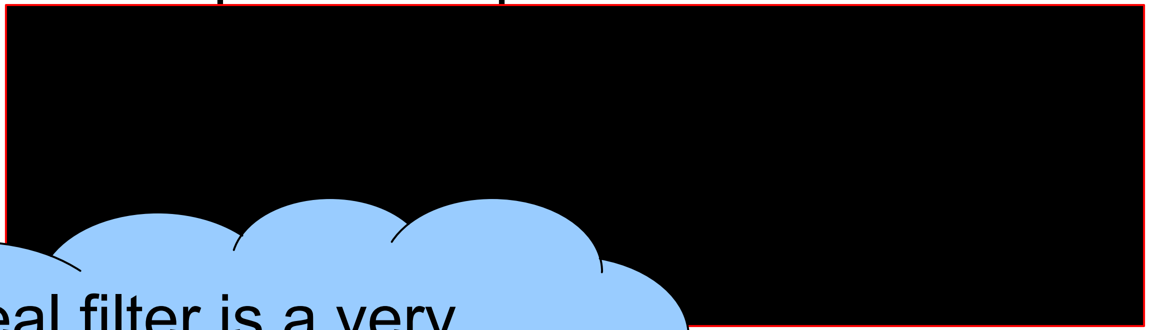
- For a band-pass system (filter), the bandwidth is defined as the range of positive frequencies over which the amplitude response $|H(\omega)|$ remains within $1/\sqrt{2}$ times its value at the mid-band frequency



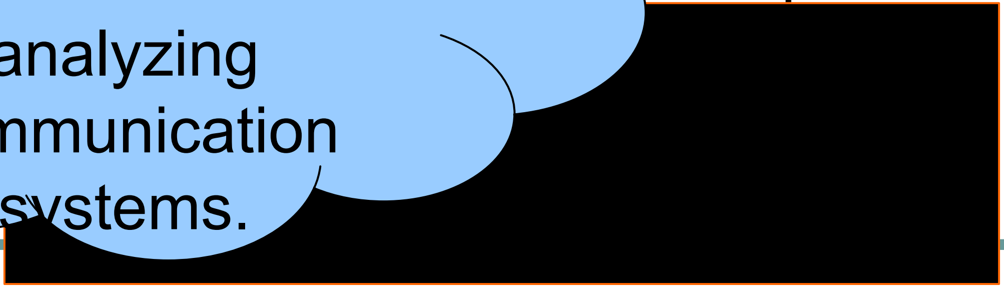
- ω_u=upper frequency, ω_M=middle frequency, and ω_L=Lower frequency

Ideal low-pass filter

- Magnitude response has ideal (sharp) stopband, has ideal bandwidth f_0 .
  - Every frequency lower than f_0 can pass, others can not.
- Phase response can have linear phase response or non-linear phase response



er response may look like



Ideal filter is a very useful tool for analyzing communication systems.

Ideal Band-pass filter

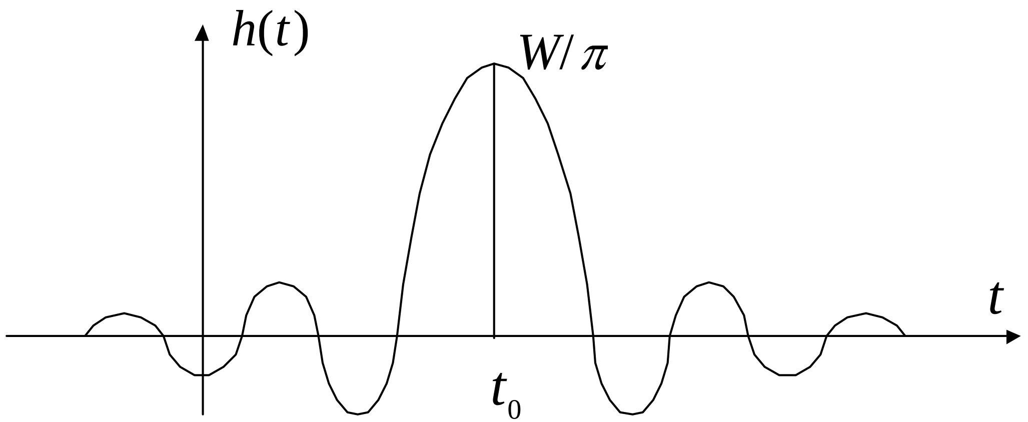
- An ideal filter passes without distortion all frequency components between its lower frequency and its upper frequency. Outside this range, the ideal filter is assumed to have zero magnitude

