EECE 340 – Signals and Systems
Homework # 1

1.1 Consider the following voltage signal across a 50 Ω resistance.

$$f(t) = 2rect\left[\frac{t - T/2}{T}\right] V$$

Determine the energy dissipated by *f(t)* across the resistance.

Hint: $rect(t) = \begin{cases} 1, & -\frac{1}{2} \le t \le \frac{1}{2} \\ 0, & elsewhere \end{cases}$

1.2 Consider the signal $f(t) = e^{-t}$ for all *t*.

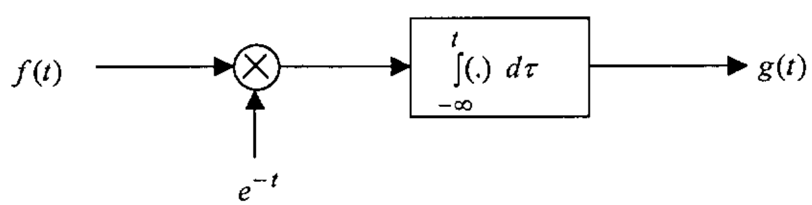
Apply the time-domain integration formulas for energy and average power to determine the nature of the signal *f(t)*.

1.3 Consider the following system:

$$g(t) = T[f(t)] = 4\int_{-\infty}^{t} f(\tau)d\tau + u(t).$$

Determine the impulse response *h(t)* of the system and the system output *g(t)* when the input *f(t)* is given by $f(t) = e^{-2t}u(t)$.

1.4 Consider the system shown in the figure below.



In this system, *f(t)* is the input signal and *g(t)* is the output signal. Determine the impulse response, *h(t)*, of the system and the system response, *g(t)*, for an input $f(t)=\delta(t-t_0)$, where $\delta(t)$ is the unit impulse function. Conclude whether the system is time-invariant or time-varying.

1.5 Consider an LTI system with impulse response $h(t)$ given by

$$h(t) = e^{2t}u(-t).$$

Determine the output $g(t)$ of this system when the input $f(t) = u(t)$ and use g(t) to determine whether the system is or is not physically realizable. Explain.

1.6 Consider the following system:

$$g(t) = T[f(t)] = 2f(t) + u(t).$$

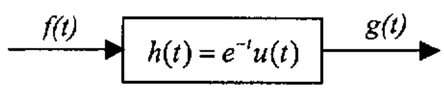
Determine whether this system is linear or non-linear and time-invariant or time-varying.

1.7 Consider the system represented by the following input-output transformation:

$$g(t) = T[f(t)] = e^{-t}f(t)$$

Examine the linearity, time-invariance and causality of this system.

1.8



For the above LTI system, determine the energy or average power, whichever applies, of the system output $g(t)$, when $f(t) = u(t)$. Use time-domain analysis.

EECE 340 – Signals and Systems
Homework # 1 – Solution

1.1 The voltage signal

$$f(t) = 2rect(\frac{t-T/2}{T})\text{ V}$$

is across a 50Ω resistance. Using

$$rect(t) = \begin{cases} 1, & -\frac{1}{2} \le t \le \frac{1}{2} \\ 0, & elsewhere \end{cases}$$

then

$$\begin{aligned} f(t) &= 2rect(\frac{t-T/2}{T})\text{ V} \\ &= \begin{cases} 2, 0 \le t \le T \\ 0, \text{elsewhere} \end{cases} \end{aligned}$$

$$E_f = \frac{1}{50}\int_0^T 4dt = \frac{4T}{50} = \frac{2T}{25}\text{ Joules}$$

1.2 Consider the signal

$$f(t) = e^{-t},\text{ all } t$$

$$E = \int_{-\infty}^{\infty} e^{-2t}dt = -\frac{1}{2}e^{-2t}\Big|_{-\infty}^{\infty} = \frac{1}{2}\left(e^{\infty} - e^{-\infty}\right) = \frac{1}{2}e^{\infty} \to \infty$$

$$P_f = \lim_{T\to\infty}\frac{1}{T}\int_{-T/2}^{T/2} e^{-2t}dt = \lim_{T\to\infty}\left[\frac{\frac{1}{2}\left(e^{T} - e^{-T}\right)}{T}\right]$$

$$= \lim_{T\to\infty}\frac{e^{T}}{2T} - \lim_{T\to\infty}\frac{e^{-T}}{2T} = \lim_{T\to\infty}\frac{e^{T}}{2T} = \infty$$

