AMERICAN UNIVERSITY OF BEIRUT

FACULTY OF ENGINEERING AND ARCHITECTURE
ELECTRICAL AND COMPUTER ENGINEERING DEPARTMENT
EECE 440 – Signals and Systems
FINAL EXAM
Closed book exam
NINE SHEETS OF FORMULAS WITH NO PROBLEM SOLUTIONS ARE ALLOWED
TIME: 2 hours
Saturday, June 11, 2005
INSTRUCTOR: Dr. JEAN J. SAADE

Student Name: ______________________ **ID #:** ______________________

PROBLEM # 1

An analog linear and time-invariant system has the frequency response given by

$$H(\omega)=\frac{1}{2}\left[\frac{1}{a+j(\omega-2)}+\frac{1}{a+j(\omega+2)}\right]$$

where a is real. The system is assumed to be causal and stable.

(a) Determine the transfer function $H(s)$ of the system. Locate the poles of $H(s)$ in the s-plane.
(b) Determine the system impulse response, $h(t)$.
(c) The system impulse response $h(t)$ is sampled at $t=nT$ to give the unit sample response, $h(n)$, of a discrete time system. Let $T=1sec.$ and determine $h(n)$ as well as $H(z)$. Locate the poles of $H(z)$ in the z-plane and state whether or not the system is stable. Justify your answer.

PROBLEM # 2

Consider the difference equation given below.

$$y(n)=\frac{5}{2}y(n-1)-y(n-2)+x(n)-x(n-1).$$

(a) Determine the system transfer function $H(z)$.
(b) Determine $h(n)$ for the causal system.
(c) Let the input to the causal system be $x(n)=(1/2)^n\,u(n)$. Determine the output $y(n)$ using the inverse Z-transform.
(d) Implement $H(z)$ in direct form I and direct form II.

PROBLEM # 3

Consider the discrete time sequences

$$h(n) = e^{-bn}\, u(n),$$

$$x(n) = e^{-a|n|}, \quad all \quad n$$

with $a>b>0$. $h(n)$ represents the unit sample response of a discrete time, linear and shift-invariant system.

(a) Determine the output $y(n)$ of the system if $x(n)$ is the input. Use discrete convolution.
(b) Determine whether $h(n)$ is stable. Validate your answer using time-domain analysis.
(c) Determine the frequency response $H(\omega)$ of $h(n)$. Also, determine and plot the magnitude frequency response $|H(\omega)|$.

PROBLEM # 4

Consider the sequence

$$x(n) = \begin{cases} \left(\frac{1}{2}\right)^n, & 0 \le n \le 5, \\ 0, & elsewhere. \end{cases}$$

(a) Determine the Z-transform of $x(n)$.
(b) Determine the expression of the sampled version, $\widetilde{X}(k)$, of $X(z)$ for $z = e^{j\frac{2\pi}{8}k} = W_8^{-k}$ and $k=0,1,\ldots,7$. Plot the periodic extension of $x(n)$ which admits $\widetilde{X}(k)$ as the Fourier series coefficients.
(c) Determine the DFT, $X(k)$, of $x(n)$ considered to be of length 8. Evaluate $X(k)$ for $k=0,1$ only.
(d) Use the DFT in Part (c) and plot the finite finite length sequence $x_1(n)$ whose DFT is given by $X_1(k) = W_8^{-4k} X(k)$.

EECE 440 — Final Exam Solution — Spring 2004-05

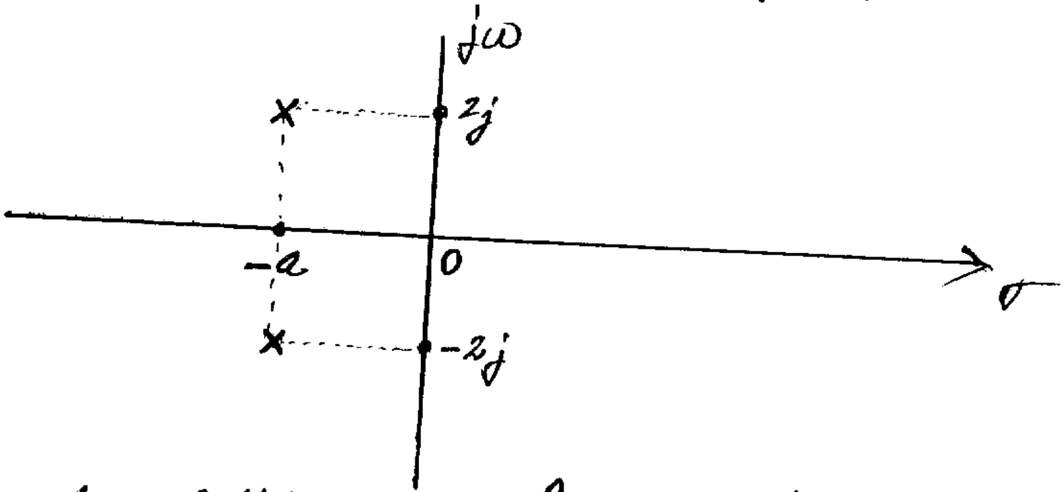
Problem # 1

$$H(\omega) = \frac{1}{2}\left[\frac{1}{a+j(\omega-2)} + \frac{1}{a+j(\omega+2)}\right]$$

(a) Since the system is causal and stable, then the region of convergence of $H(s)$ is a right half s-plane that contains the imaginary axis.

Hence, $H(s)$ can be obtained from $H(\omega)$ or vice-versa by replacing $j\omega$ by s.

$$\Rightarrow H(s) = \frac{1}{2}\left[\frac{1}{s+(a-2j)} + \frac{1}{s+(a+2j)}\right]$$



The poles of $H(s)$ are as located above and they are in the left-half s-plane.

(b) $$h(t) = \mathcal{L}^{-1}[H(s)] = \frac{1}{2}\left[e^{(-a+2j)t} + e^{(-a-2j)t}\right]$$

$$= e^{-at}\left[\frac{e^{2jt} + e^{-2jt}}{}\right] \quad -at$$

(c) $h(n) = e^{-an} \cos 2n \, u(n)$

$$H(z) = \sum_{n=0}^{\infty} h(n) z^{-n} = \sum_{n=0}^{\infty} e^{-an} \cos 2n \, z^{-n}$$

