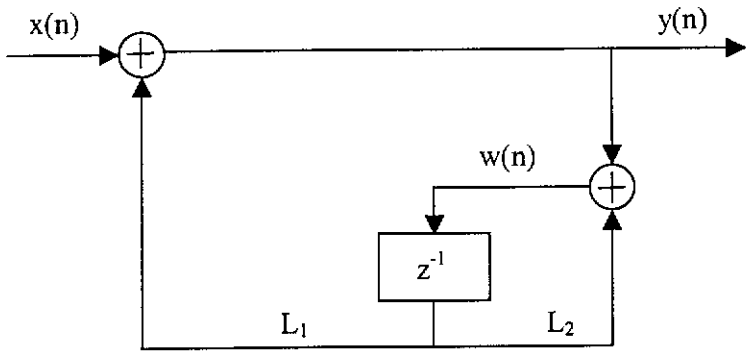
EECE 340-Signals and Systems
Homework #9

Problem # 1

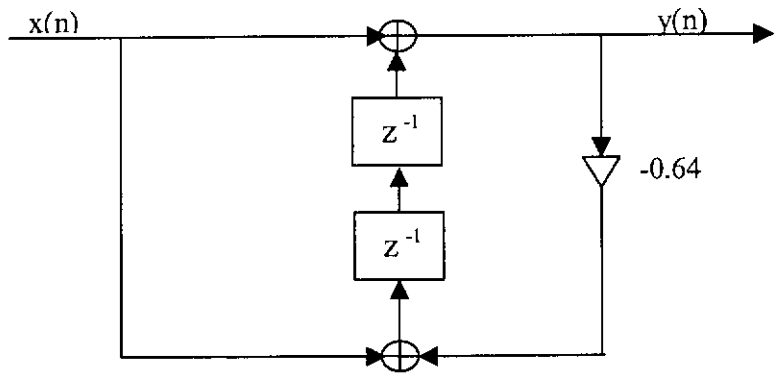
The block diagram of a discrete-time system is shown in the figure below.



a) Write the difference equations relating $x(n)$, $w(n)$ and $y(n)$.
b) Find the transfer function, H(z), of the system by applying the Z-transform to the equations determined in Part (a).
c) Plot the signal flow graph of the above block diagram.

Problem # 2

The block diagram of a discrete-time system is shown in the figure below.



a) Determine the transfer function, H (z), of the system.
b) Determine the magnitude frequency responsre, $|H(\omega)|$, of the system using the pole-zero plot of H (z).

Problem # 3

Consider an LTI discrete-time system with transfer function given by:

$$H(Z) = \frac{1 - 2\cos\theta z^{-1} + z^{-2}}{1 - 2r\cos\theta z^{-1} + r^2 z^{-2}}$$

Draw the signal flow graph in direct form (I), direct form (II) and transpose of direct form (II).

Problem # 4

Consider the following difference equation:

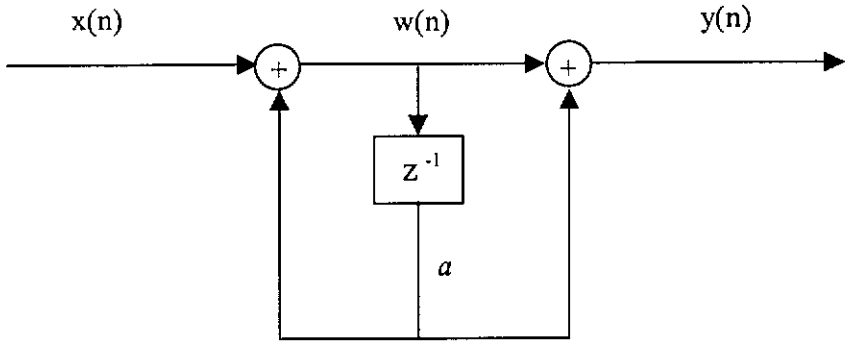
$$y(n) = x(n) + \frac{1}{3}x(n-1) + \frac{3}{4}y(n-1) + \frac{1}{8}y(n-2)$$