$\Rightarrow f(t)$ is neither an energy nor a power signal

1.3 The following system is considered:

$$g(t) = T\left[f(t)\right] = 4\int_{-\infty}^{t} f(\tau)d\tau + u(t)$$

$$h(t) = T\left[\delta(t)\right] = 4\int_{-\infty}^{t} \delta(\tau)d\tau + u(t)$$

For $t < 0$, $h(t) = 0$

For $t \geq 0$, $h(t) = 4u(t) + u(t) = 5u(t)$

For $f(t) = e^{-2t}u(t)$, the system output is:

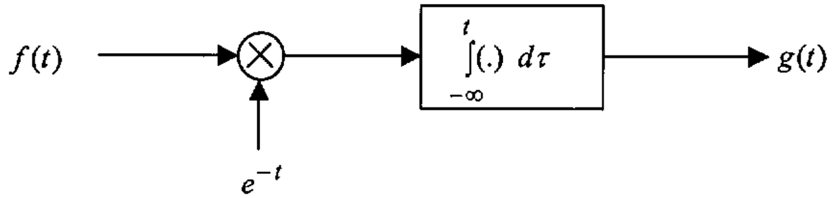
$$g(t) = 4\int_{-\infty}^{t} f(\tau)d\tau + u(t)$$

$$= 4\int_{0}^{t} e^{-2\tau}d\tau + u(t) = -2e^{-2\tau}\Big|_0^t + u(t)$$

$$= 2\left(1 - e^{-2t}\right)u(t) + u(t)$$

$$= \left(3 - 2e^{-2t}\right)u(t)$$

1.4 The system shown in the figure below is considered.



The system impulse response is the system output, denoted by $h(t)$, when the input $f(t) = \delta(t)$.

Thus, $h(t) = \int_{-\infty}^{t} e^{-\tau}\delta(\tau)d\tau = \int_{-\infty}^{t} \delta(\tau)d\tau$

$$= \begin{cases} 0, & t < 0 \\ 1, & t > 0 \end{cases}$$

$$= u(t)$$

For $f(t) = \delta(t - t_0)$,

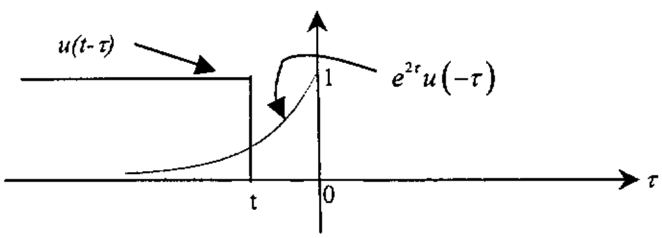
$$g(t) = \int_{-\infty}^{t} e^{-\tau}\delta(\tau - t_0)d\tau = e^{-t_0}\int_{-\infty}^{t} \delta(\tau - t_0)d\tau$$

$$= e^{-t_0}u(t - t_0)$$

Obviously, the system, which is formed by the cascaded connection of a multiplier and integrator is a time-varying system although the integrator is time-invariant.

1.5 The LTI system with impulse response *h(t)* given by $h(t) = e^{2t}u(-t)$ is considered.

The output *g(t)* of this system when the input *f(t) = u(t)* can be obtained using convolution.



$$g(t) = f(t) * h(t) = u(t) * e^{2t}u(-t)$$

For $t \le 0$, $g(t) = \int_{-\infty}^{t} e^{2\tau}d\tau = \frac{1}{2}e^{2\tau}\Big|_{-\infty}^{t}$

$$= \frac{1}{2}\left(e^{2t} - 0\right) = \frac{1}{2}e^{2t}$$

For $t \ge 0$, $g(t) = \int_{-\infty}^{0} e^{2\tau}d\tau = \frac{1}{2}e^{2\tau}\Big|_{-\infty}^{0} = \frac{1}{2}$

$$\Rightarrow g(t) = \begin{cases} \frac{1}{2}e^{2t}, & t \le 0 \\ \frac{1}{2}, & t \ge 0 \end{cases}$$

The system input is *f(t) = u(t)*. So, it applied for $t \ge 0$. But, the system produces an output for $t < 0$; i.e., before the input is applied. Hence, the system is not physically realizable.