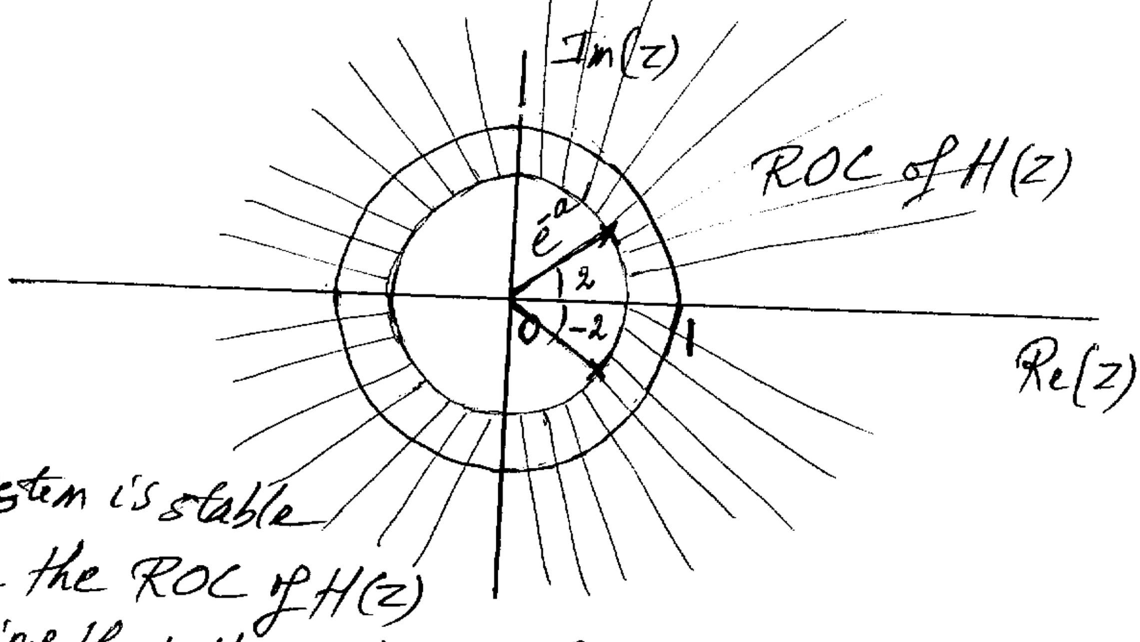
$$= \sum_{n=0}^{\infty} \frac{e^{-an} e^{j2n} z^{-n}}{2} + \sum_{n=0}^{\infty} \frac{e^{-an} e^{-j2n} z^{-n}}{2}$$

$$= \sum_{n=0}^{\infty} \frac{1}{2} \left(e^{-a+2j} z^{-1}\right)^n + \sum_{n=0}^{\infty} \frac{1}{2} \left(e^{-a-2j} z^{-1}\right)^n$$

$$= \frac{1}{2} \cdot \frac{1}{1 - e^{-a+2j} z^{-1}} + \frac{1}{2} \frac{1}{1 - e^{-a-2j} z^{-1}}$$

With $|e^{-a} z^{-1}| < 1$ or $|z| > e^{-a}$.

Since $a > 0$ (see (a)), then $e^{-a} < 1$.



The system is stable since the ROC of $H(z)$ contains the unit circle

z-plane

Problem #2

$$y(n) = \frac{5}{2}y(n-1) - y(n-2) + x(n) - x(n-1).$$

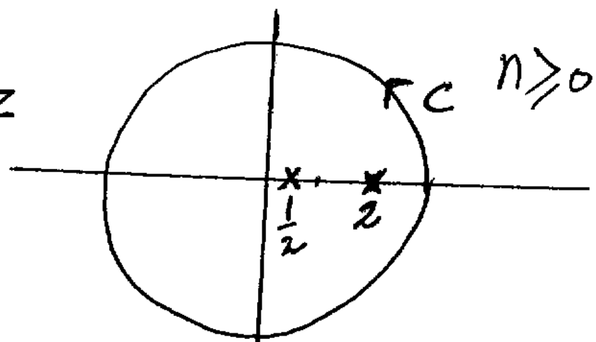
(a) $Y(z) - \frac{5}{2}z^{-1}Y(z) + z^{-2}Y(z) = X(z) - z^{-1}X(z)$

$$\Rightarrow Y(z)\left[1 - \frac{5}{2}z^{-1} + z^{-2}\right] = X(z)\left[1 - z^{-1}\right]$$

$$H(z) = \frac{Y(z)}{X(z)} = \frac{1 - z^{-1}}{1 - \frac{5}{2}z^{-1} + z^{-2}}$$

$$= \frac{z^2 - z}{z^2 - \frac{5}{2}z + 1} = \frac{z(z-1)}{(z-2)\left(z - \frac{1}{2}\right)}.$$

(b) $$h(n) = \frac{1}{2\pi j}\oint_C H(z) z^{n-1} dz$$



Z-plane

$$= \left.\frac{z(z-1)}{(z-2)} z^{n-1}\right|_{z=\frac{1}{2}}$$

$$+ \left.\frac{z(z-1)\, z^{n-1}}{\left(z - \frac{1}{2}\right)}\right|_{z=2}$$

$$= \frac{\left(\frac{1}{2} - 1\right)}{\left(\frac{1}{2} - 2\right)}\left(\frac{1}{2}\right)^n + \frac{(2-1)}{\left(2 - \frac{1}{2}\right)} 2^n$$

(c) $x(n) = \left(\frac{1}{2}\right)^n u(n)$

$$X(z) = \sum_{n=0}^{\infty} \left(\frac{1}{2} z^{-1}\right)^n = \frac{1}{1 - \frac{1}{2} z^{-1}}$$

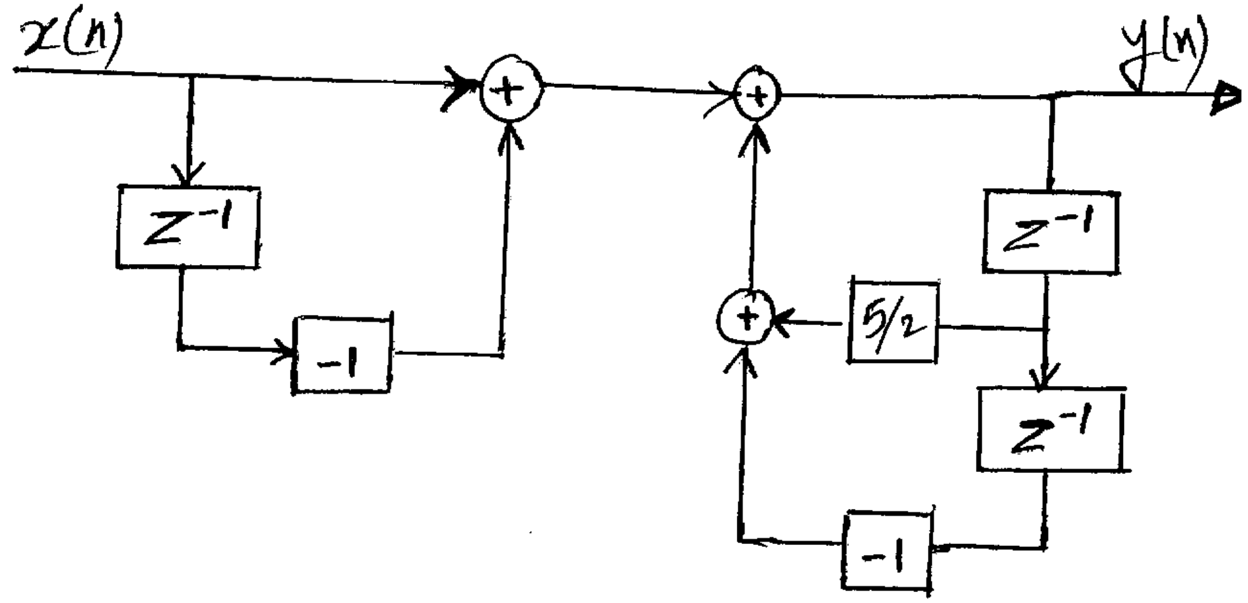
$$= \frac{z}{z - \frac{1}{2}}; \ |z| > \frac{1}{2}.$$

$$Y(z) = H(z)X(z) = \frac{z^2(z-1)}{(z-2)\left(z-\frac{1}{2}\right)^2}; \ |z| > 2.$$

