EECE 340-Signals and Systems

Homework # 4

Problem 1

Consider the unstable system whose transfer function, $H(s)$, is given by:

$$H(s) = \frac{-1}{s(s+1)}, \quad -1 < \sigma < 0.$$

(a) Determine the impulse response of the system.

(b) Determine the differential equation of the system

(c) Determine the Direct form I and Direct form II block diagram representation of the system.

Problem 2

Repeat problem # 1 for the causal system with transfer funcion given by:

$$H(s) = \frac{6}{(s+2)(s+3)}$$

Problem 3

Repeat problem # 1 for the causal systems with transfer functions given by:

$$H(s) = \frac{2s+3}{(s+2)(s+3)}$$

and $$H(s) = \frac{2s+3}{(s+2)(s+3)(s-1)^2}$$

For question (a), use partial fraction expansion and also the Laplace inversion formula.

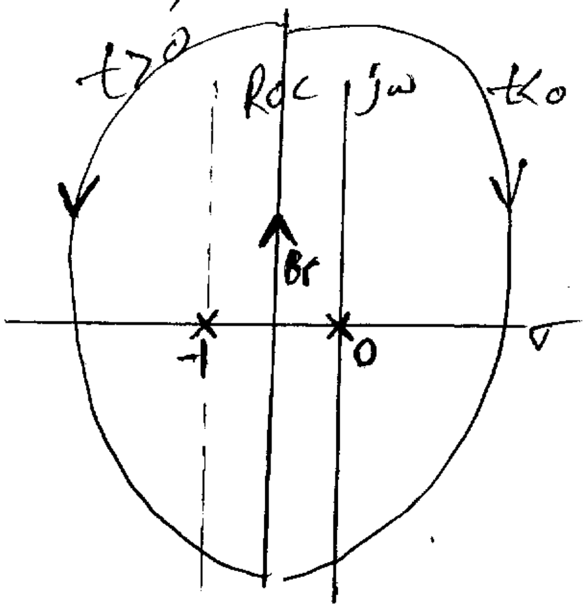
EECE340 Homework # 4 Solution

Problem # 1

$$H(s) = \frac{-1}{s(s+1)}, \quad -1 < \sigma < 0.$$

(a) Since $H(s)$ has a region of convergence represented by a strip in the s-plane not a right half of the s-plane, then the system is not causal. Hence, we determine $h(t)$ using the Laplace inversion formula.

$$h(t) = \frac{1}{2\pi j} \int_{Br} \frac{-e^{st}}{s(s+1)} \, ds$$



For $t > 0$, $h(t) = -\left.\frac{e^{st}}{s}\right|_{s=-1}$

$$= +e^{-t}$$

For $t < 0$, $h(t) = \left.\frac{e^{st}}{s+1}\right|_{s=0} = 1$

$$\Rightarrow h(t) = \begin{cases} e^{-t}, & t > 0 \\ 1, & t < 0 \end{cases}$$

(b) $H(s) = \frac{G(s)}{F(s)} = \frac{-1}{s^2 + s}$

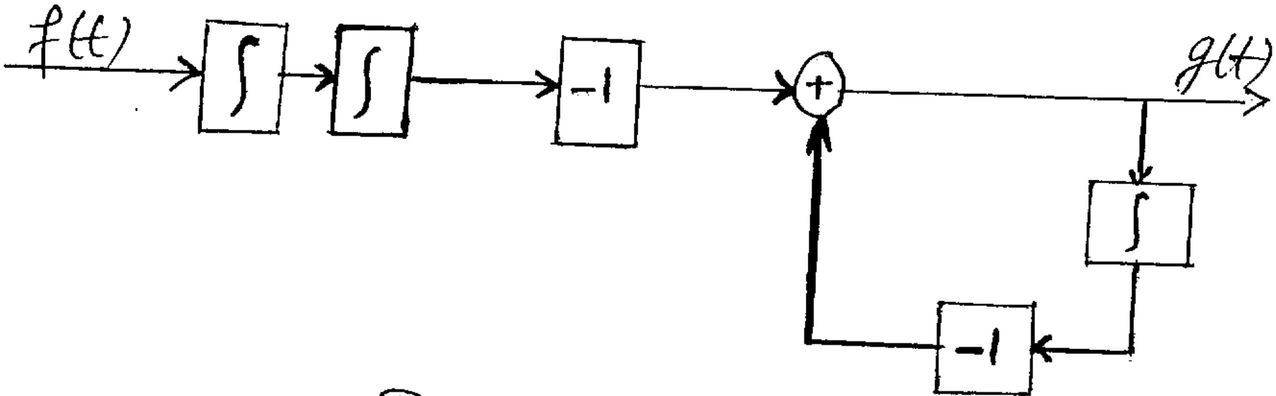
$$\Rightarrow G(s)\left[s^2 + s\right] = -F(s)$$