Draw the signal flow graph of the above difference equation in:

a) Direct form I
b) Direct form II
c) Transpose of direct form I
d) Transpose of direct form II

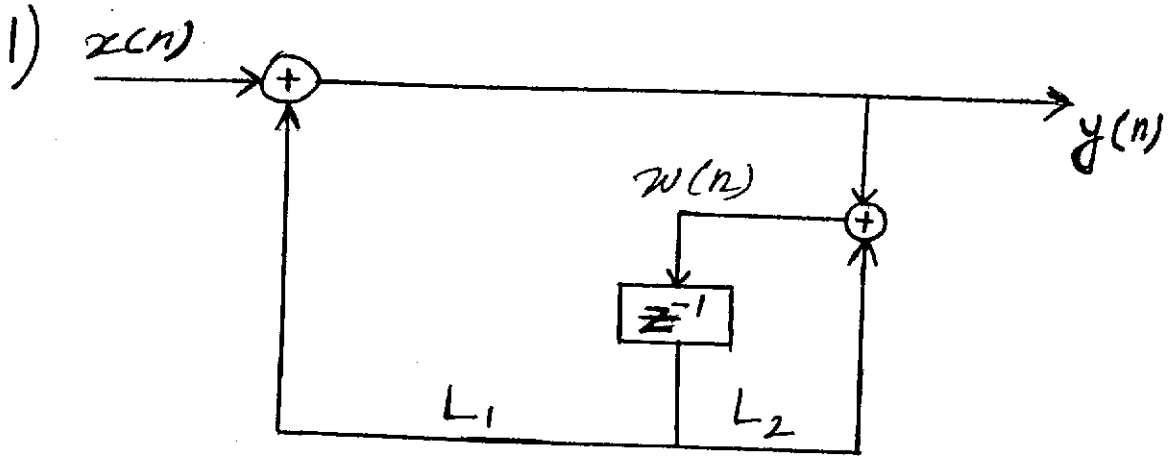
Problem # 5

Consider an LTI discrete-time system represented by the block diagram shown below.



Determine the transfer function, H(z), and the unit sample (impulse) response of the system.

EECE 340 Homework #9 Solution

1) 

a) $y(n) = x(n) + L_1 w(n-1)$

$w(n) = y(n) + L_2 w(n-1)$

b) $Y(z) = X(z) + L_1 z^{-1} W(z)$ ①

$W(z) = Y(z) + L_2 z^{-1} W(z)$ ②

$\Rightarrow$

$W(z)[1 - L_2 z^{-1}] = Y(z)$

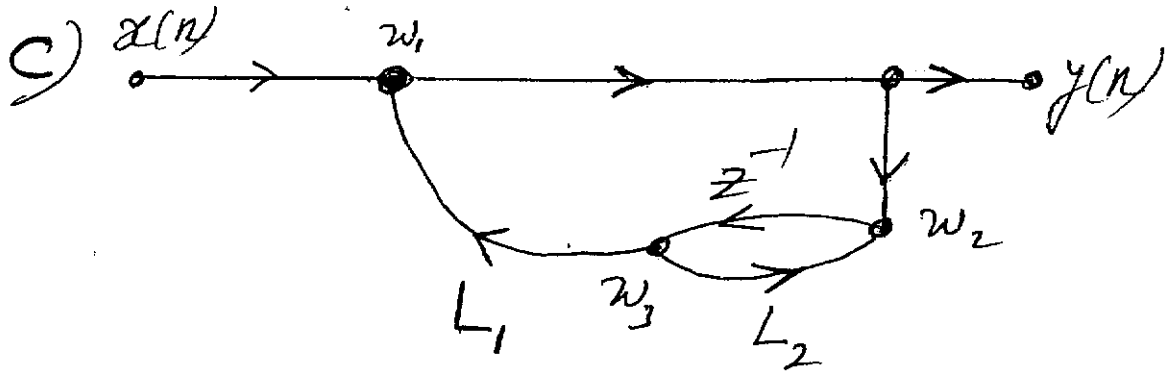
or $W(z) = \dfrac{Y(z)}{1 - L_2 z^{-1}}$

