EECE 340-Signals and Systems
Homework #5

Problem # 1

An analog system has the impulse response given by

$$h_c(t) = \begin{cases} e^{-t/\tau}, & t \geq 0 \\ 0, & t < 0 \end{cases}$$

Let a discrete-time system be obtained by sampling $h_c(t)$; i.e., having $h(n) = h_c(nT)$.
Find and plot $|H(\omega)|$.

Problem # 2

The impulse response of a discrete-time system is given by

$$h(n) = \begin{cases} 1, & 0 \leq n \leq N-1 \\ 0, & otherwise \end{cases}$$

Find and plot $|H(\omega)|$ and $\arg(H(\omega))$ for N=5.

Problem # 3

Find and sketch the output $y(n)$ for an LTI discrete-time system with input $x(n) = u(n) - u(n-6)$ and impulse response: $h(n) = a^n u(n)$.

Problem # 4

Let $h(n) = a^n u(n)$

$x(n) = b^n u(n)$

Find the output $y(n) = x(n) * h(n)$ for $a \neq b$. Repeat for $a = b$.

Problem # 5

The impulse responses of two discrete-time LTI systems are given by

$$h_1(n) = u(n) - u(n-6)$$
$$h_2(n) = u(n+4) - u(n)$$

a) Find the impulse response $h(n)$ for the parallel combination of $h_1(n)$ and $h_2(n)$.
b) Find the impulse response $h(n)$ for the cascaded combination of $h_1(n)$ and $h_2(n)$.

Problem # 6

Repeat Parts (a) and (b) in Problem 5 for the case where the systems impulse responses are given by

$$h_1(n) = (ja)^n u(n)$$
$$h_2(n) = (-ja)^n u(n)$$

Problem # 7

The unit step response for an LTI discrete-time system is given by

$$s(n) = u(n) * h(n)$$

(a) Express *s(n)* for a causal system as a summation of *h(n)*.
(b) Give the inverse relationship for *h(n)* in terms of *s(n)*.
(c) Find *s(n)* if $h(n) = u(n) - u(n-6)$.
(d) Find and sketch *s(n)* when $h(n) = (-\frac{1}{2})^n u(n)$. Find $\lim_{n \to \infty} s(n)$.

Problem # 8

Determine the conditions on the parameters of the following systems for stability:

a) $h(n) = a^n u(-n)$
b) $h(n) = a^n [u(n) - u(n-100)]$
c) $h(n) = r^n \sin(n\omega_0 T) u(n)$
d) $h(n) = a^{|n|}$

EECE 340 Homework 5 Solution

1) $h(n) = \begin{cases} e^{-nT/2} & , n \geq 0 \\ 0 & ; n < 0 \end{cases}$

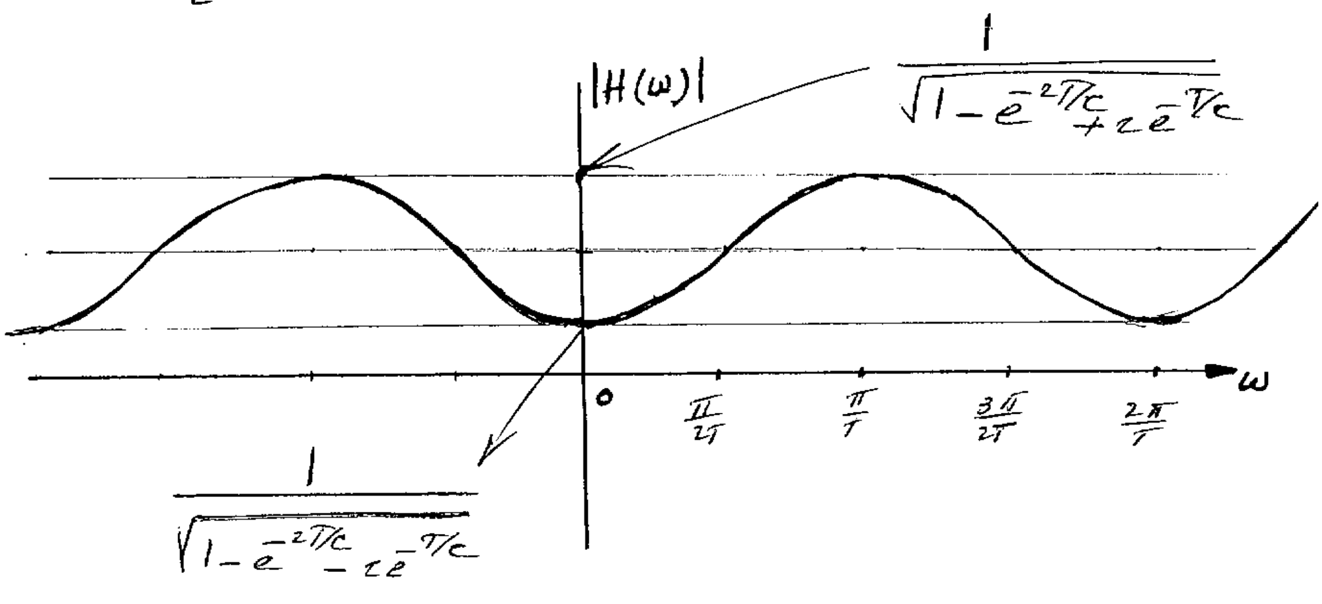
$$H(\omega) = \sum_{n=0}^{\infty} e^{-nT/2} e^{-j\omega n T} = \sum_{n=0}^{\infty} e^{-(T/2 + j\omega T)n}$$

