

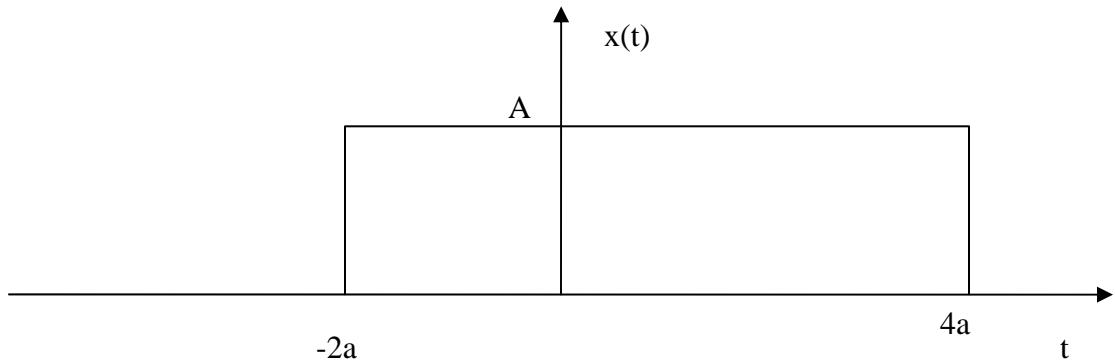
Homework 1

Solution

Problem 1:

Determine if each of the following signals is an energy signal or a power signal by deriving the energy or the time averaged power of the signal:

a) $x(t) = A[u(t+2a) - u(t-4a)] \quad a > 0;$



$$E = \int_{-2a}^{4a} A^2 dt = 6aA^2 \rightarrow \text{Energy signal}$$

b) $x(t) = \sin(t)$

$x(t)$ is periodic with $T = 2\pi$

$$E = \int_{-\infty}^{+\infty} \sin^2(t) dt = \int_{-\infty}^{+\infty} \frac{1 - \cos(2t)}{2} dt = \infty \rightarrow \text{Not an energy signal.}$$

$$P = \lim_{n \rightarrow \infty} \frac{1}{nT} \int_{-nT}^{+nT} \sin^2(t) dt = \lim_{n \rightarrow \infty} \frac{1}{nT} \int_{-nT}^{+nT} \frac{1 - \cos(2t)}{2} dt = \lim_{n \rightarrow \infty} \frac{1}{nT} nT + 0 = 1 \rightarrow \text{Power}$$

Signal.

Problem 2:

Determine if the following systems are:

1. Memoryless
2. Time-invariant
3. Linear
4. Causal
5. Stable

Justify your answers.

$$a) \quad y(t) = \frac{x(t)}{1 - x(t-1) - x(t-2)}$$

1. Not memoryless since the output depends on previous input values.
2. The system S is time invariant since:

$$y_1(t) = S\{x(t-T)\} = \frac{x(t-T)}{1 - x(t-T-1) - x(t-T-2)} = y(t-T)$$

3. Not linear since it doesn't have the superposition property:

$$y_1(t) = \frac{x_1(t)}{1 - x_1(t-1) - x_1(t-2)};$$

$$y_2(t) = \frac{x_2(t)}{1 - x_2(t-1) - x_2(t-2)};$$

Let: $x(t) = x_1(t) + x_2(t)$. Then,

$$\begin{aligned} y(t) &= \frac{x(t)}{1 - x(t-1) - x(t-2)} = \frac{x_1(t) + x_2(t)}{1 - x_1(t-1) - x_2(t-1) - x_1(t-2) - x_2(t-2)} \\ &\neq \frac{x_1(t)}{1 - x_1(t-1) - x_1(t-2)} + \frac{x_2(t)}{1 - x_2(t-1) - x_2(t-2)} \end{aligned}$$