replace in ① $\Rightarrow$

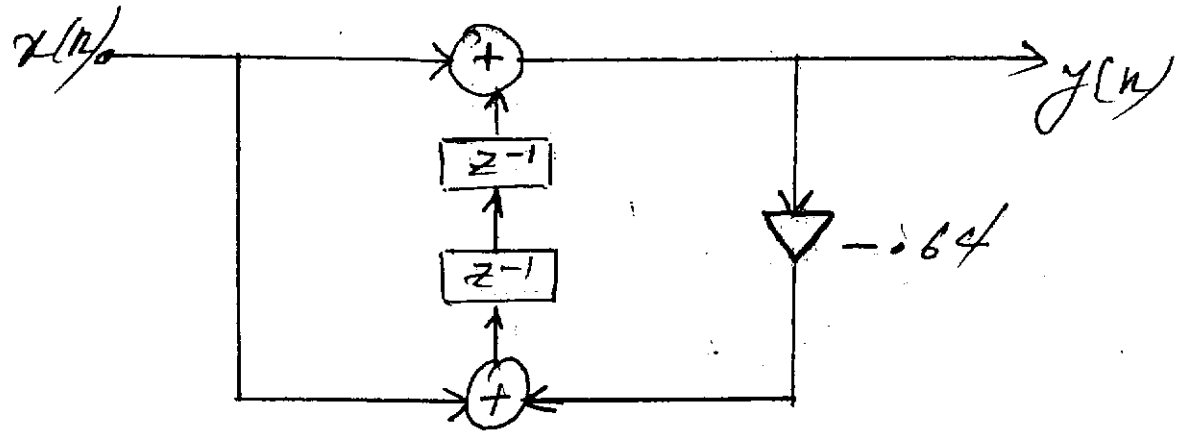
$$Y(z)\left[1 - \frac{L_1 z^{-1}}{1 - L_2 z^{-1}}\right] = X(z)$$

$$\Rightarrow H(z) = \frac{Y(z)}{X(z)} = \frac{1 - L_2 z^{-1}}{1 - (L_1 + L_2) z^{-1}}$$

c)



2)

