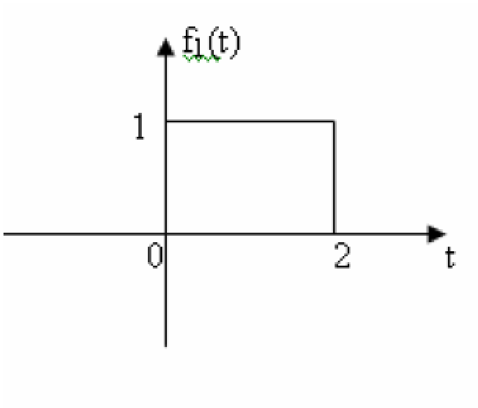
Homework 2

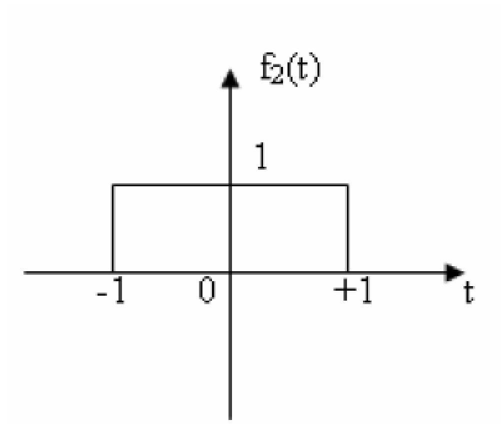
Solution

Problem 1

Determine and plot the convolution integral $f_1(t)*f_2(t)$ for:

a)