4. Causal, since the output is a function of past and current values of the input.

5. Not stable, since for example, if $x(t-1) = -1$ and $x(t-2) = 2$, then

$$|y(t)| = \infty$$

b) $y(t) = e^{-kt} x(t)$ $k > 0$

1. Memoryless since the output at time t depends only on the input value at time t .

2. The system S is not time invariant since:

$$y_1(t) = S\{x(t-T)\} = e^{-kt} x(t-T) \neq y(t-T) = e^{-k(t-T)} x(t-T)$$

3. The system is linear since:

$$y_1(t) = e^{-kt} x_1(t)$$

$$y_2(t) = e^{-kt} x_2(t)$$

$$x(t) = ax_1(t) + bx_2(t) \Rightarrow$$

$$y(t) = e^{-kt} x(t) = e^{-kt} ax_1(t) + e^{-kt} bx_2(t) = ay_1(t) + by_2(t)$$

4. The system is Causal, since the output is a function of only the current value of the input.

5. The system is stable, since if the input is finite the output will be finite and with a magnitude decreasing with time.

Problem 3:

Check whether or not the following signals are periodic. Determine the fundamental periods T for the periodic ones.

a) $x(t) = 3\cos(2\pi t)\sin(8\pi t)$

$$x(t+T) = 3\cos(2\pi t + 2\pi T)\sin(8\pi t + 8\pi T) \rightarrow \text{Periodic with } T = 1$$

b) $x(t) = \cos(5\pi t) + \sin(3\pi t)$

$$x(t+T) = \cos(5\pi t + 5\pi T) + \sin(3\pi t + 3\pi T)$$

$$x(t+T) = x(t) \text{ if}$$

$$\exists k, p \in \mathbb{Z} \text{ such that:}$$

$$5\pi T = 2\pi k$$

$$3\pi T = 2\pi p$$

$$\Rightarrow T = \frac{2}{5}k = \frac{2}{3}p \Rightarrow \frac{p}{k} = \frac{3}{5}$$

Take $p=3$ and $k=5$; since p and k are coprimes, then $T=2$.

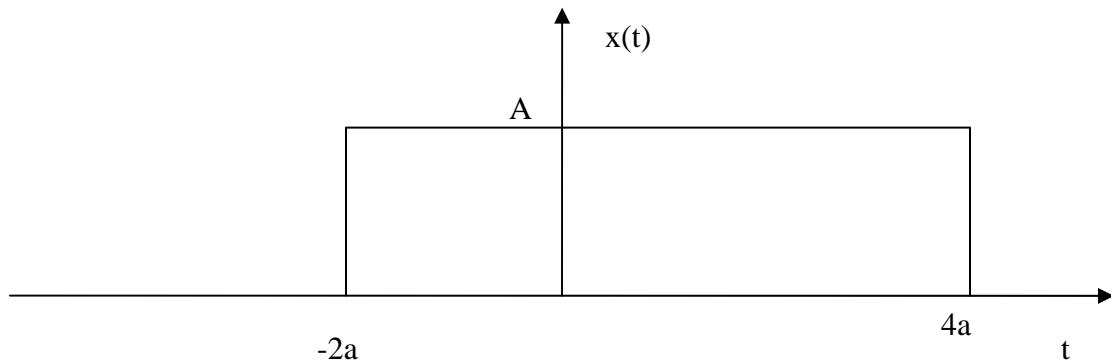
c) $x(t) = e^{-t}$

$$x(t+T) = e^{-(t+T)} \neq x(t) = e^{-t} \forall T \neq 0 \rightarrow \text{Not periodic}$$

Problem 4:

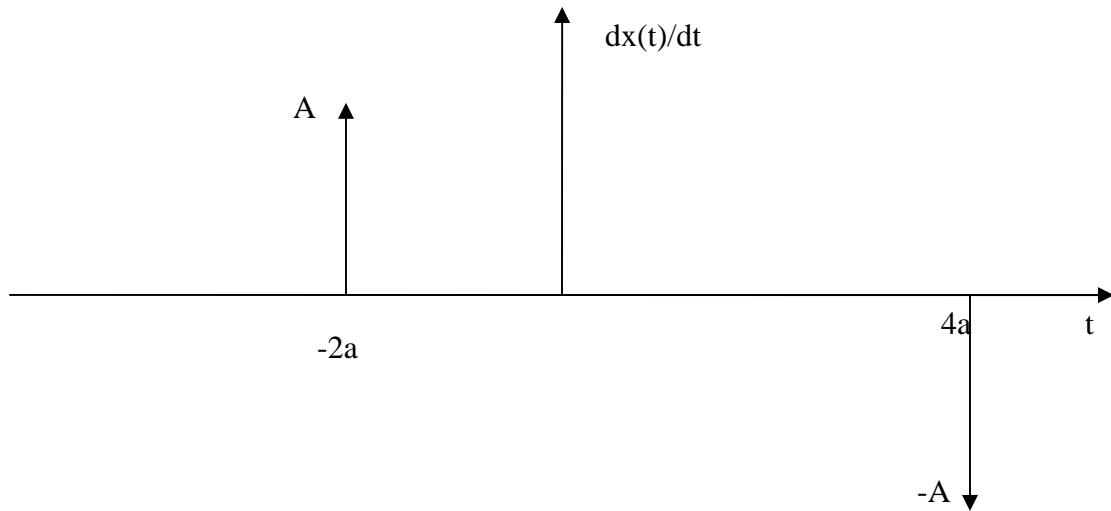
Compute and Graph the derivatives of the following functions:

a)

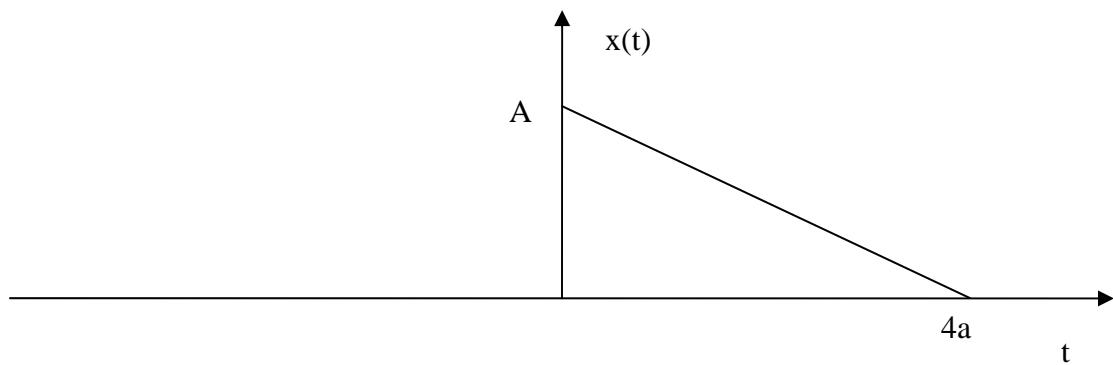


$$x(t) = A[u(t + 2a) - u(t - 4a)] \Rightarrow$$

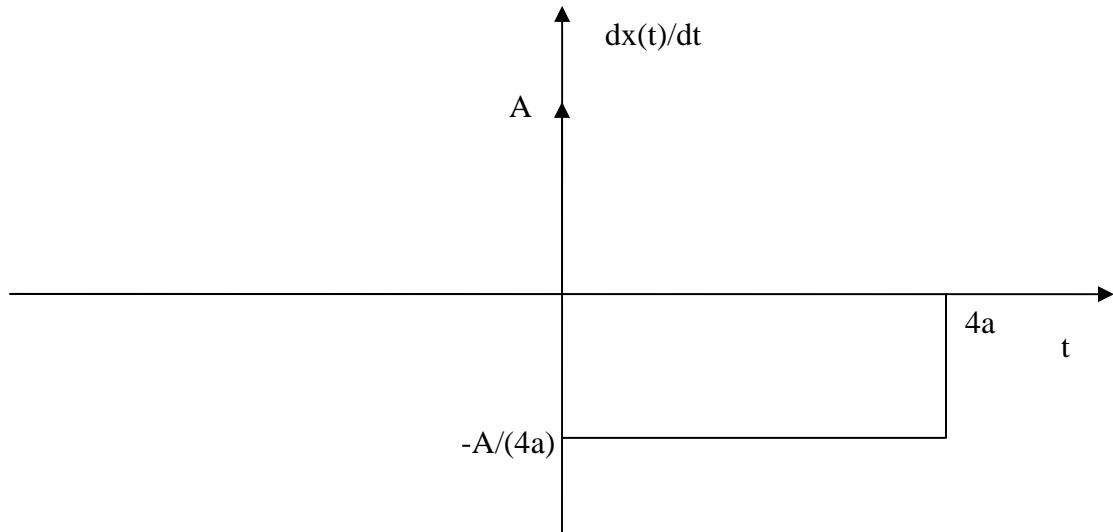
$$\frac{dx(t)}{dt} = A\delta(t + 2a) - A\delta(t - 4a)$$