$$\Rightarrow d^2 g(t) + dg(t) = -f(t)$$

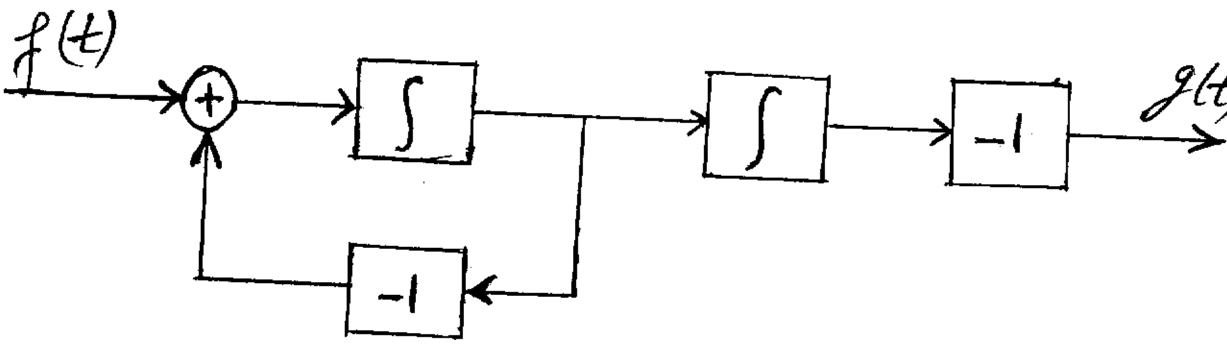
(C) $\dfrac{d^2 g(t)}{dt^2} + \dfrac{dg(t)}{dt} = -f(t)$

$\Rightarrow a_2 = 1,\ a_1 = 1,\ a_0 = 0$

$b_2 = 0,\ b_1 = 0,\ b_0 = -1$



Direct form I



Direct form II

Problem # 2

$$H(s) = \frac{6}{(s+2)(s+3)}$$

(a) $H(s) = \dfrac{A}{s+2} + \dfrac{B}{(s+3)} = \dfrac{6}{(s+2)} - \dfrac{6}{s+3}$