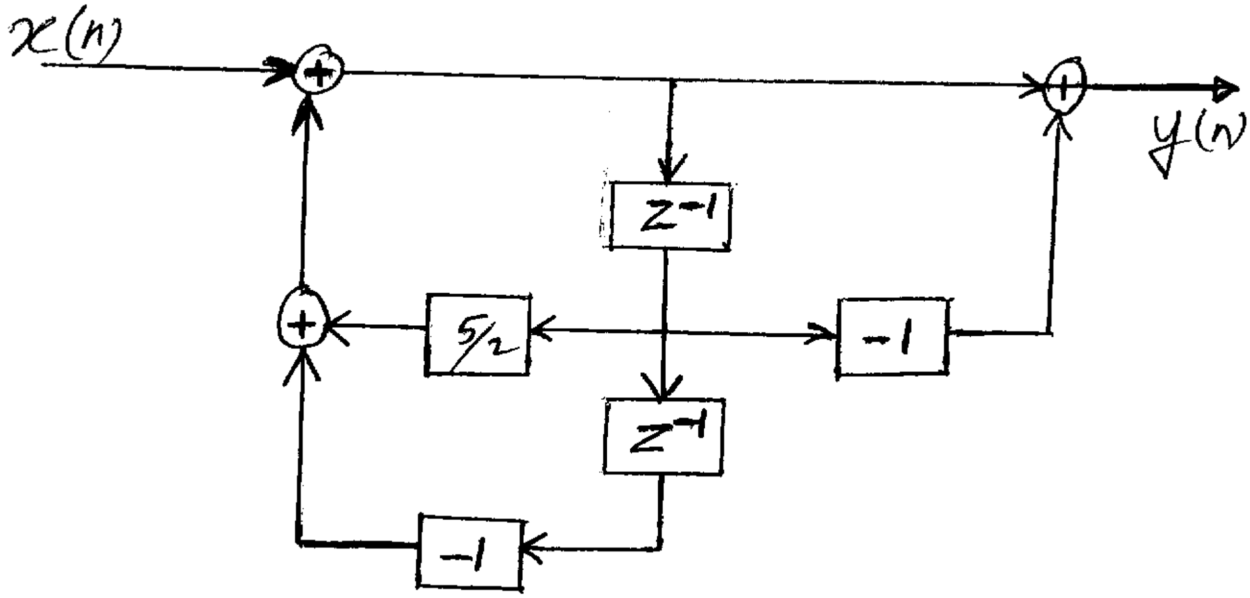
$$y(n) = \left.\frac{z^2(z-1)}{\left(z-\frac{1}{2}\right)^2} z^{n-1}\right|_{z=2} + \frac{1}{1!}\left[\frac{d}{dz} \frac{z^2(z-1)z^{n-1}}{(z-2)}\right]_{z=1/2}$$

$$= \frac{1}{9}\left[2^{n+3} + \left(\frac{1}{2}\right)^n (3n+1)\right] u(n)$$

(d)



Direct form I

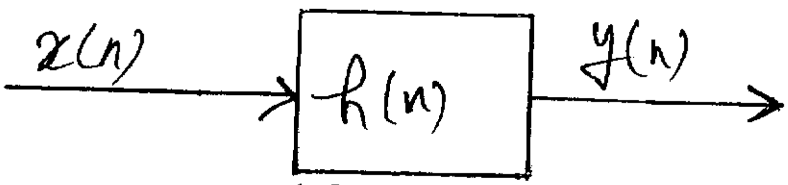


Direct form II

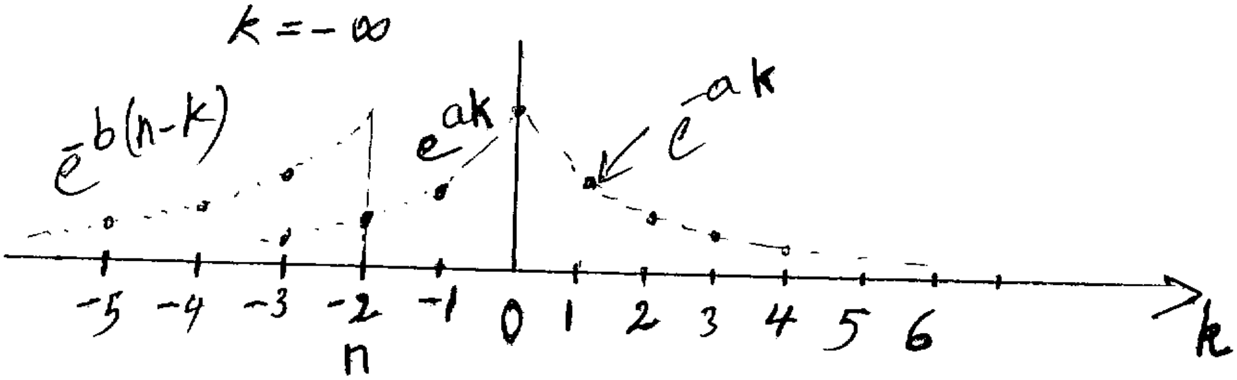
Problem #3

$$h(n) = e^{-bn}u(n) \; ; \; x(n) = e^{-a|n|}, \text{ all } n.$$

$a > b > 0.$



(a) $$y(n) = \sum_{k=-\infty}^{\infty} x(k)\,h(n-k)$$



For $n \le 0$,

$$y(n) = \sum_{k=-\infty}^{n} e^{-b(n-k)} e^{ak} = e^{-bn} \sum_{k=-\infty}^{n} e^{(a+b)k}$$

$$= e^{-bn}\left[\sum_{k=-\infty}^{0} e^{(a+b)k} - \sum_{k=n+1}^{0} e^{(a+b)k}\right]$$

$$= e^{-bn}\left[\sum_{k=0}^{\infty} e^{-(a+b)k} - \sum_{k=0}^{-n-1} e^{-(a+b)k}\right]$$

$$= e^{-bn}\left[\frac{1}{1-e^{-(a+b)}} - \frac{1-\left(e^{+(a+b)}\right)^{n}}{1-e^{-(a+b)}}\right]$$

$$= e^{-bn} \times \frac{e^{(a+b)n}}{1-e^{-(a+b)}}$$

$$= \frac{e^{an}}{1-e^{-(a+b)}}$$

For $n \ge 0$,

$$y(n) = \sum_{k=-\infty}^{0} e^{-b(n-k)} e^{ak} + \sum_{k=1}^{n} e^{-b(n-k)} e^{-ak}$$

$$= e^{-bn} \sum_{k=-\infty}^{0} e^{(a+b)k} + e^{-bn} \sum_{k=1}^{n} e^{-(a-b)k}$$

$$= e^{-bn}\left[\sum_{k=0}^{\infty} e^{-(a+b)k} + \sum_{k=1}^{n} e^{-(a-b)k}\right]$$

$$y(n) = e^{-bn}\left[\frac{1}{1-e^{-(a+b)}} + \frac{1-e^{-(a-b)(n+1)}}{1-e^{-(a-b)}} - 1\right]$$

$$= e^{-bn}\left[\frac{1}{1-e^{-(a+b)}} + \frac{e^{-(a-b)} - e^{-(a-b)(n+1)}}{1-e^{-(a-b)}}\right]$$

$$= e^{-bn}\left[\frac{1}{1-e^{-(a+b)}} + \frac{e^{-(a-b)}\left[1-e^{-(a-b)n}\right]}{1-e^{-(a-b)}}\right]$$

$$= e^{-bn}\left[\frac{1}{1-e^{-(a+b)}} - \frac{1-e^{-(a-b)n}}{1-e^{(a-b)}}\right]$$

$$= \frac{e^{-bn}}{1-e^{-(a+b)}} + \frac{e^{-an} - e^{-bn}}{1-e^{(a-b)}}$$

(b)

$$h(n) = e^{-bn}\,u(n)$$

$$\sum_{n=0}^{\infty} |h(n)| = \sum_{n=0}^{\infty} e^{-bn} = \frac{1}{1-e^{-b}} < \infty.$$