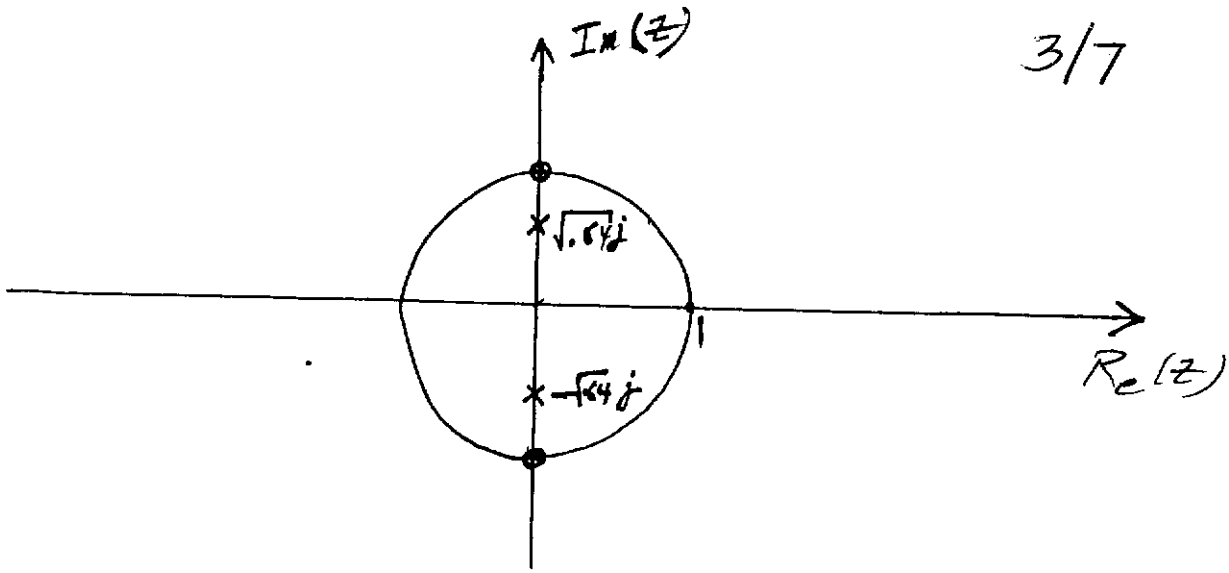


$$y(n) = x(n) - .64\,y(n-2) + x(n-2)$$

$$Y(z) = X(z) - .64\,z^{-2}Y(z) + z^{-2}X(z)$$

$$Y(z)\left[1 + .64\,z^{-2}\right] = X(z)\left[1 + z^{-2}\right]$$

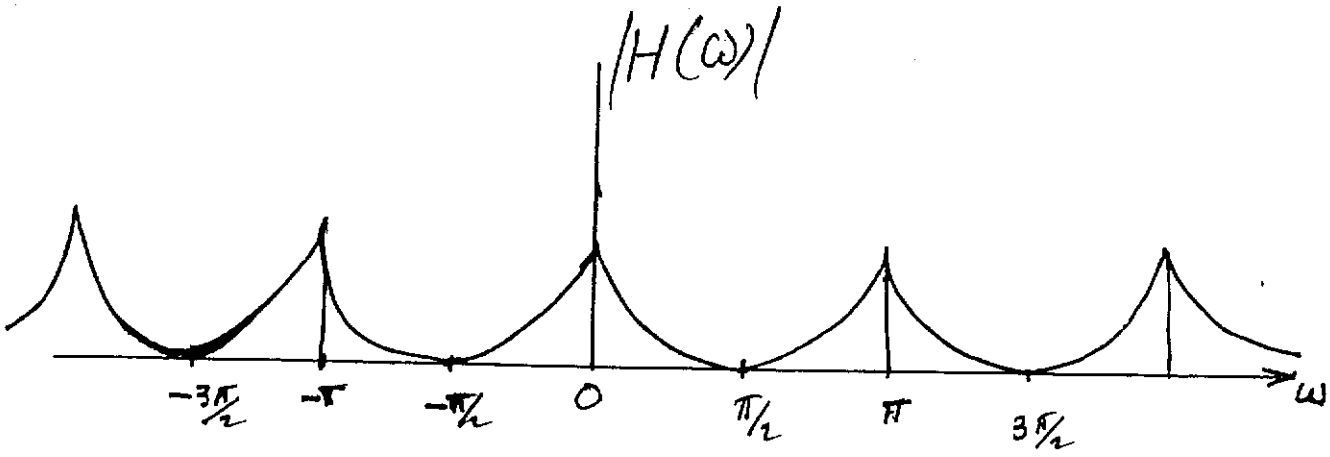
$$\Rightarrow H(z) = \frac{1 + z^{-2}}{1 + .64\,z^{-2}} = \frac{z^2 + 1}{z^2 + .64}$$



z-plane

$z^2+1=0 \Rightarrow z=\pm j$ (zeros)