$$= \frac{1}{1 - e^{-T/2} e^{-j\omega T}} \quad , \text{with } T/2 > 0$$

$$= \frac{1}{1 - e^{-T/2}(\cos\omega T - j\sin\omega T)} = \frac{1}{1 - e^{-T/2}\cos\omega T + j e^{-T/2}\sin}$$

$$|H(\omega)| = \frac{1}{\left[(1 - e^{-T/2}\cos\omega T)^2 + (e^{-T/2}\sin\omega T)^2\right]^{1/2}}$$

$$= \frac{1}{\left[1 - e^{-2T/2} - 2e^{-T/2}\cos\omega T\right]^{1/2}}$$

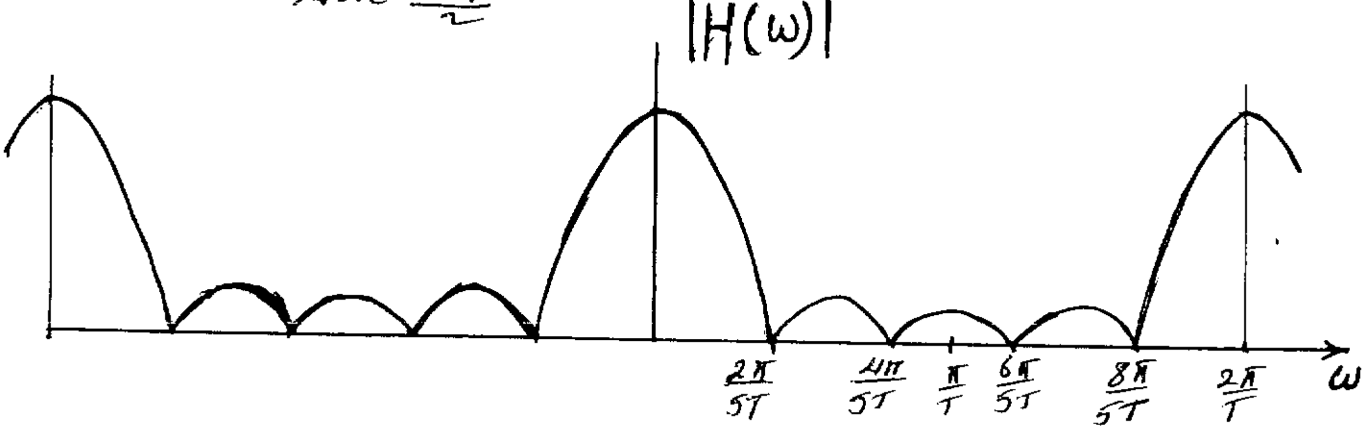


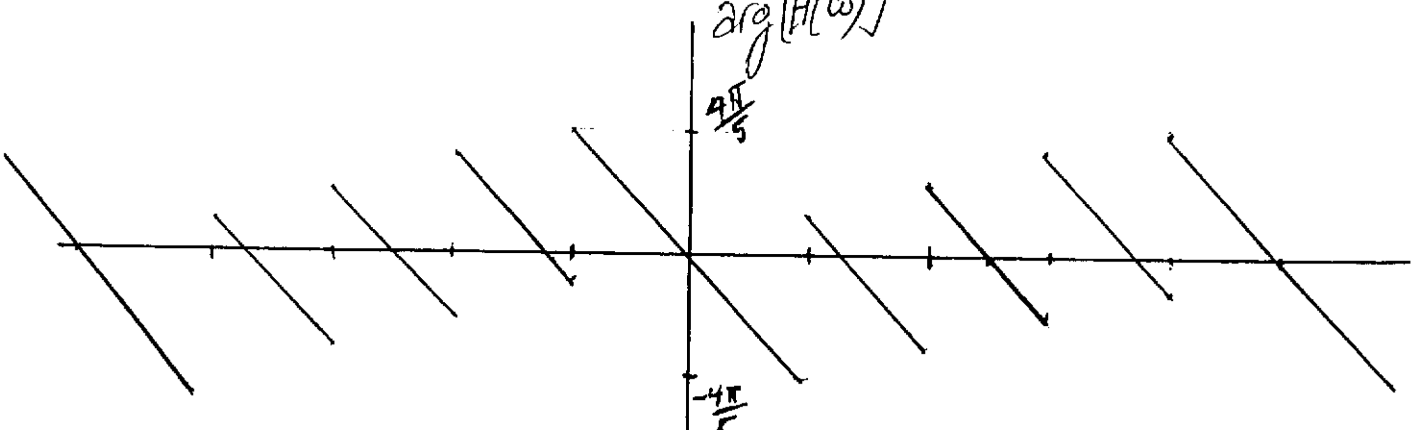
2) $h(n) = \begin{cases} 1, & 0 \le n \le N-1 \\ 0, & \text{otherwise} \end{cases}$

$$H(\omega) = \sum_{n=0}^{N-1} e^{-j\omega n T} = \sum_{n=0}^{N-1} \left(e^{-j\omega T}\right)^n$$

$$= \frac{1 - e^{-j\omega N T}}{1 - e^{-j\omega T}} = \frac{e^{-j\frac{\omega N T}{2}}\left(e^{j\frac{\omega N T}{2}} - e^{-j\frac{\omega N T}{2}}\right)}{e^{-j\frac{\omega T}{2}}\left(e^{j\frac{\omega T}{2}} - e^{-j\omega T/2}\right)}$$

$$= \frac{\sin \frac{\omega N T}{2}}{\sin \frac{\omega T}{2}} \, e^{-j\frac{\omega T}{2}(N-1)}$$





3)

$$x(n) = u(n) - u(n-6)$$

$$h(n) = a^n u(n)$$

$$n < 0 \,, \quad y(n) = 0$$

$$0 \le n < 5 \,, \quad y(n) = \sum_{k=0}^{n} a^{n-k} = a^n \sum_{k=0}^{n} a^{-k}$$