$\Rightarrow$ The system $h(n)$ is stable,

$$\text{(c)}\ H(\omega) = \sum_{n=0}^{\infty} e^{-bn} e^{-j\omega n} = \sum_{n=0}^{\infty} e^{-(b+j\omega)n}$$

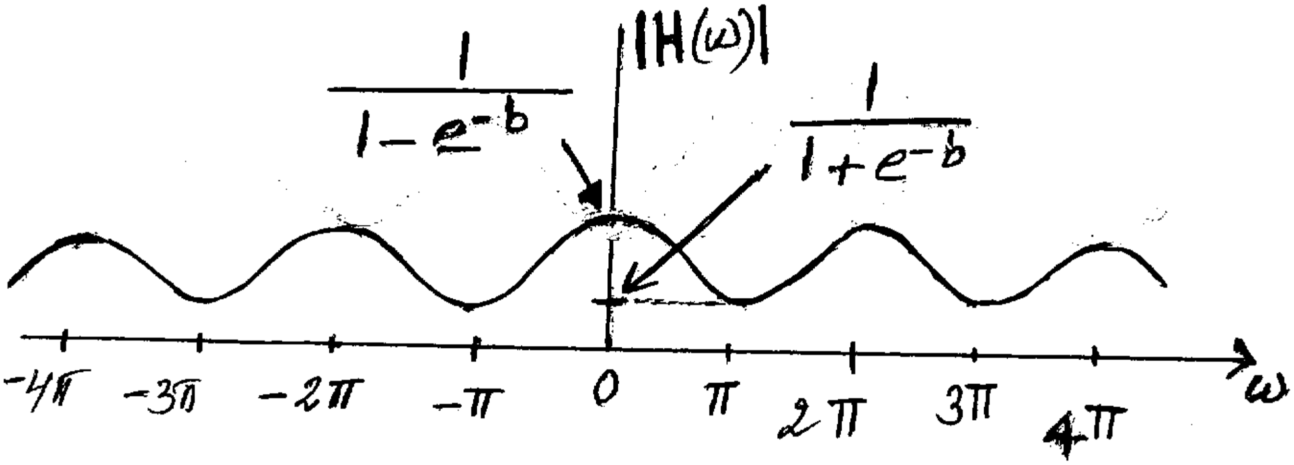
$$= \frac{1}{1 - e^{-b}\,e^{-j\omega}}$$

$$|H(\omega)| = \frac{1}{\left|1 - e^{-b}(\cos\omega - j\sin\omega)\right|}$$

$$= \frac{1}{\left[(1 - e^{-b}\cos\omega)^2 + (e^{-b}\sin\omega)^2\right]^{1/2}}$$

$$= \frac{1}{\left[1 - 2e^{-b}\cos(\omega) + e^{-2b}\cos^2\omega + e^{-2b}\sin^2\omega\right]^{1/2}}$$

$$= \frac{1}{\left[1 - 2e^{-b}\cos\omega + e^{-2b}\right]^{1/2}}$$



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Problem # 4

$$x(n) = \begin{cases} \left(\frac{1}{2}\right)^n, & 0 \le n \le 5 \\ 0, & \text{elsewhere.} \end{cases}$$

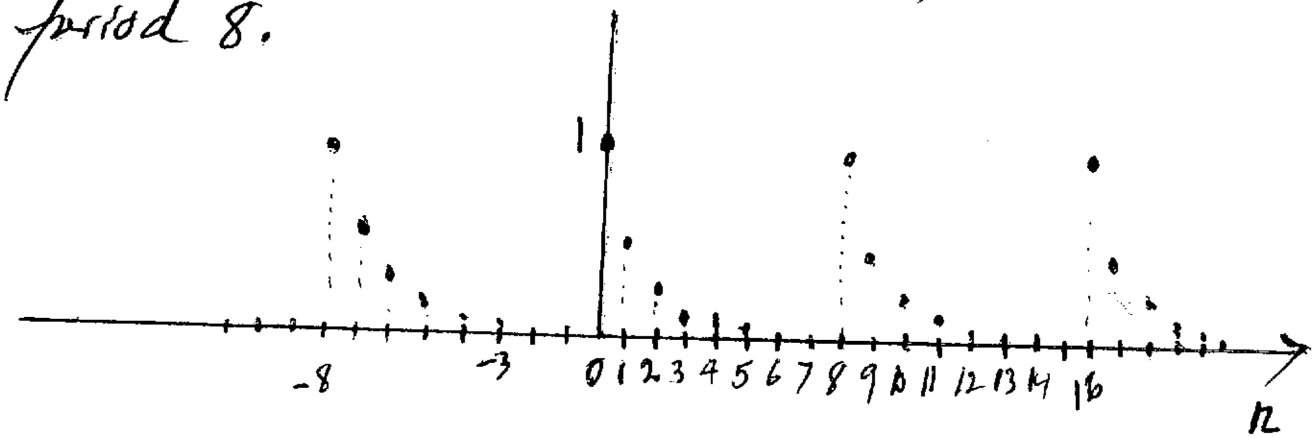
(a) $$X(z) = \sum_{n=0}^{5} \left(\frac{1}{2}\right)^n z^{-n} = \sum_{n=0}^{5} \left(\frac{1}{2} z^{-1}\right)^n$$

$$= \frac{1 - \left(\frac{1}{2} z^{-1}\right)^6}{1 - \frac{1}{2} z^{-1}} = \frac{1 - \frac{1}{64} z^{-6}}{1 - \frac{1}{2} z^{-1}}$$

(b) $$\tilde{X}(k) = X(z)\Big|_{z = e^{j\frac{2\pi}{8}k}}$$

$$= \frac{1 - \frac{1}{64} e^{-j\frac{12\pi}{8}k}}{1 - \frac{1}{2} e^{-j\frac{2\pi}{8}k}}$$

$\tilde{X}(k)$ as expressed above is the Fourier Series coefficients of the periodic extension of $x(n)$ with period 8.



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(c) $X(k)$ of $x(n)$ considered to be of length 8 is:

$$X(k) = \sum_{n=0}^{8} x(n) W_8^{kn} = X(z)\Big|_{z=W_8^{-k}}$$

$\Rightarrow X(k) = \tilde{X}(k)$ as determined in (b).