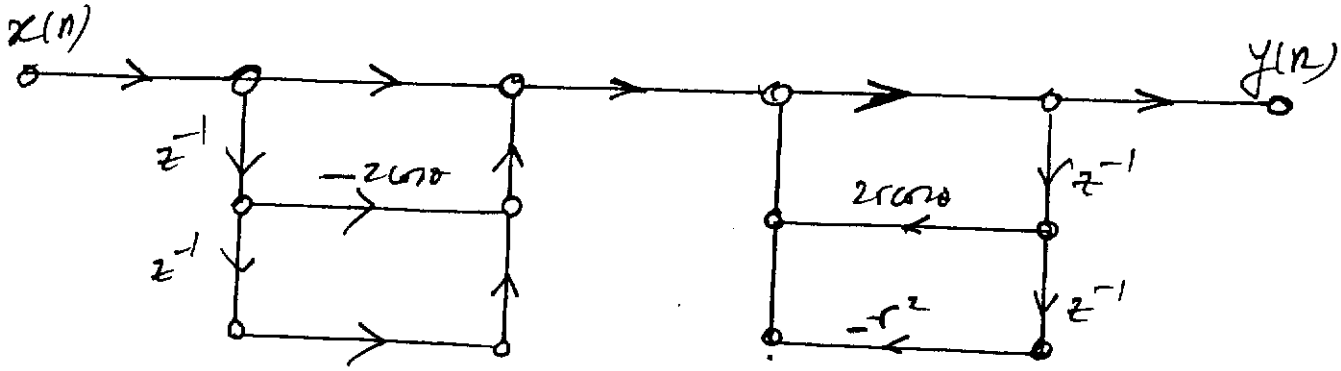
$z^2+.64=0 \Rightarrow z=\pm\sqrt{.64}\,j$ (poles).



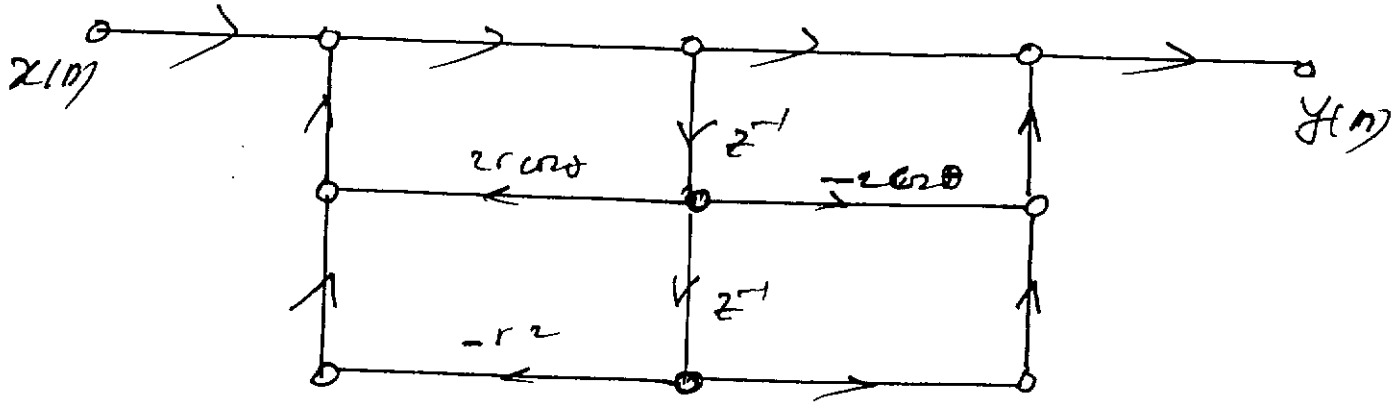
3) $$H(z) = \frac{1 - 2\cos\theta\, z^{-1} + z^{-2}}{1 - 2r\cos\theta\, z^{-1} + r^2 z^{-2}}$$

$$Y(z)\left[1 - 2r\cos\theta\, z^{-1} + r^2 z^{-2}\right] = X(z)\left[1 - 2\cos\theta\, z^{-1} + z^{-2}\right]$$