$$= \frac{1 - a^{n+1}}{1 - a}$$

$$n \geq 5 \,, \quad y(n) = \sum_{k=0}^{5} a^{n-k} = a^n \sum_{k=0}^{5} a^{-k}$$

$$= \frac{a^6 - 1}{a - 1} \, a^{n-5}$$

4)

$$y(n) = \sum_{k=0}^{n} b^k a^{n-k} = a^n \sum_{k=0}^{n} b^k a^{-k}$$

$$= a^n \sum_{k=0}^{n} \left(\frac{b}{a}\right)^k = a^n \left(\frac{1 - \left(\frac{b}{a}\right)^{n+1}}{1 - \frac{b}{a}} \right),$$

$$= \frac{a^{n+1} - b^{n+1}}{a - b} \,, \quad n \geq 0 \,, \quad a \neq b \,.$$

$$y(n) = 0, \quad n < 0 \,.$$

For $a = b$, $y(n) = a^n \sum_{k=0}^{n} 1 = (n+1)a^n, \quad n \geq 0$

$$y(n) = 0, \quad n < 0.$$

5) $h_1(n) = u(n) - u(n-6)$

$h_2(n) = u(n+4) - u(n).$

a) $h(n) = h_1(n) + h_2(n)$

$$= \begin{cases} 1 & , -4 \leq n \leq 5 \\ 0 & , \text{else} \end{cases}$$

b) $h(n) = 0, \quad n < -4$

$$h(n) = \sum_{k=-4}^{n} 1 = n + 5, \quad -4 \leq n \leq -1$$

$$h(n) = \sum_{k=-4}^{-1} 1 = 4, \quad 0 \leq n \leq 1$$

$$h(n) = \sum_{k=n-5}^{-1} 1 = -n + 5, \quad 1 < n \leq 4$$

$$h(n) = 0, \quad n > 4$$

6) $h_1(n) = (ja)^n u(n)$
$h_2(n) = (-ja)^n u(n)$

a) If $n = 2k$, $k = 0, 1, 2, \ldots$
then $(ja)^n = (ja)^{2k} = (-1)^k a^{2k}$
$(-ja)^n = (-ja)^{2k} = (-1)^k a^{2k}$