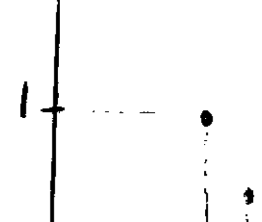
$$\Rightarrow X(k) = \frac{1 - \frac{1}{64} e^{-j\frac{12\pi}{8}k}}{1 - \frac{1}{2} e^{-j\frac{2\pi}{8}k}}.$$

$$X(0) = 1 - \frac{1}{64} \Big/ 1 - \frac{1}{2} = 2 \times \frac{63}{64} = \frac{63}{32}.$$

$$X(1) = \frac{1 - \frac{1}{64}\left(\cos\frac{12\pi}{8} - j\sin\frac{12\pi}{8}\right)}{1 - \frac{1}{2}\left(\cos\frac{2\pi}{8} - j\sin\frac{2\pi}{8}\right)}$$

$=$

(d) The sequence $x_1(n)$ such that $X_1(k) = W_8^{-4k} X(k)$ is obtained from $x(n)$ by a cyclic shift of amount 4. $x(n)$ is considered of length 8.



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EECE 440 – SIGNALS and SYSTEMS
Final
SPRING 2005-2006
June 6, 2006
TIME: 2 Hours
CLOSED BOOK EXAM
TWELVE SHEETS OF FORMULAS ARE ALLOWED
INSTRUCTOR: Dr. JEAN SAADE

NAME:______________________ **ID #:** _________________

INSTRUCTIONS

- WRITE YOUR ID # AND NAME ON THE COMPUTER CARD AND ON THIS SHEET IN THE PROVIDED SPACES.
- PROVIDE YOUR ANSWER ON THE COMPUTER CARD **and solution of each problem on the scratch booklet**
- **Random checking will be done to find out about any inconsistency between the problem solutions and the provided answers on the computer card.**
- RETURN THE COMPUTER CARD ATTACHED ON TOP OF THE QUESTION SHEET AND SCRATCH BOOKLET.
- USE PENCIL FOR MARKING YOUR ANSWERS AND ID # ON THE COMPUTER CARD.
- **ONLY YOUR ANSWER PROVIDED ON THE COPMUTER CARD WILL BE CONSIDERED IN GRADING.**
- **All QUESTIONS ARE EQUALLY WEIGHTED IN GRADING.**

Problem # 1

Consider a linear shift-invariant discrete time system with impulse response given by

$$h(n) = a^n u(n).$$

Let $x(n) = u(n)$ be the input to the system. Determine the output $y(n)$ of the system and then identify in this output the transient response, $y_t(n)$. Note that the transient response has the form of the system impulse response. Also, determine the condition that makes the transient response decay to zero when n tends to infinity.

(a) $y_t(n) = \dfrac{a^{n+1}}{a-1}u(n),\ |a| > 1.$

(b) $y_t(n) = \dfrac{a^{n+1}}{a-1}u(n),\ |a| < 1.$

(c) $y_t(n) = \dfrac{a^{n}}{a-1}u(n),\ |a| < 1.$

(d) $y_t(n) = \dfrac{a^{n}}{a-1}u(n),\ |a| > 1.$

(e) $y_t(n) = a^n\ u(n),\ |a| < 1.$

Problem # 2

Consider the linear and time invariant (LTI) finite impulse response (FIR) system whose impulse response is given by

$$h(n) = a^n u(n) - a^n u(n-3)$$

with a being a finite real number larger than 1. Determine the transfer function, $H(z)$, of the system and specify whether the system is or is not stable.

(a) $H(z) = \dfrac{z^3 - a^3}{z^2}$, and the system is stable.

(b) $H(z) = \dfrac{z^3 - (1/a)^3}{z^2[z-(1/a)]}$, and the system is stable.

(c) $H(z) = \dfrac{z^3 - a^3}{z^2(z-a)}$, and the system is *not* stable.

(d) $H(z) = \dfrac{z^3 - a^3}{z^2(z-a)}$, and the system is stable.

(e) $H(z) = \dfrac{z^3 - a^3}{z^2}$, and the system is *not* stable.

Problem # 3

Consider again the FIR system given in Problem # 2 and determine the difference equation for this system.

(a) $y(n)+ay(n-1)=x(n)+a^2x(n-2)$

(b) $y(n)-y(n-1)=x(n)+a^2x(n-2)$

(c) $y(n)=x(n)+ax(n-1)+a^2x(n-2)$

(d) $y(n)=x(n)+ax(n-1)$

(e) $y(n)-ay(n-1)=x(n)+a^2x(n-2)$

Problem # 4

Consider the following two discrete time sequences:

$$x(n)=a^n u(n) \text{ and } h(n)=b^{-n}u(n).$$

Let the sequence $y(n)$ be given by the multiplication of $x(n)$ and $h(n)$. That is,

$$y(n)=x(n)h(n).$$

Determine the Fourier transform of $y(n)$; i.e., $Y(\omega)$, and the condition under which this transform exists.

(a) $Y(\omega)=(1-\frac{a}{b}e^{-j\omega})^{-1},\ \left|\frac{a}{b}\right|>1$

(b) $Y(\omega)=(1-ab\,e^{j\omega})^{-1},\ |ab|<1$

(c) $Y(\omega)=(1-ab\,e^{-j\omega})^{-1},\ |ab|<1$

(d) $Y(\omega)=(1-\frac{a}{b}e^{j\omega})^{-1},\ \left|\frac{a}{b}\right|<1$

(e) $Y(\omega)=(1-\frac{a}{b}e^{-j\omega})^{-1},\ \left|\frac{a}{b}\right|<1$

Problem # 5

Consider the following Z-transform, $X(z)$, of a discrete time sequence, $x(n)$:

$$X(z)=\frac{z^4+2z^3-z+2}{z^2},\ 0<|z|<\infty.$$

Determine the sequence $x(n)$.

(a) $x(n)=\begin{cases}1, n=-2\\ 2, n=-1\\ -1, n=1\\ 2, n=2\\ 0, \text{elsewhere}\end{cases}$ (b) $x(n)=\begin{cases}-1, n=-2\\ -2, n=-1\\ 1, n=1\\ -2, n=2\\ 0, \text{elsewhere}\end{cases}$ (c) $x(n)=\begin{cases}1, n=-2\\ 2, n=0\\ -1, n=1\\ 2, n=2\\ 0, \text{elsewhere}\end{cases}$

(d) $x(n)=\begin{cases}-1, n=-2\\ -2, n=0\\ 1, n=1\\ -2, n=2\\ 0, \text{elsewhere}\end{cases}$ (e) $x(n)=\begin{cases}-1, n=-1\\ -2, n=0\\ 1, n=1\\ -2, n=2\\ 0, \text{elsewhere}\end{cases}$

Problem # 6

Consider a linear and time-invariant system whose transfer function, *H(z)*, is given by

$$H(z)=\frac{z}{(z-a)(z-b)},\ \text{with } |a|>|b|>1.$$

Determine the impulse response, *h(n)*, of the system assuming that the system is causal.