$\Rightarrow h(t) = 6\left(e^{-2t} - e^{-3t}\right)u(t).$

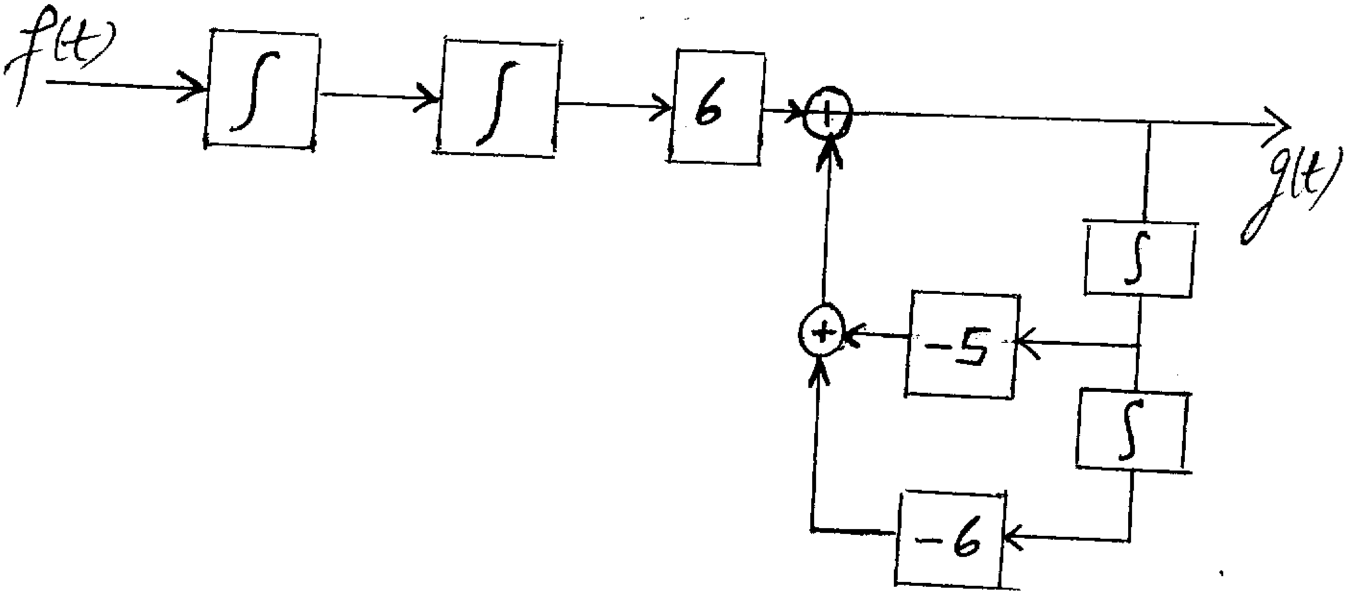
(b) $H(s) = \dfrac{G(s)}{F(s)} = \dfrac{6}{s^2+5s+6}$

$\Rightarrow s^2 G(s) + 5s\,G(s) + 6G(s) = 6F(s)$

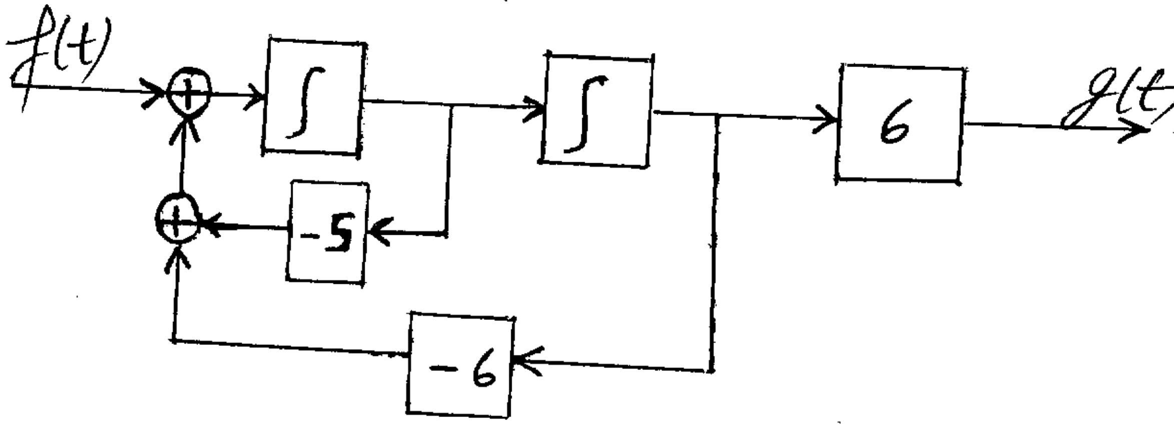
$\Rightarrow \dfrac{d^2g(t)}{dt^2} + 5\dfrac{dg(t)}{dt} + 6g(t) = 6f(t).$

