AMERICAN UNIVERSITY OF BEIRUT
FACULTY OF ENGINEERING AND ARCHITECTURE
ELECTRICAL AND COMPUTER ENGINEERING DEPARTMENT
EECE 340 – Signals & Systems
QUIZ # 1
Open book exam
TIME: 1 HOUR and 30 MINUTES
March 28, 2009
INSTRUCTOR: Dr. JEAN J. SAADE

NAME : ______________________ **ID # :** ____________________

INSTRUCTIONS

- **Write your Name and ID # on this sheet, the computer card and the scratch booklet in the provided spaces.**
- **Provide your answer on the computer card and solution of each problem on the scratch booklet.**
- **Return the computer card, this question sheet and the scratch booklet when you finish the test.**
- **All questions are equally weighted in grading.**

Classify the signals given in Problems #1 and # 2 into energy-type, power-type and signals that are neither energy nor power-type. For each energy or power-type signal, find the energy or average power of the signal.

PROBLEM # 1

The following continuous-time signal is considered:

$$f(t) = 4e^{j\omega_0 t} + 3e^{j2\omega_0 t} \ Volts$$

(a) Neither energy nor power signal.
(b) Energy signal with energy equal 25 Joules.
(c) Power signal with average power equal to 25 Watts.
(d) Periodic signal, but neither energy nor power signal.
(e) None of the above.

PROBLEM # 2

The following continuous-time signal is considered:

$$f(t) = e^{-2|t|}\ Volts$$

(a) Neither energy nor power signal.
(b) Energy signal with energy equal 0.5 Joules.
(c) Power signal with average power equal to 0.5 Watts.
(d) Periodic signal but neither energy nor power signal.
(e) None of the above.

PROBLEM # 3

In this problem, x(t) denotes the input and y(t)=T[x(t)] denotes the output of a continuous time system. Let the system be given by

$$y(t)=T[x(t)]=x(t)\cos[x(t)]\times\sin[x(t)]$$

Select from what is given below only one property that is **not** satisfied by the system.

(a) Non-linearity
(b) Time-invariance
(c) Time-varying
(d) Causality

PROBLEM # 4

In this problem, x(t) denotes the input and y(t)=T[x(t)] denotes the output of a continuous time system. Let the system be given by

$$y(t)=T[x(t)]=x(t-1)+2$$

Select from what is given below only one property that is **not** satisfied by the system.

(a) Linearity
(b) Non-linearity
(c) Time-invariance
(d) Causality

PROBLEM # 5

In this problem, x(t) denotes the input and y(t)=T[x(t)] denotes the output of a continuous time system. Let the system be given by

$$y(t)=T[x(t)]=x(2t-2)$$

Determine the time instants at which the system output depends on future values of the system input.

(a) -3<t<3 secs.
(b) t<3 secs.
(c) -2<t<2 secs.
(d) $-\infty<t<\infty$
(e) t>2 secs.

PROBLEM # 6

Consider the following system with f(t) being the system input and g(t) the system output:

$$g(t) = T[f(t)] = \int_{-\infty}^{t} f(\tau)d\tau + \int_{-\infty}^{\infty} e^{(t-\tau)} f(\tau)d\tau, \ -\infty < t < \infty$$

Determine the impulse response, h(t), of the system.

$(a)\ h(t) = \begin{cases} e^{-t}, t<0 \\ e^{-t}+1, t\geq 0 \end{cases}$ $(b)\ h(t) = \begin{cases} e^{-t}, t<0 \\ e^{t}+1, t\geq 0 \end{cases}$ $(c)\ h(t) = \begin{cases} e^{t}, t<0 \\ e^{-t}+1, t\geq 0 \end{cases}$

$(d)\ h(t) = \begin{cases} e^{t}, t<0 \\ e^{t}+1, t\geq 0 \end{cases}$ $(e)\ h(t) = \begin{cases} e^{t}+1, t<0 \\ e^{t}, t\geq 0 \end{cases}$

PROBLEM # 7

Consider again the system in Problem # 6. Determine the system output when the input is the unit step signal, u(t).