Impulse response of an ideal low-pass filter:

$$H(\omega) = |H(\omega)| . e^{j\theta_H(\omega)}$$

$$|H(\omega)| = \begin{cases} 1 & -W < \omega < W \\ 0 & \text{Otherwise} \end{cases} \text{ and } \theta_H(\omega) = \begin{cases} -\omega . t_0 & -W < \omega < W \\ 0 & \text{Otherwise} \end{cases}$$

$$h(t) = \frac{W}{\pi} Sa[W(t_0 - t)]$$

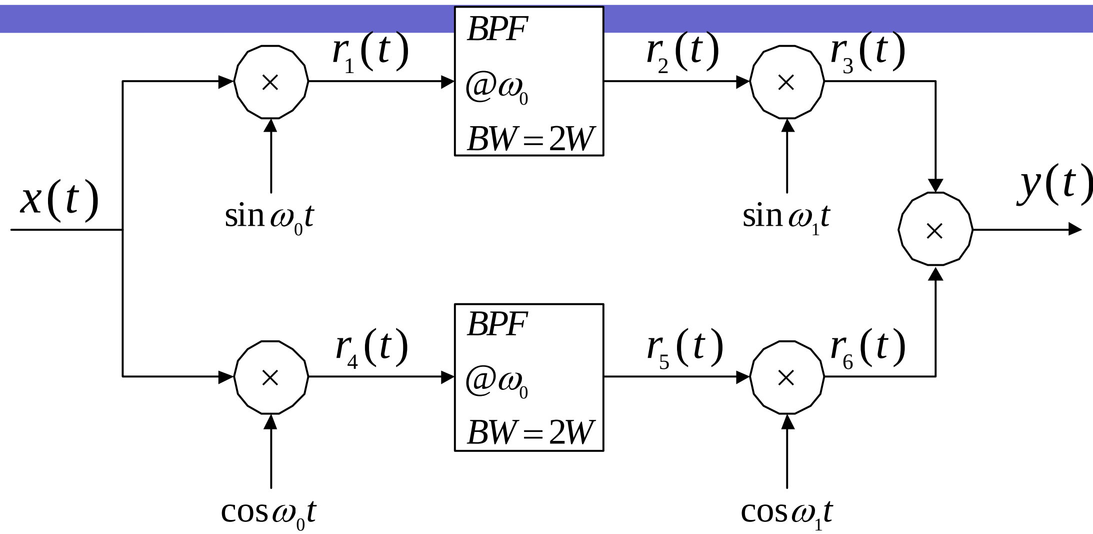


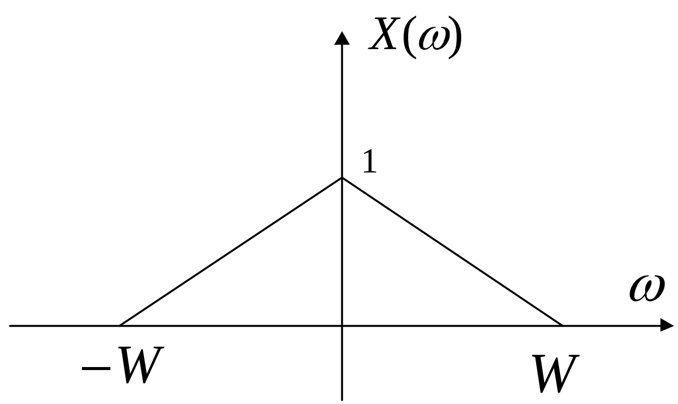
As h(t)#0 for t<0 Therefore the ideal low-pass filter is not a causal system.