(a) $h(n)=\frac{a^n-b^n}{a-b}, n\geq 2$ (b) $h(n)=\frac{a^n+b^n}{a-b}, n\geq 0$ (c) $h(n)=\frac{a^n-b^n}{a-b}, n\geq 0$

(d) $h(n)=\frac{a^n+b^n}{a-b}, n\geq 2$ (e) $h(n)=\frac{a^n b^n}{a-b}, n\geq 0$

Problem # 7

Consider again the given in Problem # 6 and determine the impulse response, *h(n)*, of the system assuming that the system is stable.

(a) $h(n)=\frac{a^n b^n}{b-a}, n\leq 0$ (b) $h(n)=\frac{a^n+b^n}{b-a}, n\leq 0$ (c) $h(n)=\frac{a^n-b^n}{a-b}, n\leq 0$

(d) $h(n)=\frac{a^n-b^n}{b-a}, n\leq 0$ (e) $h(n)=\frac{a^n+b^n}{a-b}, n\leq 0$

Problem # 8

Consider again the system whose transfer function is given in Problem # 6. Let this system be implemented using the block diagram form obtained from the system difference equation. The implemented system would turn out to be:

(a) Causal but not stable
(b) Causal and stable
(c) Stable but not causal
(d) Causal and stable but not time invariant
(e) Causal and stable and time invariant

Problem # 9

Consider the cascaded connection of two causal, linear and time-invariant analog systems with transfer functions given by:

$$H_1(s) = \frac{s+1}{(s-3)(s+5)} \quad \text{and} \quad H_2(s) = \frac{P(s)}{(s+2)(s+3)}.$$

P(s) is a polynomial in s of degree less than or equal 2. Select from what is given below the polynomial *P(s)* that makes the system that is equivalent to the cascaded connection of $H_1(s)$ and $H_2(s)$ stable and having the transient response that decays the fastest possible to zero as time goes to infinity.

(a) P(s)=(s-3)
(b) P(s)=(s-3)(s+2)
(c) P(s)= (s-3)(s+5)
(d) P(s)= (s-3)(s+3)
(e) P(s)= (s+2)(s+3).

Problem # 10

Consider the cascaded connection of two causal, linear and time-invariant discrete systems with transfer functions given by:

$$H_1(z) = \frac{z+1}{\left(z-\frac{1}{2}\right)(z-2)} \quad \text{and} \quad H_2(z) = \frac{P(z)}{\left(z-\frac{1}{4}\right)\left(z-\frac{3}{4}\right)}.$$

P(z) is a polynomial in z of degree less than or equal 2. Select from what is given below the polynomial *P(z)* that makes the system that is equivalent to the cascaded connection of $H_1(z)$ and $H_2(z)$ stable and having the transient response that decays the fastest possible to zero as time goes to infinity.

$(a)\ P(z) = (z-2)$

$(b)\ P(z) = (z-2)\left(z-\frac{1}{2}\right)$

$(c)\ P(z) = (z-2)\left(z-\frac{1}{4}\right)$

$(d)\ P(z) = \left(z-\frac{1}{2}\right)\left(z-\frac{3}{4}\right)$

$(e)\ P(z) = (z-2)\left(z-\frac{3}{4}\right)$

Problem # 11

Consider a linear and time-invariant analog system with impulse response given by:

$$h(t) = 2e^{-t}u(t) + e^{2t}u(-t)$$

Determine the ROC of the system transfer function, *H(s)*, and specify if the system is or is not stable.

$(a)\ -1 < \sigma < 2$ and the system is *not* stable.
$(b)\ -1 < \sigma < 2$ and the system is stable.
$(c)\ -2 < \sigma < 1$ and the system is *not* stable.
$(d)\ -2 < \sigma < 1$ and the system is stable.
$(e)\ -1 < \sigma < 1$ and the system is stable.

Problem # 12

Consider the linear and time-invariant discrete system whose impulse response is obtained by sampling the impulse response of the analog system given in Problem # 11. Determine the ROC of the discrete system transfer function, *H(z)*, and specify if the system is or is not stable.

$(a)\ e^{-2} < |z| < e$ and the system is *not* stable.

$(b)\ e^{-2} < |z| < e$ and the system is stable.

$(c)\ e^{-1} < |z| < e^{2}$ and the system is *not* stable.

$(d)\ e^{-1} < |z| < e^{2}$ and the system is stable.

$(e)\ e^{-1} < |z| < e$ and the system is stable.

Problem # 13

Consider the following finite duration discrete time sequence:

$$x(n) = \begin{cases} 2^n, 0 \le n \le 2 \\ 0, \text{ elsewhere} \end{cases}$$

Use the DFT, $X(k)$, of $x(n)$, that represents the coefficients of the DFS representation of the periodic repetition of $x(n)$ with period equal to 3 to determine $X(0)$ and $X(1)$.

(a) $X(0) = 7,\ X(1) = -3 + 2.732j$
(b) $X(0) = 7,\ X(1) = -2 + 1.732j$.
(c) $X(0) = 7,\ X(1) = 2 - 1.732j$
(d) $X(0) = 7,\ X(1) = 3 - 2.732j$
(e) $X(0) = 7,\ X(1) = -4 + 2.732j$

Problem # 14

Consider the same discrete time sequence, $x(n)$, given in Problem # 13. Use the DFT, $X(k)$, that represents samples from the Fourier transform of $x(n)$ at $\omega=(2\pi/10)k$, $k=0, 1, \ldots, 9$ and determine $X(0)$ and $X(1)$.

(a) $X(0) = 7,\ X(1) = -3.854 + 4.979j$
(b) $X(0) = 7,\ X(1) = 3.854 - 4.979j$
(c) $X(0) = 7,\ X(1) = 4.854 - 5.979j$
(d) $X(0) = 7,\ X(1) = -4.854 + 5.979j$
(e) $X(0) = 7,\ X(1) = -5.854 + 6.979j$

Problem # 15

The comparison of the results; i.e., X(0) and X(1), in Problems # 13 and 14 shows that X(0) is the same but X(1) differs. Determine the reason for this difference in the values of X(1).

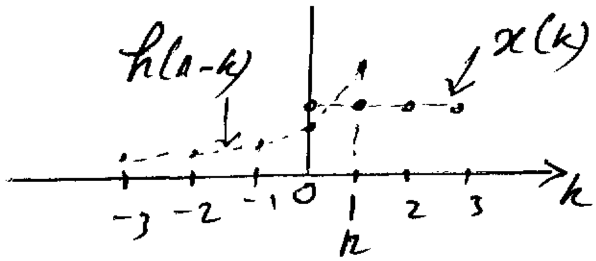
(a) $X(k)$ in Problem # 13 represents samples from $X(\omega)$ at $\omega=(2\pi/4)k$, $k=0,1,2,3$.
(b) $X(k)$ in Problem # 13 represents samples from $X(\omega)$ at $\omega=(2\pi/6)k$, $k=0,1,2,3,4,5$.
(c) $X(k)$ in Problem # 13 represents samples from $X(\omega)$ at $\omega=(2\pi/3)k$, $k=0,1,2$.
(d) $X(k)$ in Problem # 13 represents samples from $X(\omega)$ at $\omega=(2\pi/5)k$, $k=0,1,2,3,4$.
(e) $X(k)$ in Problem # 13 represents samples from $X(\omega)$ at $\omega=(2\pi/7)k$, $k=0,1,2,3,4,5,6$.