$(a)\ h(t) = \begin{cases} e^{t}, t<0 \\ e^{t}+t, t\geq 0 \end{cases}$ $(b)\ h(t) = \begin{cases} e^{-t}, t<0 \\ e^{t}+t, t\geq 0 \end{cases}$ $(c)\ h(t) = \begin{cases} e^{t}, t<0 \\ e^{-t}+t, t\geq 0 \end{cases}$

$(d)\ h(t) = \begin{cases} e^{-t}, t<0 \\ e^{-t}+t, t\geq 0 \end{cases}$ $(e)\ h(t) = \begin{cases} e^{t}+t, t<0 \\ e^{t}, t\geq 0 \end{cases}$

PROBLEM # 8

Consider an LTI continuous-time system whose response to the input $f(t) = [e^{-t} + e^{-3t}]\,u(t)$ is $g(t) = [2e^{-t} - 2e^{-4t}]\,u(t)$. Determine the impulse response, h(t), of the system assumed causal.

$(a)\ h(t) = 1.5\,(e^{-2t} + e^{-t})\,u(t)$ $(b)\ h(t) = 1.5\,(e^{-t} + e^{-4t})\,u(t)$
$(c)\ h(t) = 1.5\,(e^{-2t} + e^{-4t})\,u(t)$ $(d)\ h(t) = 1.5\,(e^{-3t} + e^{-5t})\,u(t)$
$(e)\ h(t) = 1.5\,(e^{-6t} + e^{-5t})\,u(t)$

PROBLEM # 9

Consider again Problem # 8 and determine the differential equation relating the output, g(t), to the input, f(t), of the system.

$$(a)\ \frac{d^2g(t)}{dt^2}+\frac{dg(t)}{dt}+8g(t)=3\frac{df(t)}{dt}+9f(t)$$

$$(b)\ \frac{d^2g(t)}{dt^2}+6\frac{dg(t)}{dt}+4g(t)=3\frac{df(t)}{dt}+9f(t)$$

$$(c)\ \frac{d^2g(t)}{dt^2}+6\frac{dg(t)}{dt}+8g(t)=\frac{df(t)}{dt}+9f(t)$$

$$(d)\ \frac{d^2g(t)}{dt^2}+6\frac{dg(t)}{dt}+8g(t)=3\frac{df(t)}{dt}+9f(t)$$

$$(e)\ \frac{d^2g(t)}{dt^2}+6\frac{dg(t)}{dt}+g(t)=2\frac{df(t)}{dt}+f(t)$$

PROBLEM # 10

Consider an LTI system with transfer function given by

$$H(s)=\frac{2e^{-2s}}{(s+4)(s+2)}$$

Determine the impulse response, h(t), of the system considered to be causal.

$(a)\ h(t)=(e^{-2(t-2)}-e^{-4(t-2)})u(t-2)$ $(b)\ h(t)=(e^{-2(t-2)}-e^{-4(t-2)})u(t)$

$(c)\ h(t)=(e^{-4t}-e^{-2t})u(t-2)$ $(d)\ h(t)=(e^{-2t}-e^{-4t})u(t)$

$(e)\ h(t)=(e^{-4(t+2)}-e^{-2(t+2)})u(t+2)$

PROBLEM # 11

Consider the asynchronous detection of the following received DSB-SC signal:

$$s_r(t)=A_r f(t)\cos(\omega_c t-\pi/2)$$

The received signal is multiplied by $c(t)=A_c\cos[(\omega_c+\pi/2)t]$ and then the output of the multiplier is inputted to an ideal low-pass filter. Determine the positive time instants at which the signal at the output of the LPF is null.