(c) $a_2 = 1,\ a_1 = 5,\ a_0 = 6$

$b_2 = 0,\ b_1 = 0,\ b_0 = 6$



Direct form I



Direct form II

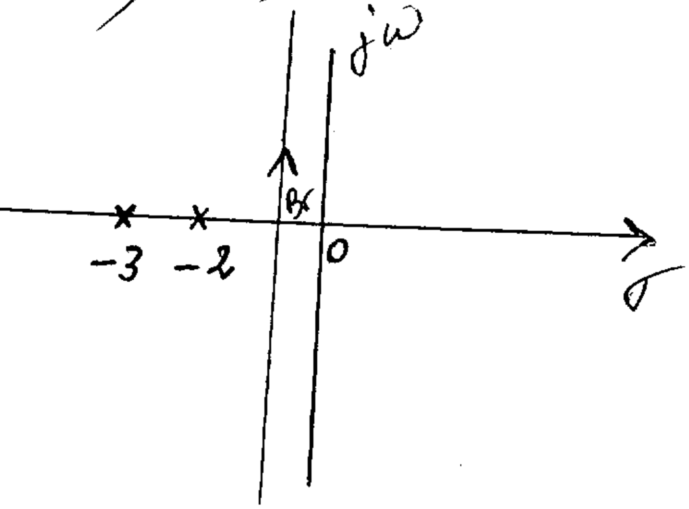
Problem # 3

a) $$H(s) = \frac{2s+3}{(s+2)(s+3)} = \frac{-1}{s+2} + \frac{3}{s+3}$$

$$\Rightarrow h(t) = \left(-e^{-2t} + 3e^{-3t}\right)u(t)$$

For $t>0$,

$h(t) = \sum$ residues at -3 and -2.



$$= \left.\frac{(2s+3)e^{st}}{(s+2)}\right|_{s=-3} + \left.\frac{(2s+3)e^{st}}{(s+3)}\right|_{s=-2}$$