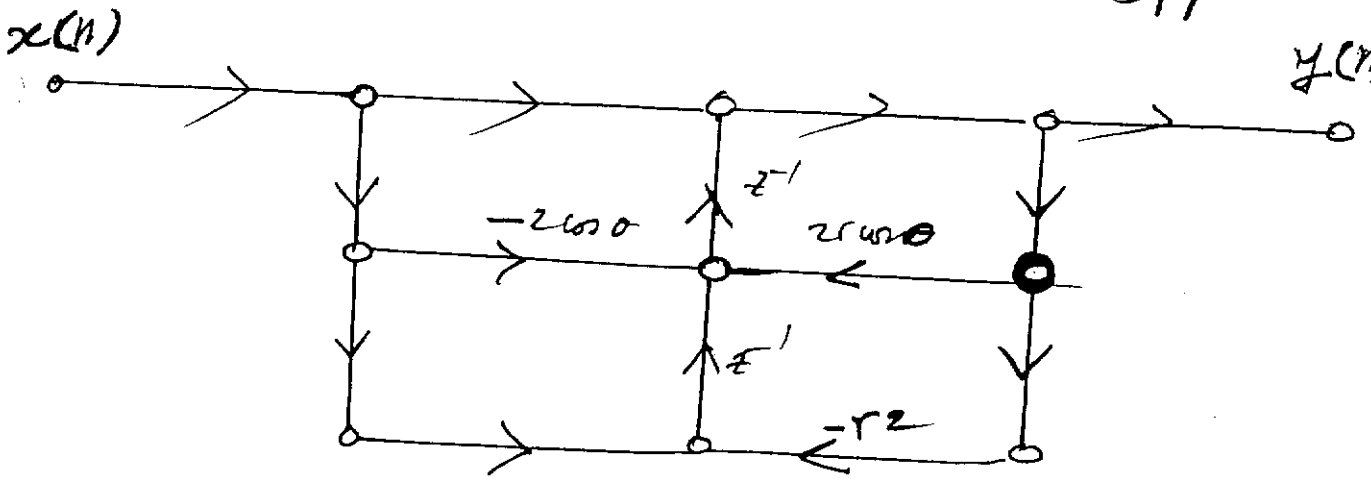
$$\Rightarrow y(n) = x(n) - 2\cos\theta\, x(n-1) + x(n-2) + 2r\cos\theta\, y(n-1) - r^2 y(n-2).$$



Direct form (I).



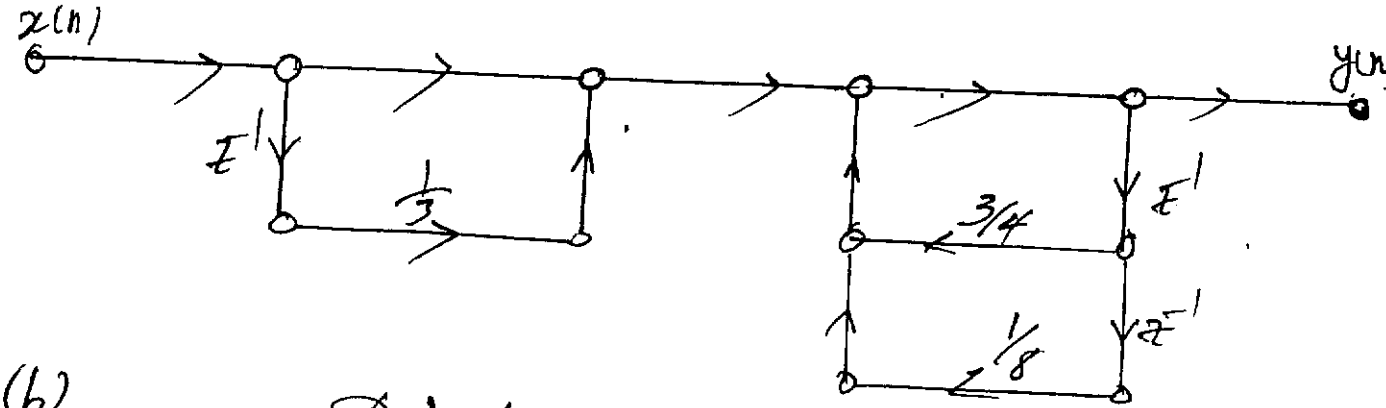
Direct form (II).