(a) 3, 5, 7, 9, … secs
(b) 1, 2, 3, 4, … secs
(c) 1, 3, 5, 7, … secs
(d) 0, 2, 4, 6, … secs
(e) 4, 6, 8, 10, … secs

PROBLEM # 12

Consider an LTI system with impulse response given by

$$h(t) = 8e^{-2t}\sin(\omega_c t)\,u(t)$$

Determine the transfer function, H(s), of this system and then conclude if this system is stable or not.

$(a)\ H(s) = \dfrac{8\pi\omega_c}{s^2 + 2s + 2 + \omega_c^2}$, stable

$(b)\ H(s) = \dfrac{8\omega_c}{s^2 + 2s + 4 + \omega_c^2}$, unstable

$(c)\ H(s) = \dfrac{8\omega_c}{s^2 + 4s + 2 + \omega_c^2}$, unstable

$(d)\ H(s) = \dfrac{8\omega_c}{s^2 + 4s + 4 + \omega_c^2}$, stable

$(e)\ H(s) = \dfrac{16\pi\omega_c}{4s^2 + 4s + 4 + \omega_c^2}$, unstable

EECE 340 Quiz #1 Solution Spring 2009

1/10

Problem #1

$$f(t) = 4e^{j\omega_0 t} + 3e^{j2\omega_0 t} \text{ Volts.}$$

The signal $f(t)$ is periodic with period equal to $\frac{2\pi}{\omega_0}$. It is also a power signal with finite average power given by:

$$P = \frac{\omega_0}{2\pi}\int_{-\frac{\pi}{\omega_0}}^{\frac{\pi}{\omega_0}} |f(t)|^2 dt = \frac{\omega_0}{2\pi}\int_{-\frac{\pi}{\omega_0}}^{\frac{\pi}{\omega_0}} [25 + 24\cos(\omega_0 t)]dt$$

$$= \frac{\omega_0}{2\pi}\int_{-\frac{\pi}{\omega_0}}^{\pi/\omega_0} 25\,dt = 25 \text{ Watts.}$$

In the above we have used

$$|f(t)|^2 = [\mathrm{Re}\, f(t)]^2 + [\mathrm{Im}\, f(t)]^2 = 25 + 24\cos(\omega_0 t)$$

and $\int_{-\frac{\pi}{\omega_0}}^{\frac{\pi}{\omega_0}} \cos(\omega_0 t)\,dt = 0.$

Problem #2

$$f(t) = e^{-2|t|} \text{ Volts.}$$

$f(t)$ is an energy signal with finite energy given by:

$$E = \int_{-\infty}^{\infty} f^2(t)\,dt = \int_{-\infty}^{\infty} e^{-4|t|}\,dt = 2\int_0^{\infty} e^{-4t}\,dt$$

$$= \frac{1}{2} \text{ Joule.}$$

Problem # 3 (2/10)

$$y(t) = T[x(t)] = x(t)\cos[x(t)]\sin[x(t)]$$

is the transformation defining a continuous-time system with input $x(t)$ and output $y(t)$.

* Linearity:

$$T[a_1 x_1(t) + a_2 x_2(t)] = [a_1 x_1(t) + a_2 x_2(t)] \times \cos[a_1 x_1(t) + a_2 x_2(t)] \times \sin[a_1 x_1(t) + a_2 x_2(t)]$$

$$=?? \; a_1 T[x_1(t)] + a_2 T[x_2(t)]$$

$$= a_1 x_1(t)\cos[x_1(t)]\sin[x_1(t)] + a_2 x_2(t)\cos[x_2(t)]\sin[x_2(t)] \rightarrow \text{NO.}$$

$\Rightarrow$ The system is non-linear.

* Time invariance

Is $y(t-t_0) = T[x(t-t_0)]$??

Yes, since $T[x(t-t_0)] = x(t-t_0)\cos[x(t-t_0)] \times \sin[x(t-t_0)]$

$= y(t-t_0)$.

$\Rightarrow$ The system is time-invariant.

* Causality $y(t_0) = x(t_0)\cos[x(t_0)]\sin[x(t_0)]$

So, the output at time t_0 depends on value of the input at time t_0. The system is causal.