$\Rightarrow h(n) = h_1(n) + h_2(n)$
$= 2a^{2k}(-1)^k = 2a^n(-1)^{n/2}$, n is even

If $n = 2k+1$, $k = 0, 1, 2, \ldots$
then $(ja)^n = (ja)^{2k+1} = j(-1)^k a^{2k+1}$
$(-ja)^n = (-ja)^{2k+1} = -j(-1)^k a^{2k+1}$
$\Rightarrow h(n) = h_1(n) + h_2(n) = 0$, n is odd.

Thus, $h(n) = \begin{cases} 2(-1)^{n/2} a^n, & n \text{ is even} \\ 0, & n \text{ is odd.} \end{cases}$

b) $$y(n) = \sum_{k=0}^{n} (ja)^{n-k} (-ja)^{k}$$

$$= (ja)^{n} \sum_{k=0}^{n} (ja)^{-k} (-ja)^{k}$$

$$= (ja)^{n} \sum_{k=0}^{n} \left(\frac{1}{j}\right)^{2k}$$

$$= (ja)^{n} \sum_{k=0}^{n} (-1)^{k} = \frac{(ja)^{n}}{2}\left(1-(-1)^{n+1}\right)$$

$$= \begin{cases} (-1)^{n/2} a^{n} & , n \text{ is even} \\ 0 & , n \text{ is odd.} \end{cases}$$

7) $$s(n) = u(n) * h(n)$$

a) $h(n) = 0$, $n<0$

$$s(n) = \sum_{k=0}^{n} u(k) h(n-k) = \sum_{k=0}^{n} h(n-k)$$

$$= \sum_{k=0}^{n} h(k)$$

b) $s(0) = h(0)$

$s(1) = h(0) + h(1)$

$\vdots$

$s(n-1) = h(0) + h(1) + \cdots + h(n-1)$

$s(n) = h(0) + h(1) + \cdots + h(n-1) + h(n)$.

$\Rightarrow$

$h(n) = s(n) - s(n-1)$.

c) $h(n) = u(n) - u(n-6)$

$$s(n) = \sum_{k=0}^{n} 1 = n+1, \quad 0 \leq n < 5$$

$$s(n) = \sum_{k=0}^{5} 1 = 6, \quad n \geq 5$$

d) $h(n) = \left(-\frac{1}{2}\right)^n u(n)$.

$$s(n) = \sum_{k=0}^{n} \left(-\frac{1}{2}\right)^k = \frac{1 - \left(-\frac{1}{2}\right)^{n+1}}{1 + \frac{1}{2}}$$

$$\underset{n \to}{\longrightarrow} \frac{2}{3}$$

8)

a) $h(n) = a^n u(-n)$

$$\sum_{n=-\infty}^{0} |a|^n = \sum_{n=0}^{\infty} |a|^{-n} = \sum_{n=0}^{\infty} \left(\frac{1}{|a|}\right)^n$$

$$= \frac{1}{1 - \frac{1}{|a|}}, \quad |a| > 1$$

b) $h(n) = a^n [u(n) - u(n-100)]$

Always stable

c) $h(n) = r^n \sin(n\omega_0 T) u(n)$

$$\sum_{n=0}^{\infty} |r|^n = \frac{1}{1-|r|}, \quad |r| < 1$$

d) $h(n) = a^{|n|} = \begin{cases} a^n, & n \geq 0 \\ a^{-n}, & n < 0 \end{cases}$

$$\sum_{n=-\infty}^{\infty} |h(n)| = \sum_{n=-\infty}^{-1} |a|^{-n} + \sum_{n=0}^{\infty} |a|^{n}$$

$$= \sum_{n=0}^{\infty} |a|^{n} + \sum_{n=0}^{\infty} |a|^{n} - 1$$

goes to a finite sum if $|a| < 1$