b)



$$\begin{aligned}x(t) &= \left[A - \frac{A}{4a}t \right] (u(t) - u(t-4a)) \Rightarrow \\ \frac{dx(t)}{dt} &= A(\delta(t) - \delta(t-4a)) - \frac{A}{4a}(u(t) - u(t-4a)) - \frac{A}{4a}t(\delta(t) - \delta(t-4a)) \\ &= A\delta(t) - \frac{A}{4a}(u(t) - u(t-4a))\end{aligned}$$



Problem 5:

Evaluate the following integrals:

$$a) \int_{-\infty}^{+\infty} e^{\cos(\pi t)} \delta(t) dt$$

$$\int_{-\infty}^{+\infty} e^{\cos(\pi t)} \delta(t) dt = e^{\cos(\pi 0)} = e^1 = e$$

$$b) \int_5^{+\infty} e^{\cos(\pi t)} \delta(t - 2) dt$$

$$\int_5^{+\infty} e^{\cos(\pi t)} \delta(t - 2) dt = 0 \text{ since } 2 \notin [5, +\infty)$$

MATLAB Solution

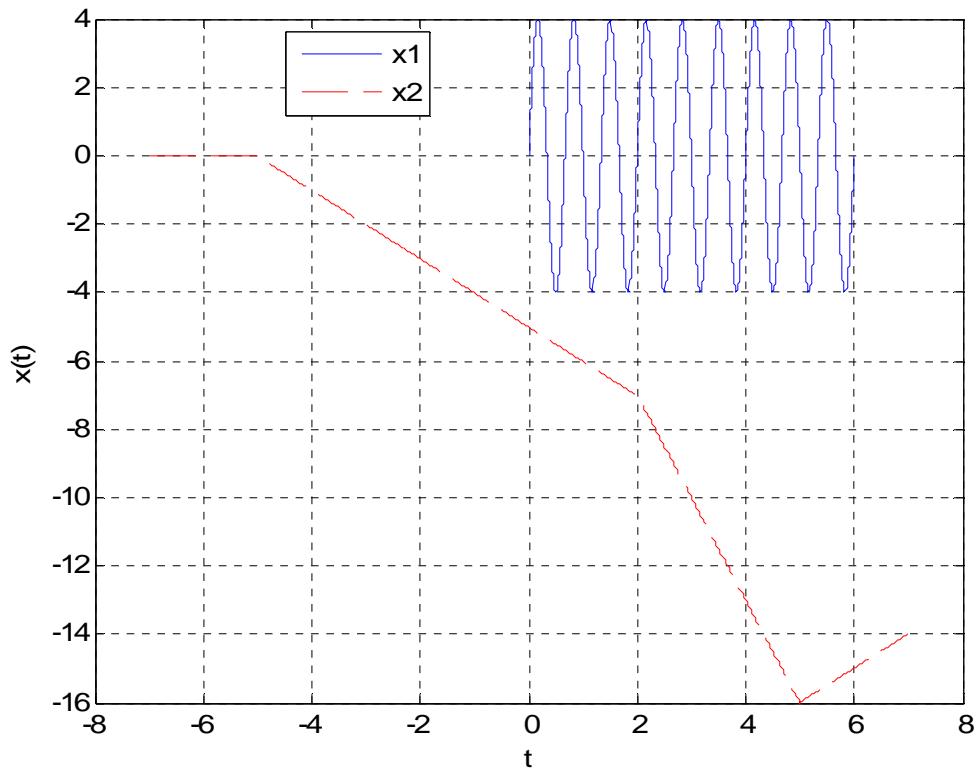
Problem 1:

Sketch the following functions using Matlab:

$$a) x_1(t) = 4 \sin(3\pi t) [u(t) - u(t - 6)]$$

$$b) x_2(t) = -(t + 5)u(t + 5) - 2(t - 2)u(t - 2) + 4(t - 5)u(t - 5)$$

```
clear;clc;
t = -7:0.001:7;
x1 = 4*sin(3*pi.*t).*(qstep(t)-qstep(t-6));
x2 = -(t+5).*qstep(t+5)-2*(t-2).*qstep(t-2)+4*(t-5).*qstep(t-5);
plot(t,x1,'-b',t,x2,'--r');
xlabel('t');ylabel('x');legend('x1','x2');
```



N.B.: if your Matlab version doesn't support "qstep", you can use the equivalent code below to solve the problem:

```

clear;
t=0:0.001:6;
x1=4*sin(3*pi.*t);
plot(t,x1,'-b'); hold on;
t=-7:0.001:7;
x2 = zeros(1,size(t,2));
for i = 1:size(t,2)
    if t(i) >= -5
        x2(i) = x2(i)-(t(i)+5);
    end
    if t(i) >= 2
        x2(i) = x2(i)-2*(t(i)-2);
    end
    if t(i) >= 5
        x2(i) = x2(i)+4*(t(i)-5);
    end
end
plot(t,x2,'--r'); grid on;
xlabel('t'); ylabel('x(t)'); legend('x1','x2');

```

Problem 2

Sketch on the same figure, the following functions using Matlab:

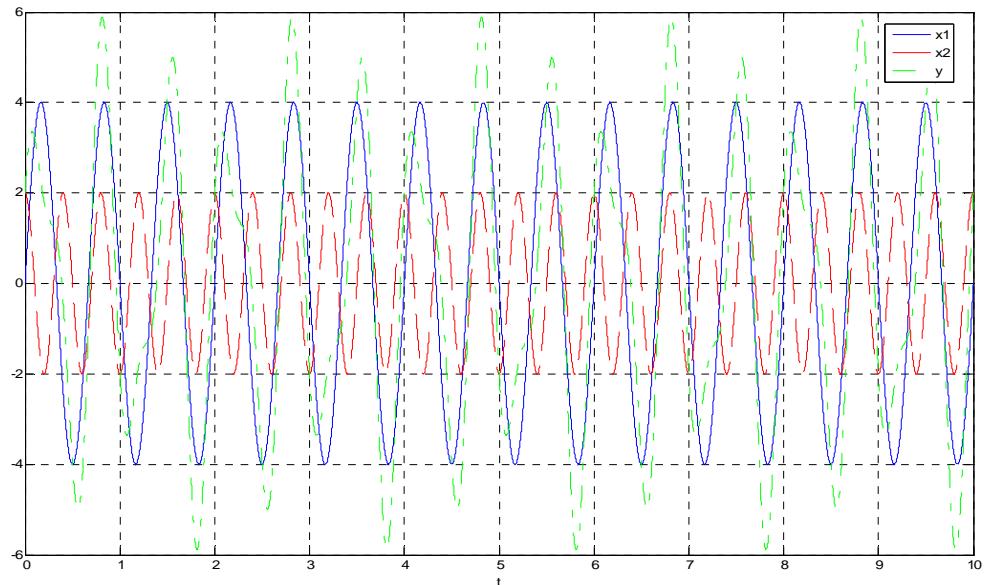
a) $x_1(t) = 4 \sin(3\pi t)$

b) $x_2(t) = 2 \cos(5\pi t)$

c) $y(t) = x_1(t) + x_2(t)$

Determine the period of $y(t)$.

```
clear;clc;
t=0:0.001:10;
x1 = 4*sin(3*pi.*t);
x2 = 2*cos(5*pi.*t);
y = x1 + x2;
plot(t,x1,'-b',t,x2,'--r',t,y,'-.g');
xlabel('t');legend('x1','x2','y');grid on;
```



$y(t)$ periodic with $T = 2$.