EECE 440 Final Exam Solution Spring 2006

Problem #1

$$h(n) = a^n u(n); \quad x(n) = u(n).$$

$$y(n) = \sum_{k=-\infty}^{\infty} x(k) h(n-k)$$



$n<0, \; y(n) = 0.$

$$n \geq 0, \; y(n) = \sum_{k=0}^{n} a^{n-k}$$

$$= a^n \sum_{k=0}^{n} (a^{-1})^k = a^n \left[\frac{1-(a^{-1})^{n+1}}{1-a^{-1}} \right]$$

$$= \frac{a^{n+1}-1}{a-1}$$

$$= \frac{a^{n+1} u(n)}{a-1} - \frac{u(n)}{a-1} = y_t(n) + y_s(n).$$

$\Rightarrow$ the transcient response of the system is:

$$y_t(n) = \frac{a^{n+1}}{a-1} u(n).$$

If $|a|<1$, then $y_t(n) \rightarrow 0$ when $n \rightarrow \infty$.

Note that $y_t(n)$ has the same form of the system impulse response. Also, $|a|<1$ is the condition

Problem # 2

$$h(n) = a^n u(n) - a^n u(n-3)$$

$$= \begin{cases} 1, & n=0 \\ a, & n=1 \\ a^2, & n=2, \text{ and } 0 \text{ elsewhere.} \end{cases}$$

$$H(z) = \sum_{n=0}^{2} h(n) z^{-n} = 1 + a z^{-1} + a^2 z^{-2}$$

$$= \frac{z^2 + a z + a^2}{z^2} = \frac{(z-a)(z^2+az+a^2)}{z^2(z-a)}$$

$$= \frac{z^3 - a^3}{z^2(z-a)}, \text{ for } 0 < |z| \leq \infty$$

The region of convergence contains the unit circle. Hence, the system is stable.

Problem # 3

$$H(z) = \frac{Y(z)}{X(z)} = \frac{z^2 + a z + a^2}{z^2} = 1 + a z^{-1} + a^2 z^{-2}$$

$$\Rightarrow Y(z) = X(z) + a z^{-1} X(z) + a^2 z^{-2} X(z)$$

$$\Rightarrow y(n) = x(n) + a\, x(n-1) + a^2 x(n-2)$$

Problem # 4

$$x(n) = a^n u(n), \; h(n) = b^{-n} u(n)$$

$$y(n) = x(n) h(n)$$

$$= (a b^{-1})^n u(n)$$

$$Y(\omega) = \sum_{n=0}^{\infty} (a b^{-1})^n e^{-j\omega n} = \sum_{n=0}^{\infty} \left[\frac{a}{b} e^{-j\omega}\right]^n$$

$$= \frac{1}{\left(1 - \frac{a}{b} e^{-j\omega}\right)} \quad \text{with } \left|\frac{a}{b}\right| < 1.$$

Problem # 5

$$X(z) = \frac{z^4 + 2z^3 - z + 2}{z^2}, \; 0 < |z| < \infty.$$

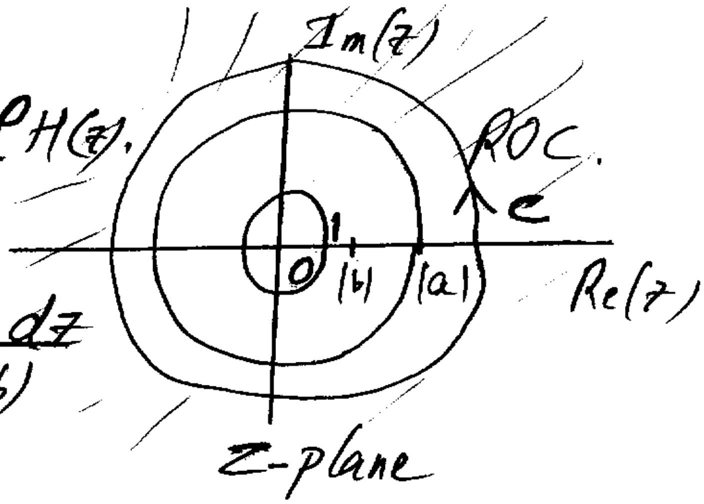
$$X(z) = z^2 + 2z - z^{-1} + 2z^{-2} = \sum_{n=-\infty}^{\infty} x(n) z^{-n}$$

$$\Rightarrow x(n) = \begin{cases} 1, & n = -2 \\ 2, & n = -1 \\ -1, & n = 1 \\ 2, & n = 2 \\ 0, & \text{elsewhere}. \end{cases}$$

Problem # 6

$$H(z) = \frac{z}{(z-a)(z-b)}, \text{ with } |a| > |b| > 1$$

For the causal system,
$|z| > |a|$ is the ROC of $H(z)$.



$$h(n) = \frac{1}{2\pi j} \oint_c \frac{z\, z^{n-1}\, dz}{(z-a)(z-b)}$$

$$= \frac{1}{2\pi j} \oint_c \frac{z^n\, dz}{(z-a)(z-b)} = [\text{Res. at } z=a] + [\text{Res. at } z=b]$$

$$= \left.\frac{z^n}{(z-b)}\right|_{z=a} + \left.\frac{z^n}{(z-a)}\right|_{z=b}$$

$$= \frac{a^n}{a-b} + \frac{b^n}{b-a} = \frac{a^n - b^n}{a-b}, \quad n \geq 0$$

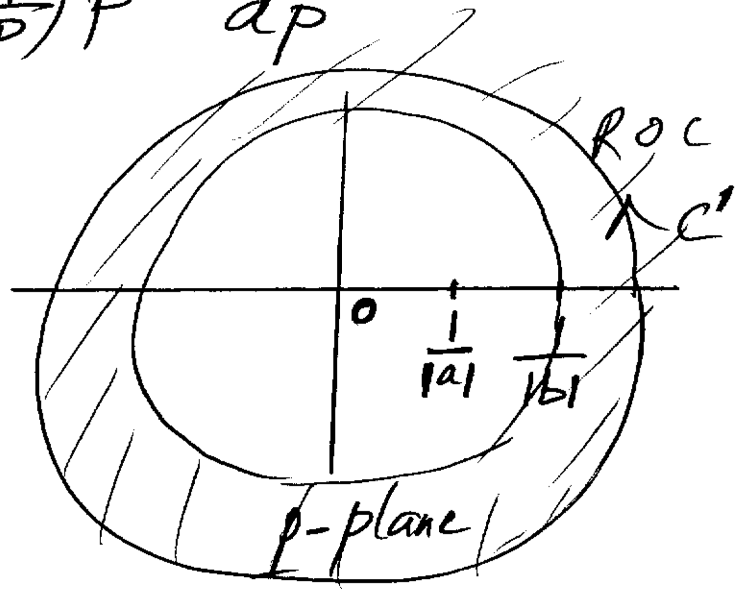
Problem # 7

The stable system has the ROC such that $|z| < |b|$.