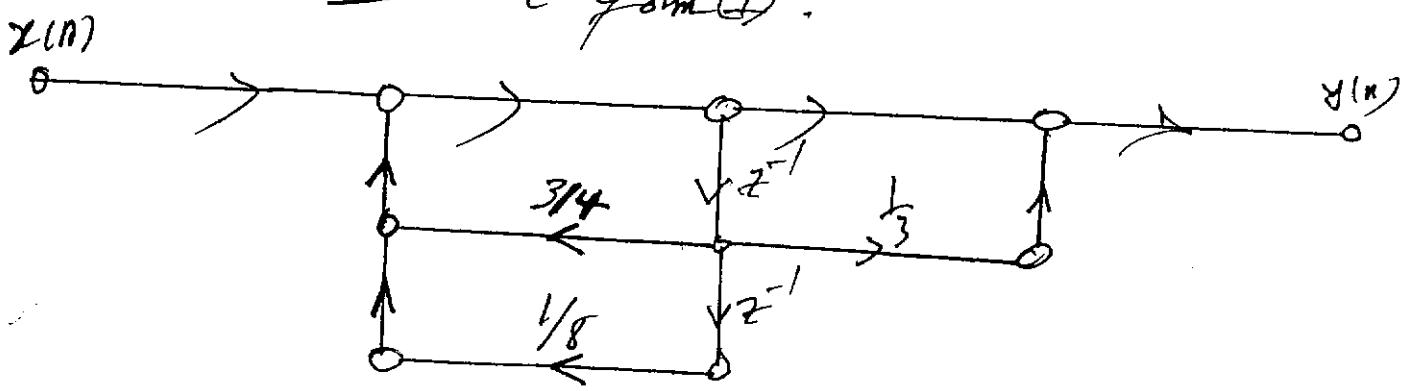
Transpose of direct form (II).

4) $y(n) = x(n) + \frac{1}{3}x(n-1) + \frac{3}{4}y(n-1) + \frac{1}{8}y(n-2)$.

(a)

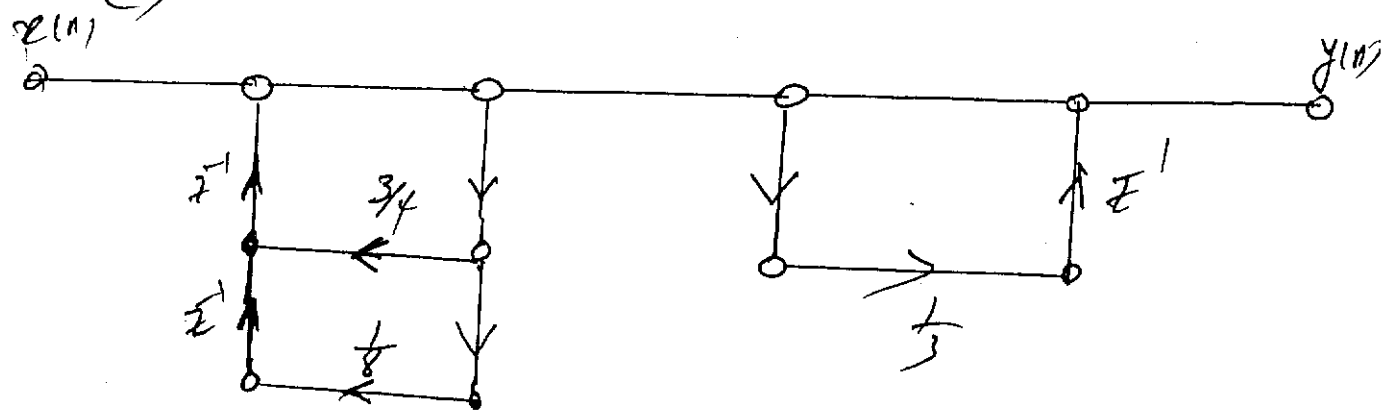


(b) Direct form (I).