$$= 3e^{-3t} - e^{-2t}.$$

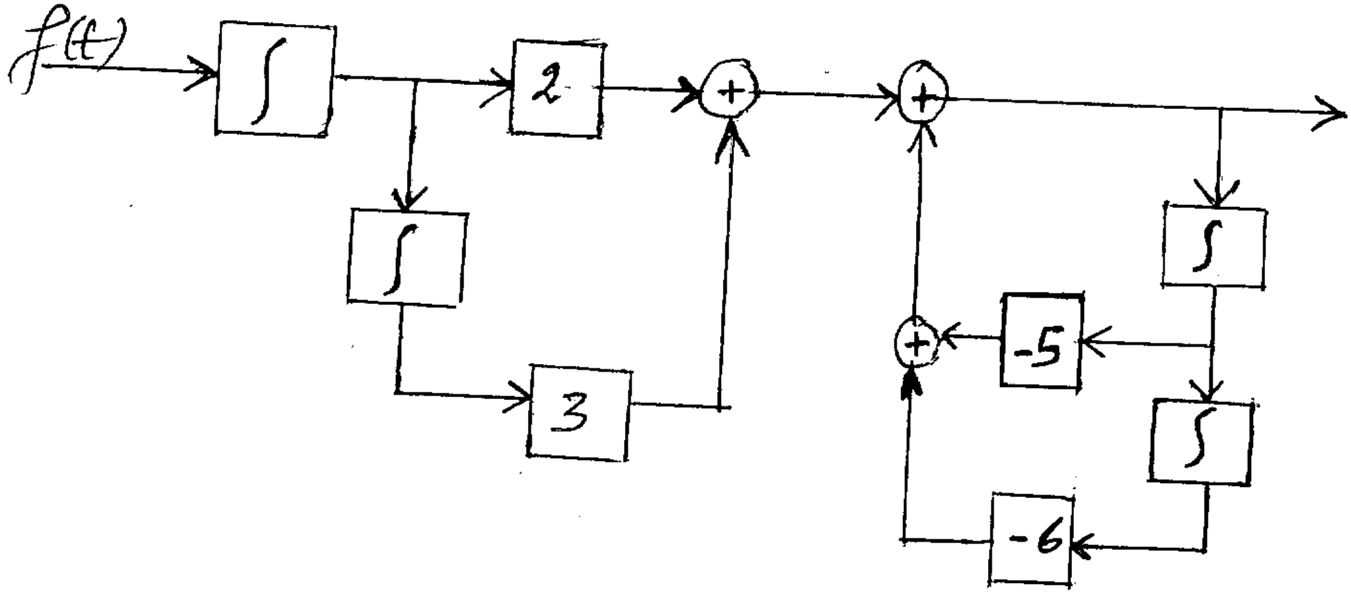
For $t<0$, $h(t) = 0$

$$\Rightarrow h(t) = \left(3e^{-3t} - e^{-2t}\right)u(t).$$

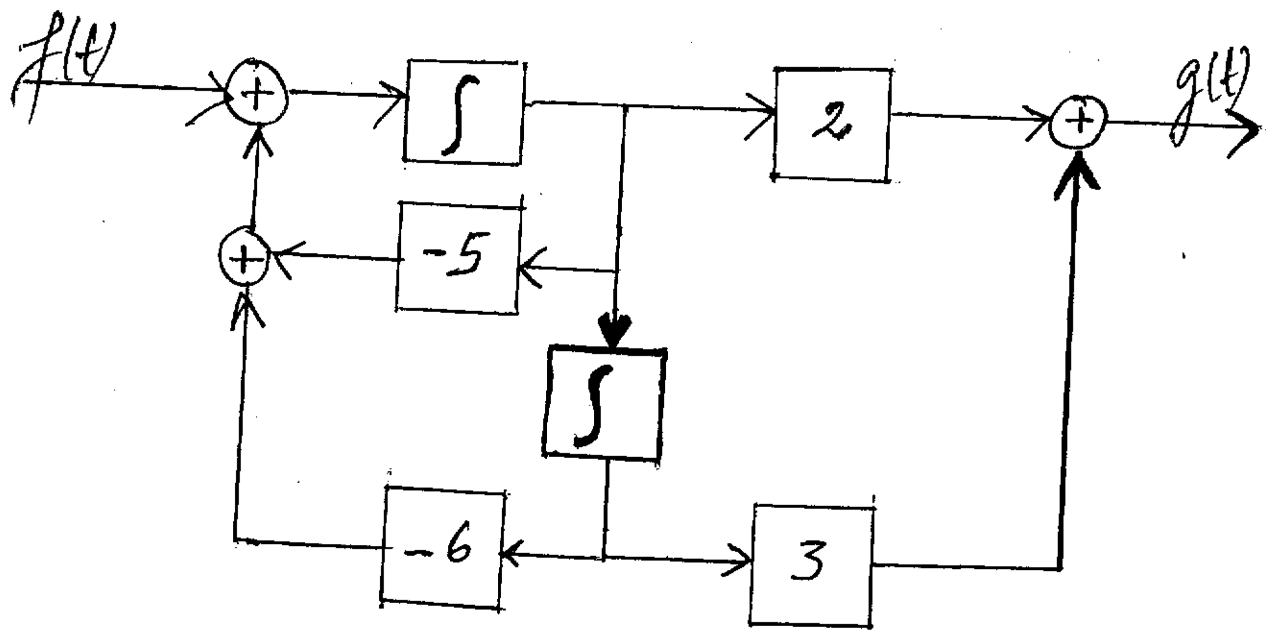
(b) $$H(s) = \frac{2s+3}{s^2+5s+6} = \frac{G(s)}{F(s)}$$

$$\Rightarrow \frac{d^2g(t)}{dt^2} + 5\frac{dg(t)}{dt} + 6g(t) = 2\frac{df(t)}{dt} + 3f(t)$$

(c) $a_2 = 1,\ a_1 = 5,\ a_0 = 6$
$b_2 = 0,\ b_1 = 2,\ b_0 = 3$



Direct form I



Direct form II

(a) $$H(s) = \frac{2s+3}{(s+2)(s+3)(s-1)^2}$$

$$= \frac{A}{(s+2)} + \frac{B}{(s+3)} + \frac{C}{(s-1)^2} + \frac{D}{(s-1)}$$

$$A = \left.\frac{2s+3}{(s+3)(s-1)^2}\right|_{s=-2} = -\frac{1}{9}$$

$$B = \left.\frac{2s+3}{(s+2)(s-1)^2}\right|_{s=-3} = \frac{3}{16}$$

$$C = \left.\frac{2s+3}{(s+2)(s+3)}\right|_{s=1} = \frac{5}{12}$$

$$D = \left.\frac{d}{ds}\left(\frac{2s+3}{(s+2)(s+3)}\right)\right|_{s=1}$$

$$= \left.\frac{2(s+2)(s+3)-(2s+5)(2s+3)}{\left[(s+2)(s+3)\right]^2}\right|_{s=1}$$

$$= \frac{-11}{144}$$

$$\Rightarrow h(t) = \left(-\frac{1}{9}e^{-2t} + \frac{3}{16}e^{-3t} + \frac{5}{12}te^{t} - \frac{11}{144}e^{t}\right)u(t)$$