Problem # 4

3/10

$$y(t) = T[x(t)] = x(t-1) + 2$$

is the transformation defining a continuous-time system.

* Linearity

$$T[a_1 x_1(t) + a_2 x_2(t)] = a_1 x_1(t-1) + a_2 x_2(t-1) + 2.$$

$$=?? \; a_1 T[x_1(t)] + a_2 T[x_2(t)]$$

$$= a_1 \{x_1(t-1) + 2\} + a_2 \{x_2(t-1) + 2\}$$

$$= a_1 x_1(t-1) + 2a_1 + a_2 x_2(t-1) + 2a_2$$

$$= a_1 x_1(t-1) + a_2 x_2(t-1) + 2(a_1 + a_2) \rightarrow \text{No.}$$

$\Rightarrow$ The system is non-linear.

* Time invariance

Is $y(t-t_0) = T[x(t-t_0)]$??

$$T[x(t-t_0)] = x(t-t_0-1) + 2$$

$$= y(t-t_0) \rightarrow \text{Yes.}$$

$\Rightarrow$ The system is time-invariant.

* Causality $y(t_0) = x(t_0 - 1) + 2$

The output at time t_0 depends on the input value at time $(t_0 - 1)$. $\Rightarrow$ The system is causal.

Problem # 5

$$y(t) = T[x(t)] = x(2t-2)$$

The time instants, t, at which the system output $y(t)$ depends on future values of the system input are such that $t < 2t-2$. Hence, $t > 2$ secs.

The system, therefore, cannot be realized for any $t > 2$ secs. Take for example $t = 3$ secs.

The input $x(3)$ produces the output $y(3) = x(4)$, which is not available at $t = 3$ secs.

The system, however, can be realized if the input signals, $x(t)$, are only restricted to $t \leq 2$ secs.

For example, $y(2) = T[x(2)] = x(2)$

$$y(1) = T[x(1)] = x(0)$$
$$y(0) = T[x(0)] = x(-2)$$
$$y(-1) = T[x(-1)] = x(-4)$$

The output is obtained from the past values of the system input. (present or)

As a whole; i.e, for input signals defined for all $t \in (-\infty, +\infty)$, the system is non-realizable. It is, therefore, non-causal.

Problem #6

$$g(t) = T[f(t)] = \int_{-\infty}^{t} f(a)\,da + \int_{-\infty}^{\infty} e^{(t-a)} f(a)\,da,$$

$$-\infty < t < \infty$$

$$h(t) = T[\delta(t)] = \int_{-\infty}^{t} \delta(a)\,da + \int_{-\infty}^{\infty} e^{(t-a)} \delta(a)\,da$$

$$= \int_{-\infty}^{t} \delta(a)\,da + e^{t} \int_{-\infty}^{\infty} \delta(a)\,da$$

$$= \begin{cases} e^{t}, & t < 0 \\ e^{t} + 1, & t \geq 0 \end{cases}$$

Problem #7

$$g(t) = T[u(t)] = \int_{-\infty}^{t} u(a)\,da + e^{t} \int_{-\infty}^{\infty} e^{-a} u(a)\,da$$

$$= \int_{0}^{t} da + e^{t} \int_{0}^{\infty} e^{-a}\,da$$

$$= t + e^{t}, \quad t \geq 0$$

For $t < 0$, $g(t) = e^{t} \int_{0}^{\infty} e^{-a}\,da = e^{t}$

$$\Rightarrow g(t) = \begin{cases} e^{t}, & t < 0 \\ t + e^{t}, & t \geq 0 \end{cases}$$

