EECE 340-Signals and Systems
Homework #10

Problem # 1

Determine the expressions of the discrete Fourier transforms of the following sequences assumed to be of length N.

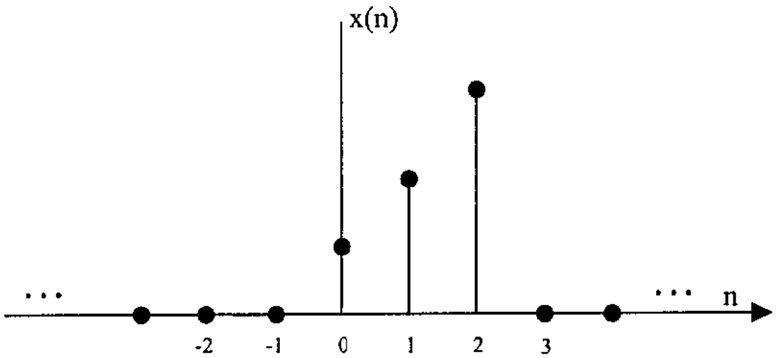
a) $x(n) = \delta(n)$

b) $x(n) = \delta(n - n_0)$

c) $x(n) = \begin{Bmatrix} a^n, & 0 \le n \le N-1 \\ 0, & otherwise \end{Bmatrix}$

Problem # 2

Let $x(n)$ be the discrete sequence shown below.



Plot $x((-n))_4$ which denotes the periodic repetition of $x(-n)$ with period equal to 4.

Problem # 3

Verify that $x((N-n))_N = x((-n))_N$, with $x((-n))_N$ being the periodic repetition of $x(-n)$ with period equal to N.

Problem # 4

Consider two finite duration sequences $x(n)$ and $y(n)$ where both are zero for $n<0$ and with $x(n)=0,\ n \ge 8$, $y(n) = 0, n \ge 20$. The 20 point DFT's of each of the sequences are multiplied and the inverse DFT is computed. Let r(n) denote the inverse DFT. Specify which points in r(n) correspond to points that would be obtained in a linear convolution of $x(n)$ and $y(n)$.

EECE340 Homework #10 Solution

1)

a) $x(n) = \delta(n)$

$$X(k) = \sum_{n=0}^{N-1} x(n) W_N^{kn} = 1, \quad k = 0, 1, 2, \ldots, N-1$$

b) $x(n) = \delta(n - n_0)$

$$X(k) = \sum_{n=0}^{N-1} \delta(n - n_0) W_N^{kn} = W_N^{kn_0}, \quad k = 0, 1, 2, \ldots, N-1$$

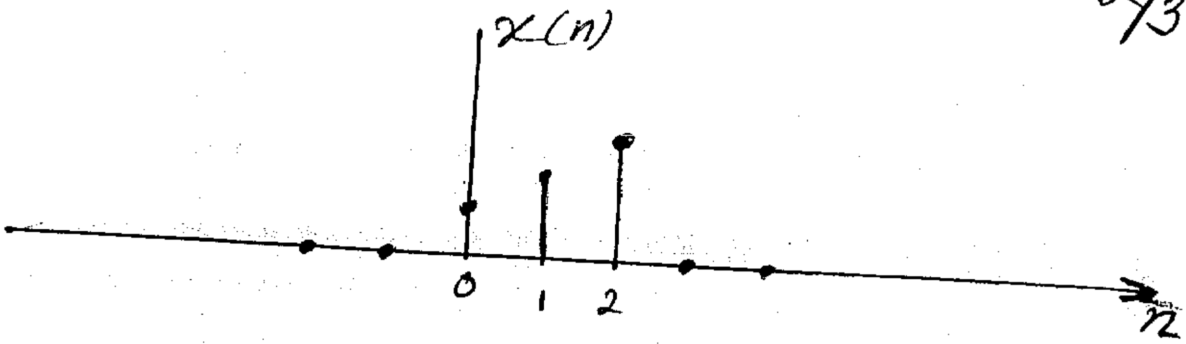
c) $x(n) = a^n, \quad 0 \leq n \leq N-1$

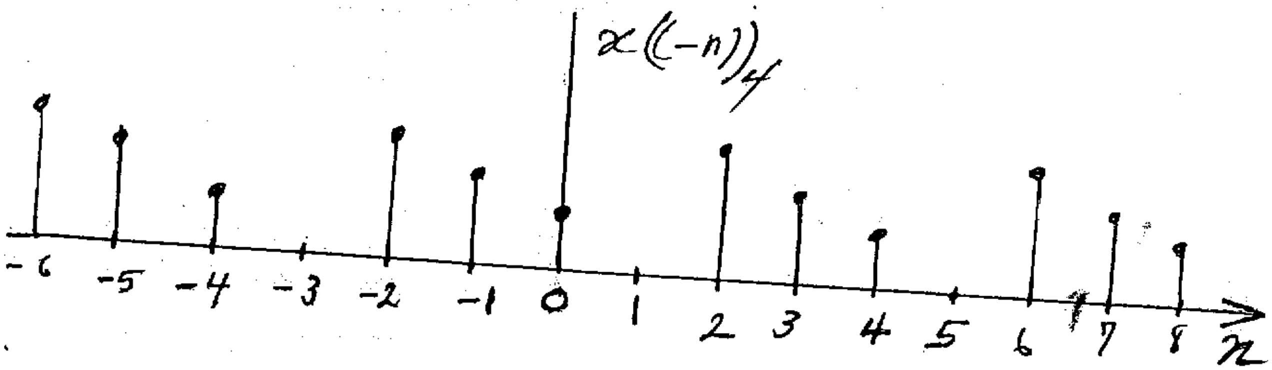
$$X(k) = \sum_{n=0}^{N-1} a^n W_N^{kn}$$

$$= \sum_{n=0}^{N-1} (a W_N^k)^n = \frac{1 - (a W_N^k)^N}{1 - a W_N^k}$$

$$= \frac{1 - a^N}{1 - a W_N^k}, \quad k = 0, 1, 2, \ldots, N-1$$

2)





3) $$x((N-n))_N = \sum_{r=-\infty}^{\infty} x(N-n+rN)$$

$$= \sum_{r=-\infty}^{\infty} x[-n+(r+1)N]$$

$$= \sum_{r=-\infty}^{\infty} x(-n+rN) = x((-n))_N$$

4) let $z(n) = x(n) * y(n)$ where $*$ denotes a linear convolution. Then $z(n)$ is of length $8+20-1 = 27$ and

$$r(n) = \left[\sum_{k=-\infty}^{\infty} z(n+20k) \right] R_{20}(n)$$

Thus, the values of $r(n)$ for $7 \leq n \leq 19$ correspond to the values of $z(n)$ or the linear convolution of $x(n)$ and $y(n)$.