Laplace inversion formula:

$$h(t) = \frac{1}{2\pi j}\int_{Br} \frac{(2s+3)\, e^{st}\, ds}{(s+2)(s+3)(s-1)^2}$$

$$= \sum \text{residue at } s = -2, -3, +1, \text{ for } t>0.$$

$$= \left.\frac{(2s+3)e^{st}}{(s+3)(s-1)^2}\right|_{s=-2} + \left.\frac{(2s+3)e^{st}}{(s+2)(s-1)^2}\right|_{s=-3}$$

$$+ \left.\left\{\frac{d}{ds}\left[\frac{(2s+3)\,e^{st}}{(s+2)(s+3)}\right]\right\}\right|_{s=1}$$

$$= \begin{cases} -\frac{1}{9}e^{-2t} + \frac{3}{16}e^{-3t} + \frac{5}{12}te^{t} - \frac{11}{144}e^{t}, & t>0 \\ 0, & t<0 \end{cases}$$

(b) $$H(s) = \frac{2s+3}{(s^2+5s+6)(s^2-2s+1)}$$

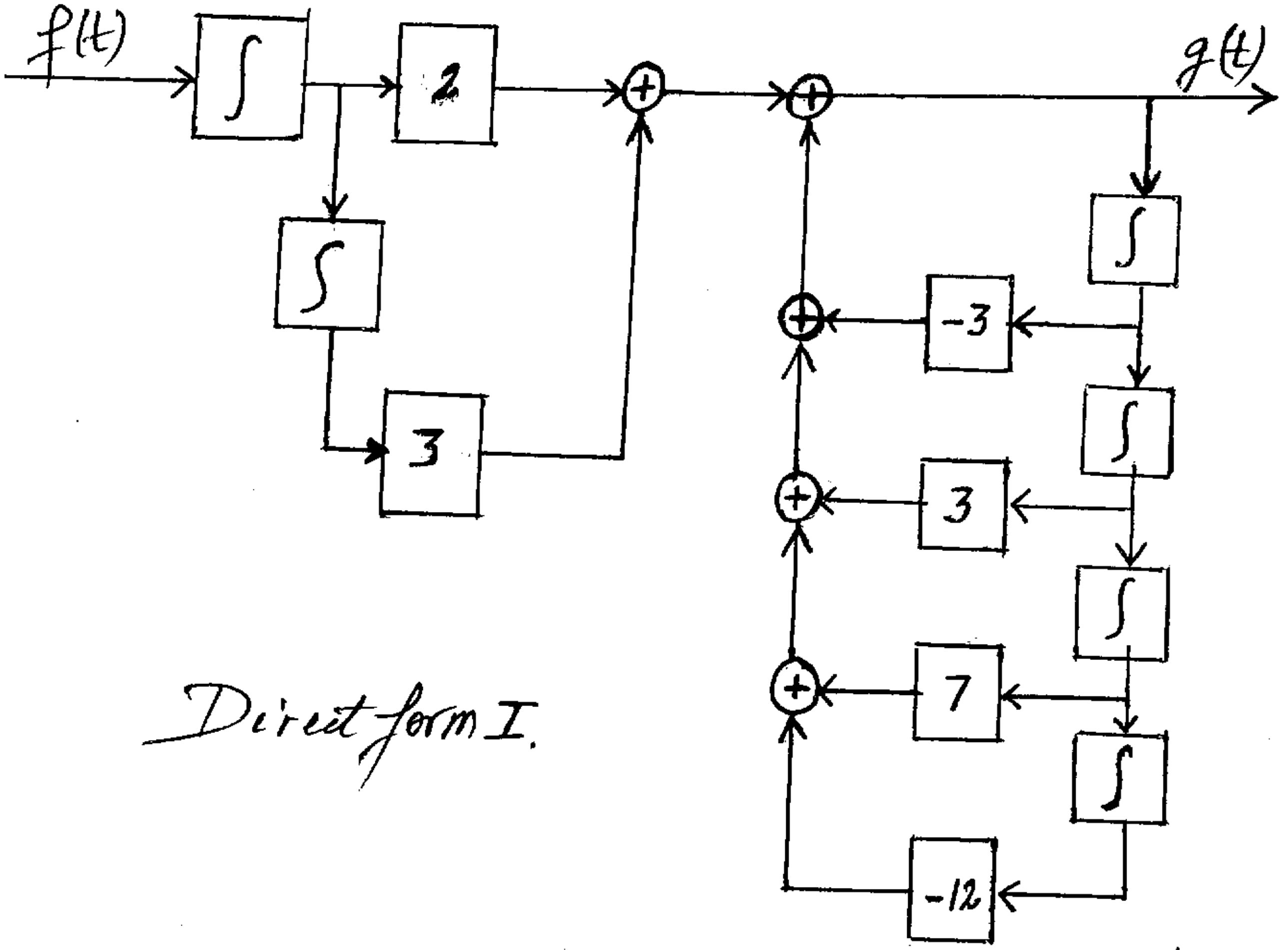
$$= \frac{2s+3}{s^4+3s-3s^2-7s+12} = \frac{G(s)}{F(s)}$$