6/10

Problem # 8

The system input is $f(t) = \left[e^{-t} + e^{-3t}\right]u(t)$

The system output is: $g(t) = \left[2e^{-t} - 2e^{-4t}\right]u(t)$

Since the system is LTI, then the system transfer function can be written as:

$$H(s) = \frac{G(s)}{F(s)}$$

$$F(s) = \mathcal{L}\left[f(t)\right] = \frac{1}{s+1} + \frac{1}{s+3}$$

$$= \frac{2s+4}{(s+1)(s+3)}, \sigma > -1$$

$$G(s) = \frac{2}{s+1} - \frac{2}{s+4} = \frac{6}{(s+1)(s+4)}, \sigma > -1$$

$$\Rightarrow H(s) = \frac{3(s+3)}{(s+2)(s+4)}, \sigma > -2 \text{ (Causal system)}$$

$$h(t) = \frac{1}{2\pi j}\int_{Br} H(s)e^{st}\,ds = \text{Residue at } -2 + \text{Residue at } -4$$

$$= \left.\frac{3(s+3)e^{st}}{(s+4)}\right|_{s=-2} + \left.\frac{3(s+3)e^{st}}{(s+2)}\right|_{s=-4}$$

$$= \frac{3}{2}e^{-2t} + \frac{3}{2}e^{-4t}, t > 0$$

$$= 0, t < 0$$

$$\Rightarrow h(t) = 1.5\left(e^{-2t} + e^{-4t}\right)u(t)$$

Problem # 9

$$H(s) = \frac{3s+9}{(s+2)(s+4)} = \frac{3s+9}{s^2+6s+8}$$

$$= \frac{G(s)}{F(s)}$$

$$\Rightarrow s^2 G(s) + 6s\,G(s) + 8\,G(s) = 3sF(s) + 9F(s)$$

$$\Rightarrow \frac{d^2 g(t)}{dt^2} + 6\frac{dg(t)}{dt} + 8\,g(t) = 3\frac{df(t)}{dt} + 9f(t)$$

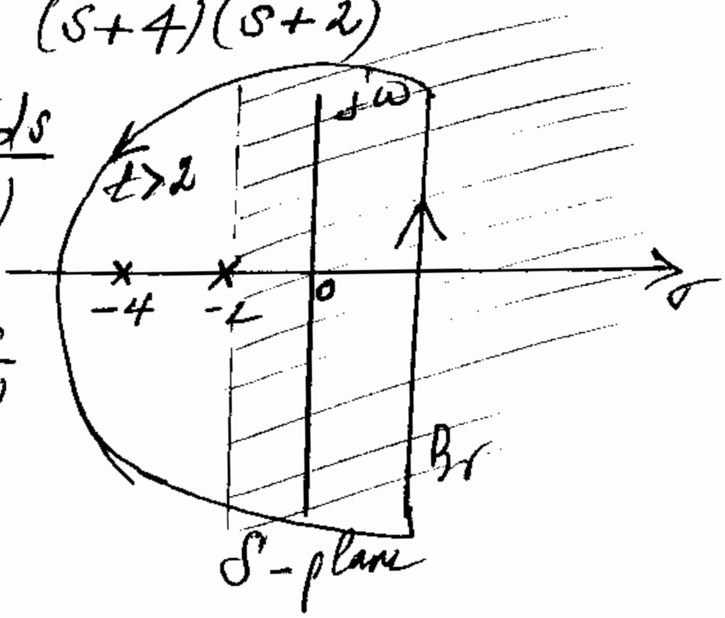
is the system differential equation.

Problem # 10

$$H(s) = \frac{2e^{-2s}}{(s+4)(s+2)}$$

$$h(t) = \frac{1}{2\pi j}\int_{Br} \frac{2e^{-2s} e^{st}\,ds}{(s+4)(s+2)}$$

$$= \frac{1}{2\pi j}\int_{Br} \frac{2e^{(s)(t-2)}\,ds}{(s+4)(s+2)}$$



$$= \left.\frac{2e^{s(t-2)}}{(s+4)}\right|_{s=-2} + \left.\frac{2e^{s(t-2)}}{(s+2)}\right|_{s=-4} = \left(e^{-2(t-2)} - e^{-4(t-2)}\right)u(t-2)$$