1.6 The following system is considered:

$$g(t) = T[f(t)] = 2f(t) + u(t).$$

Linearity:

$$\begin{aligned} T[a_1 f_1(t) + a_2 f_2(t)] &= 2a_1 f_1(t) + 2a_2 f_2(t) + u(t) \\ &\neq a_1 T[f_1(t)] + a_2 T[f_2(t)] \\ &= 2a_1 f_1(t) + u(t) + 2a_2 f_2(t) + u(t) \\ &= 2a_1 f_1(t) + 2a_2 f_2(t) + 2u(t) \end{aligned}$$

So, the system is non-linear.

Time-invariance:

$$T[f(t-t_0)] = 2f(t-t_0) + u(t) \neq g(t-t_0) = 2f(t-t_0) + u(t-t_0)$$

Hence, the system is time-varying.

1.7 The system represented by the following input-output transformation is considered:

$$g(t) = T[f(t)] = e^{-t} f(t)$$

Linearity:

$$T[a_1 f_1(t) + a_2 f_2(t)] = a_1 T[f_1(t)] + a_2 T[f_2(t)]???$$

$$\begin{aligned} T[a_1 f_1(t) + a_2 f_2(t)] &= e^{-t}[a_1 f_1(t) + a_2 f_2(t)] \\ &= a_1 e^{-t} f_1(t) + a_2 e^{-t} f_2(t) \\ &= a_1 T[f_1(t)] + a_2 T[f_2(t)] \end{aligned}$$

So, the system is linear.

Causality:

$$g(t_0) = e^{-t_0} f(t_0)$$

Hence, the system output at time t_0 depends on the input at time t_0.

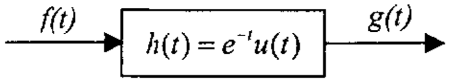
$\Rightarrow$ the system is causal

Time-invariance:

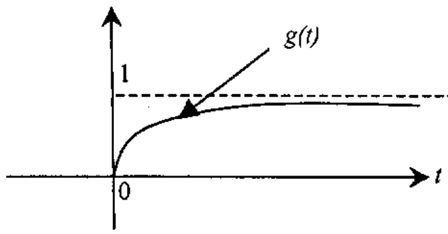
$$g(t-t_0) = e^{-(t-t_0)} f(t-t_0) \neq T[f(t-t_0)] = e^{-t} f(t-t_0)$$

Hence, the system is time-varying.

1.8



When the input to the system is $f(t)=u(t)$, then the output is $g(t)=(1-e^{-t})u(t)$. This can be verified using convolution.



It is obvious from the plot of $g(t)$ that $g(t)$ has an ∞ energy. Hence g(t) is not an energy signal.
$g(t)$ is a power signal if it has a finite non-zero average power. As shown in the computations done below, g(t) is therefore a power signal.

$$P_{av} = \lim_{T\to\infty}\frac{1}{T}\int_0^T\left(1-e^{-t}\right)^2 dt = \lim_{T\to\infty}\frac{1}{T}\left[\int_0^T\left(1+e^{-2t}-2e^{-2t}\right)dt\right]$$

$$= \lim_{T\to\infty}\frac{1}{T}\left[T-\frac{1}{2}\left(e^{-2T}-1\right)+2\left(e^{-T}-1\right)\right]$$

$$= \lim_{T\to\infty}\left[1-\frac{1}{2}\frac{e^{-2T}}{T}+2\frac{e^{-T}}{T}-\frac{3}{2T}\right]$$

$$=1$$