$\Rightarrow h(n)$ is a left-sided sequence.

$$H(P^{-1}) = \frac{\frac{1}{P}}{\left(\frac{1}{P} - a\right)\left(\frac{1}{P} - b\right)} = \frac{P^{-1}}{(1-aP)(1-bP)P^{-2}}$$

$$h(n)=\frac{1}{2\pi j}\oint_{c'} H\left(\frac{1}{p}\right)p^{-n-1}\,dp$$



$$h(n)=\frac{1}{2\pi j}\oint_{c'} \frac{p\,p^{-n-1}}{ab\left(p-\frac{1}{a}\right)\left(p-\frac{1}{b}\right)}\,dp$$

$$=\frac{1}{ab}\,\frac{1}{2\pi j}\oint_{c'} \frac{p^{-n}}{\left(p-\frac{1}{a}\right)\left(p-\frac{1}{b}\right)}\,dp$$

$$=\frac{1}{ab}\left[\left.\frac{p^{-n}}{p-\frac{1}{b}}\right|_{p=\frac{1}{a}}+\left.\frac{p^{-n}}{p-\frac{1}{a}}\right|_{p=\frac{1}{b}}\right]$$

$$=\frac{a^n}{b-a}+\frac{b^n}{a-b}=\frac{a^n-b^n}{b-a},\quad n\leqslant 0.$$

<u>Problem #8</u>

The implemented system is the causal system.

But, the causal system is not not stable.

Problem # 9

The cascaded system has a transfer function given by:

$$H(s) = H_1(s)\, H_2(s)$$
$$= \frac{(s+1)\, P(s)}{(s-3)(s+5)(s+2)(s+3)}$$

The polynomial $P(s)$ is of degree ≤ 2.

$P(s) = (s-3)(s+2)$ cancels $(s-3)$ from the denominator of $H(s)$. This makes the system stable by deleting the pole at $s=3$ which is located in the right half s-plane. Also, $P(s) = (s-3)(s+2)$ cancels $(s+2)$ from the denominator which provides the slowest decaying component in $h(t)$ or the transient response compared to $(s+3)$ and $(s+5)$. $s=-2$ is the pole that is the closest to the imaginary axis.

Problem # 10

$$H(z) = H_1(z)\, H_2(z)$$
$$= \frac{(z+1)\, P(z)}{(z-\frac{1}{2})(z-2)(z-\frac{1}{4})(z-\frac{3}{4})}.$$

$P(z) = (z-2)(z-\frac{3}{\ })$

$P(z)$ as selected, deletes the pole at $z=2$, which is outside the unit circle. This makes $H(z)$ stable. It also deletes $z=\frac{3}{4}$ which is the pole the closest to the unit circle. Hence the transient response component that decays the slowliest possible to 0 when n increases is also deleted.

Problem #11

$$h(t) = 2e^{-t}u(t) + e^{2t}u(-t)$$

$$H(s) = \int_{-\infty}^{0} e^{2t} e^{-st}\,dt + \int_{0}^{\infty} 2e^{-t} e^{-st}\,dt$$

$$= \int_{-\infty}^{0} e^{-(s-2)t}\,dt + \int_{0}^{\infty} 2e^{-(s+1)t}\,dt$$

$$= -\frac{1}{s-2} e^{-(s-2)t}\Big|_{-\infty}^{0} + 2\times\frac{-1}{s+1} e^{-(s+1)t}\Big|_{0}^{\infty}$$

$$= \frac{-1}{s-2} + \frac{2}{s+1} = \frac{-s-1+2s-4}{(s+1)(s-2)}$$

$$= \frac{s-5}{(s+1)(s-2)}, \quad -1 < \sigma < 2.$$

Problem #12

$$h(n) = 2e^{-n}u(n) + e^{2n}u(-n).$$

$$H(z) = \sum_{n=-\infty}^{0} e^{2n} z^{-n} + 2\sum_{n=0}^{\infty} e^{-n} z^{-n}$$

$$= \sum_{n=-\infty}^{0} (e^{-2} z^{+1})^{-n} + 2\sum_{n=0}^{\infty} (e^{-1} z^{-1})^{n}$$

$$= \sum_{n=0}^{\infty} (e^{-2} z)^{n} + 2\sum_{n=0}^{\infty} (e^{-1} z^{-1})^{n}$$

$$= \frac{1}{1 - e^{-2} z} + \frac{2}{1 - e^{-1} z^{-1}}$$

With $|e^{-2} z| < 1 \Rightarrow |z| < e^{2}$

and $|e^{-1} z^{-1}| < 1 \Rightarrow |z| > e^{-1}$

$\Rightarrow$ The ROC is such that: $e^{-1} < |z| < e^{2}$

The ROC contains the unit circle and the system is stable.

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Problem #13

$$x(n) = \begin{cases} 2^n, & 0 \le n \le 2 \\ 0, & \text{elsewhere} \end{cases}$$

$$X(k) = \sum_{n=0}^{2} 2^n W_3^{kn}, \quad k = 0, 1, 2$$

$$= 1 + 2W_3^k + 4W_3^{2k}$$

$$= 1 + 2e^{-j\frac{2\pi}{3}k} + 4e^{-j\frac{2\pi}{3}2k}$$

$$X(0) = 1 + 2 + 4 = 7$$

$$X(1) = 1 + 2\left(\cos\left(\frac{2\pi}{3}\right) - j\sin\left(\frac{2\pi}{3}\right)\right) + 4\left(\cos\left(\frac{4\pi}{3}\right) - j\sin\left(\frac{4\pi}{3}\right)\right)$$

$$= \left[1 + 2\cos\left(\frac{2\pi}{3}\right) + 4\cos\left(\frac{4\pi}{3}\right)\right] - j\left[2\sin\left(\frac{2\pi}{3}\right) + 4\sin\left(\frac{4\pi}{3}\right)\right]$$

$$= -2 + 1.732j.$$

Problem #14

$$X(k) = \sum_{n=0}^{2} 2^n W_{10}^{kn} = \sum_{n=0}^{2} 2^n e^{-j\frac{2\pi}{10}kn}$$

$$= \sum_{n=0}^{2} 2^n e^{-j\omega n}\Big|_{\omega = \frac{2\pi}{10}k}, \quad k = 0, 1, 2, \dots, 9.$$

$$= 1 + 2e^{-j\frac{2\pi}{10}k} + 4e^{-j\frac{2\pi}{10}2k}$$

10/10

$$X(0) = 1 + 2 + 4 = 7$$

$$X(1) = 1 + 2\left(\cos\left(\frac{2\pi}{10}\right) - j\sin\left(\frac{2\pi}{10}\right)\right) + 4\left(\cos\left(\frac{4\pi}{10}\right) - j\sin\left(\frac{4\pi}{10}\right)\right)$$

$$= 1 + 2\cos\left(\frac{2\pi}{10}\right) + 4\cos\left(\frac{4\pi}{10}\right) - j\left(2\sin\left(\frac{2\pi}{10}\right) + 4\sin\left(\frac{4\pi}{10}\right)\right)$$

$$= 3.854 - 4.979j.$$

Problem #15

$X(1)$ in problem #14 is a sample from $X(\omega)$ at $\omega = \frac{2\pi}{10}$.

In problem #13:

$$X(k) = \sum_{n=0}^{2} 2^n W_3^{kn} = \sum_{n=0}^{2} 2^n e^{-j\frac{2\pi}{3}kn}$$

$$= \sum_{n=0}^{2} 2^n e^{-j\omega n}\Big|_{\omega = \frac{2\pi}{3}k,\ k=0,1,2,}$$

$\Rightarrow X(1)$ is a sample from $X(\omega)$ at $\omega = \frac{2\pi}{3}$.