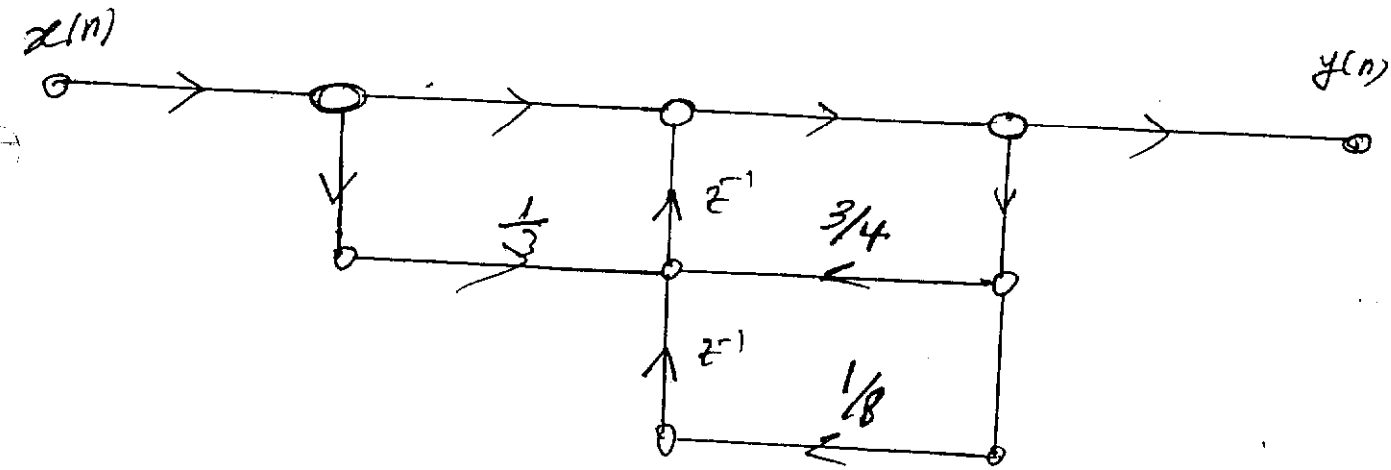
Direct form (II).

(c)



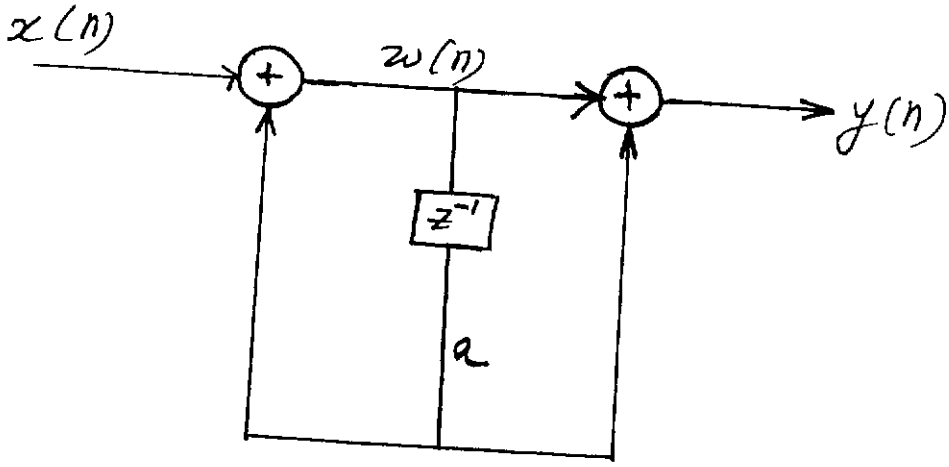
Transpose of Direct form (I).

(d)



Transpose of Direct form (I).

5)



$$y(n) = w(n) + a w(n-1)$$
$$w(n) = x(n) + a w(n-1)$$

$$Y(z) = W(z) + a z^{-1} W(z) \Rightarrow W(z)[1 + a z^{-1}] = Y(z)$$
$$W(z) = X(z) + a z^{-1} W(z) \Rightarrow W(z)[1 - a z^{-1}] = X(z)$$

$$H(z) = \frac{1 + a z^{-1}}{1 - a z^{-1}} = \frac{z + a}{z - a}$$
$$= \frac{1}{1 - a z^{-1}} + \frac{a z^{-1}}{1 - a z^{-1}}$$

$$h(n) = a^n u(n) + a a^{n-1} u(n-1)$$
$$= a^n [u(n) + u(n-1)] = \begin{cases} 1, & n = 0 \\ 2a^n, & n \geq 1 \end{cases}$$