$$\Rightarrow \frac{d^4g(t)}{dt^4} + 3\frac{d^3g(t)}{dt^3} - 3\frac{d^2g(t)}{dt^2} - 7\frac{dg(t)}{dt} + 12g(t)$$

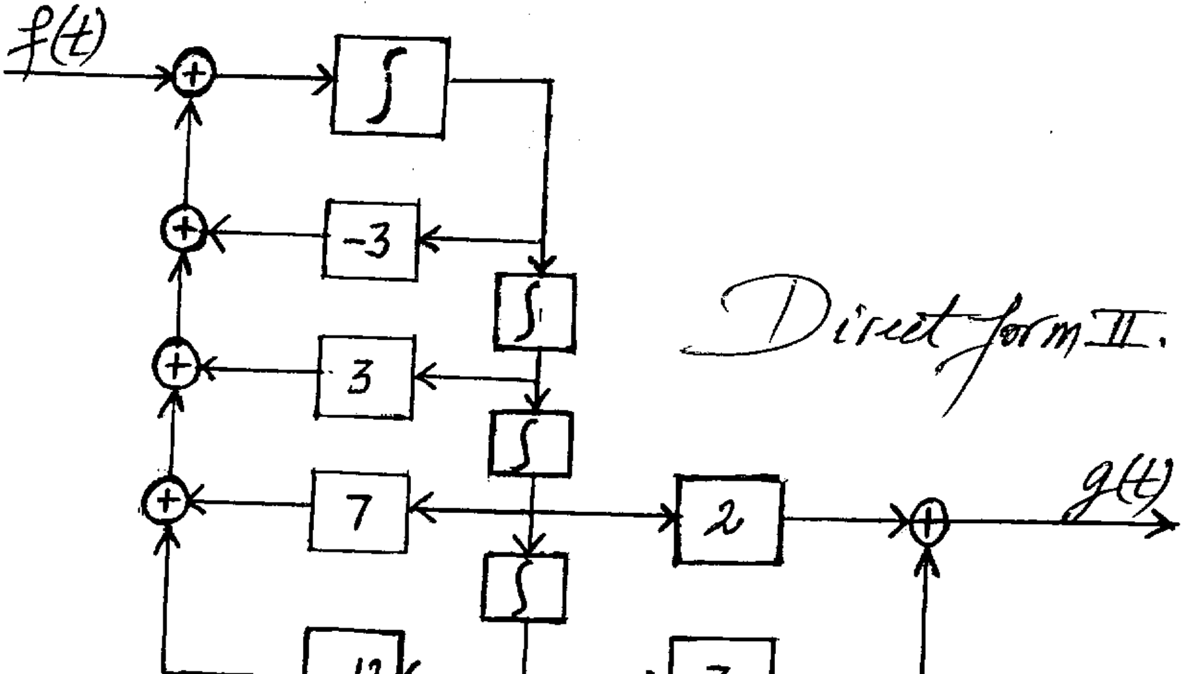
$$= 2\frac{df(t)}{dt} + 3f(t).$$

(c) $a_4 = 1$, $a_3 = 3$, $a_2 = -3$, $a_1 = -7$, $a_0 = 12$

$b_4 = 0$, $b_3 = 0$, $b_2 = 0$, $b_1 = 2$, $b_0 = 3$



Direct form I.



Direct form II.