Impulse Response of a band-pass filter

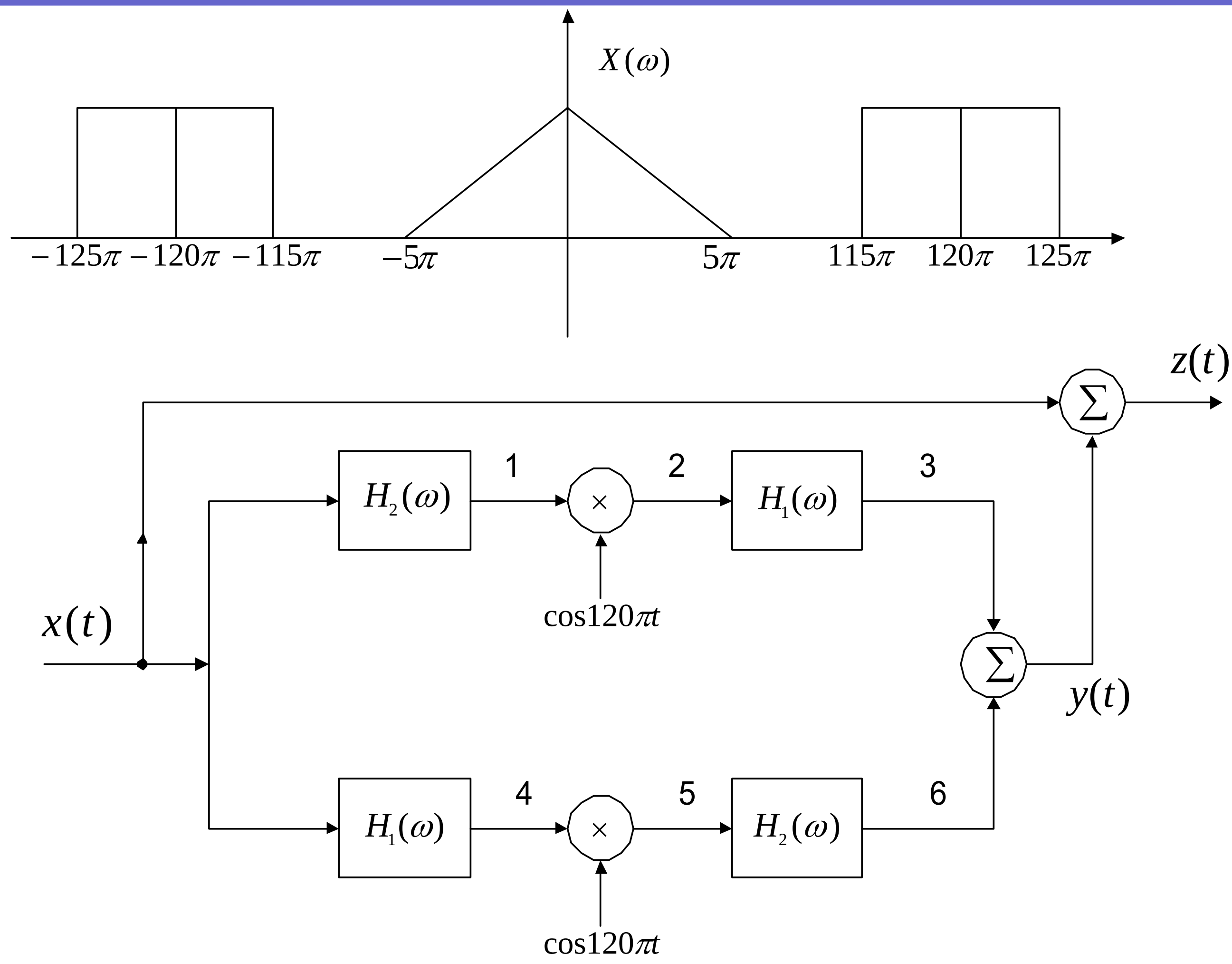
- Left for you to do

Example: Determine the spectrum of y(t).

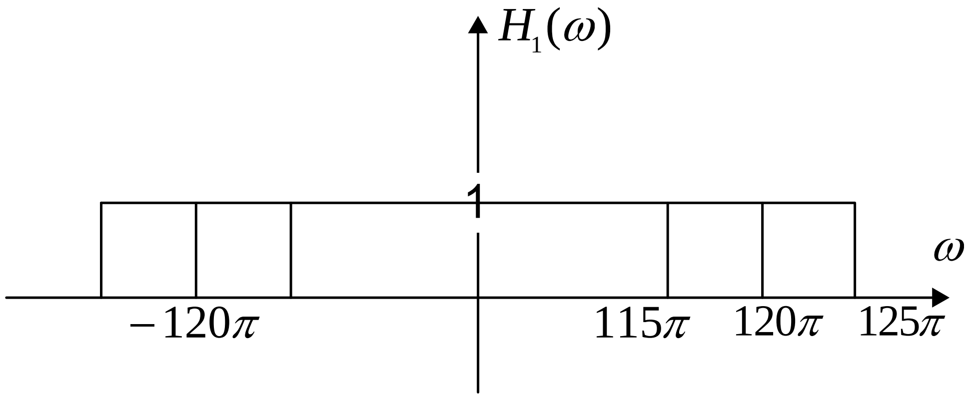


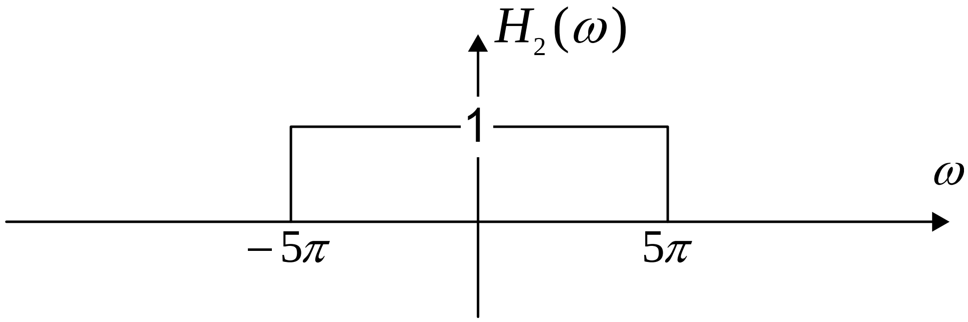


Example: Determine the spectrum of z(t)



Given


$H_1(\omega)$
1
ω
-120π
115π
120π
125π


$H_2(\omega)$
1
ω
-5π
5π