Problem # 11

8/10

$$s_r(t) = A_r f(t) \cos\left(\omega_c t - \frac{\pi}{2}\right)$$

$$c(t) = A_c \cos\left[\left(\omega_c + \frac{\pi}{2}\right)t\right]$$

$$s_r(t)c(t) = A_r A_c f(t) \cos\left(\omega_c t - \frac{\pi}{2}\right)\cos\left[\left(\omega_c + \frac{\pi}{2}\right)t\right]$$

$$= \frac{1}{2} A_r A_c f(t) \left\{ \cos\left[\left(2\omega_c + \frac{\pi}{2}\right)t - \frac{\pi}{2}\right] + \cos\left(\frac{\pi}{2}t + \frac{\pi}{2}\right) \right\}$$

The low-pass filter output is:

$$LPF_0 = \frac{1}{2} A_r A_c f(t) \cos\left(\frac{\pi}{2}t + \frac{\pi}{2}\right)$$

$LPF_0 = 0$ when $\cos\left(\frac{\pi}{2}t + \frac{\pi}{2}\right) = 0$ or

$$\frac{\pi}{2}t + \frac{\pi}{2} = (2k+1)\frac{\pi}{2}, \quad k = 0, \pm 1, \pm 2, \dots$$

$$\Rightarrow t = 2k, \qquad k = 0, \pm 1, \pm 2, \dots$$

Hence, the positive time instants at which the low-pass filter output is null are as above for $k = 0, +1, +2, \dots$

$$\Rightarrow t = 0, 2, 4, 6, \dots \text{ secs.}$$

Problem #12

9/10

$$h(t) = 8e^{-2t}\sin(\omega_c t)u(t)$$

is the impulse response of an LTI system.

Using $\sin(\omega_c t) = \dfrac{e^{j\omega_c t} - e^{-j\omega_c t}}{2j}$, then:

$$H(s) = \frac{8}{2j}\int_0^\infty e^{-2t} e^{j\omega_c t} e^{-st}\,dt - \frac{8}{2j}\int_0^\infty e^{-2t} e^{-st} e^{-j\omega_c t}\,dt$$

$$= \frac{8}{2j}\left[\int_0^\infty e^{-(s+2-j\omega_c)t}\,dt - \int_0^\infty e^{-(s+2+j\omega_c)t}\,dt\right]$$

$$\int_0^\infty e^{-(s+2-j\omega_c)t}\,dt = -\frac{1}{(s+2-j\omega_c)} e^{-(s+2-j\omega_c)t}\Big|_0^\infty$$

$$= \frac{1}{s+2-j\omega_c}, \quad \sigma > -2.$$

$$\int_0^\infty e^{-(s+2+j\omega_c)t}\,dt = \frac{1}{s+2+j\omega_c}, \quad \sigma > -2.$$

$$\Rightarrow H(s) = \frac{8}{2j}\left[\frac{1}{s+2-j\omega_c} - \frac{1}{s+2+j\omega_c}\right]$$

$$= \frac{8}{2j}\left[\frac{s+2+j\omega_c - s - 2 + j\omega_c}{(s+2-j\omega_c)(s+2+j\omega_c)}\right]$$

$$= \frac{8\omega_c}{s^2+4s+4+\omega_c^2} = \frac{8\omega_c}{(s+2)^2+\omega_c^2}$$

$H(s)$ has poles at $s = -2 \pm j\omega_c$. 10/10

These poles are in the left half s-plane and since the system is causal; $h(t) = 0$ for $t < 0$, then the system is stable.

<u>Note</u> $H(s)$ can also be obtained from the the Laplace of $\sin(\omega_c t)\,u(t) = f(t)$ by writing $H(s) = 8F(s+2)$.

From the Laplace transform tables, we have:

$$F(s) = \frac{\omega_c}{s^2 + \omega_c^2}.$$

$$\Longrightarrow H(s) = \frac{8\omega_c}{(s+2)^2 + \omega_c^2}$$