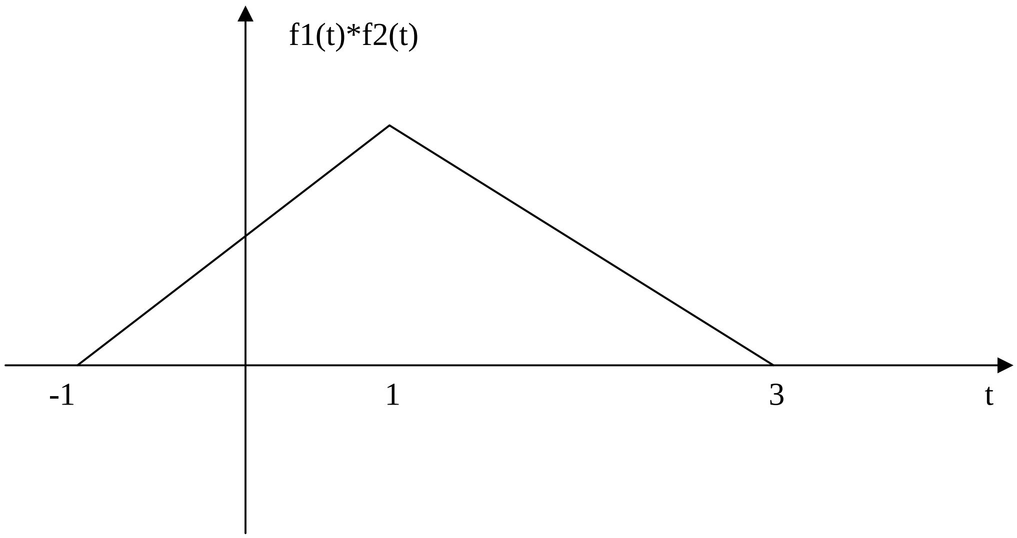


$$f_1(t) = u(t) - u(t-2) \Rightarrow F_1(s) = \frac{1}{s} - \frac{e^{-2s}}{s}$$

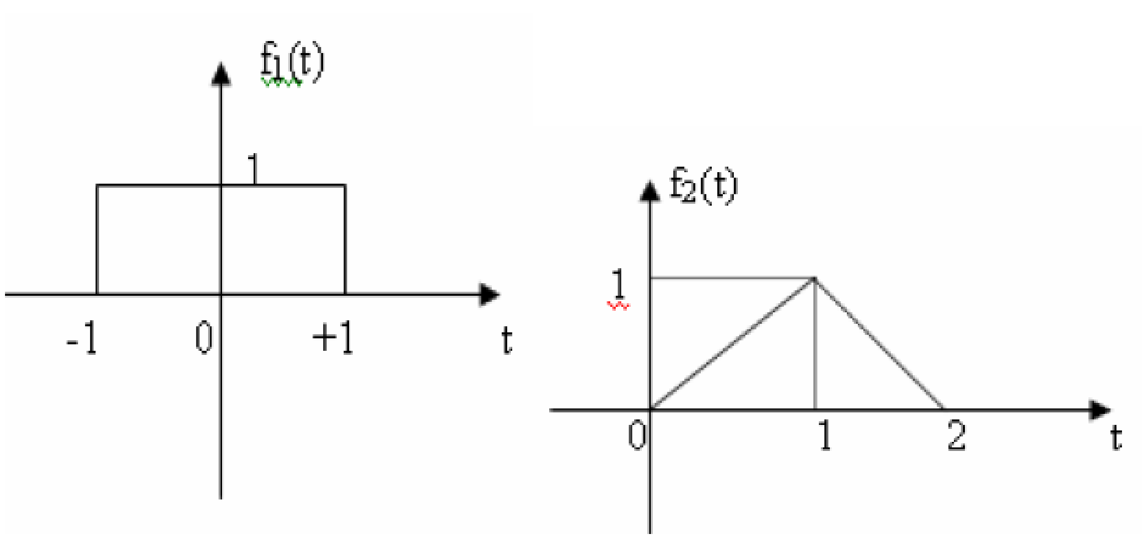
$$f_2(t) = u(t+1) - u(t-1) \Rightarrow F_2(s) = \frac{e^{+s}}{s} - \frac{e^{-s}}{s}$$

$$F_1(s) \cdot F_2(s) = \frac{e^{+s}}{s^2} - 2\frac{e^{-s}}{s^2} + \frac{e^{-3s}}{s^2} \Rightarrow$$

$$f_1(t) * f_2(t) = (t+1)u(t+1) - 2(t-1)u(t-1) + (t-3)u(t-3)$$



b)

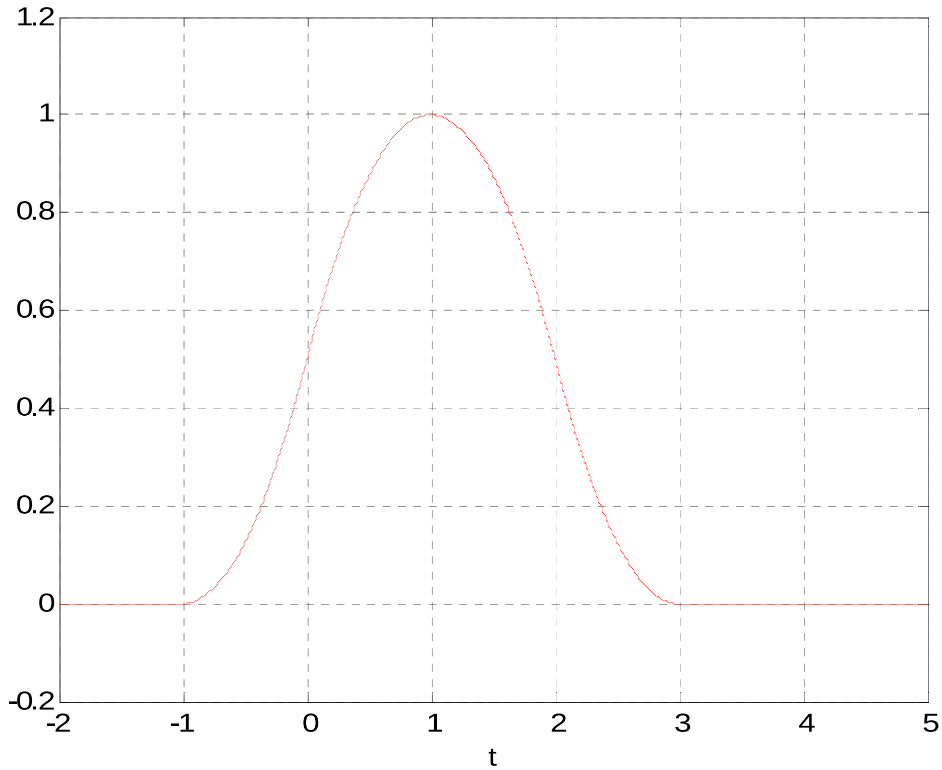


$$f_1(t) = u(t+1) - u(t-1) \Rightarrow F_1(s) = \frac{e^{+s}}{s} - \frac{e^{-s}}{s}$$

$$f_2(t) = tu(t) - 2(t-1)u(t-1) + (t-2)u(t-2) \Rightarrow F_2(s) = \frac{1}{s^2} - 2\frac{e^{-s}}{s^2} + \frac{e^{-2s}}{s^2}$$

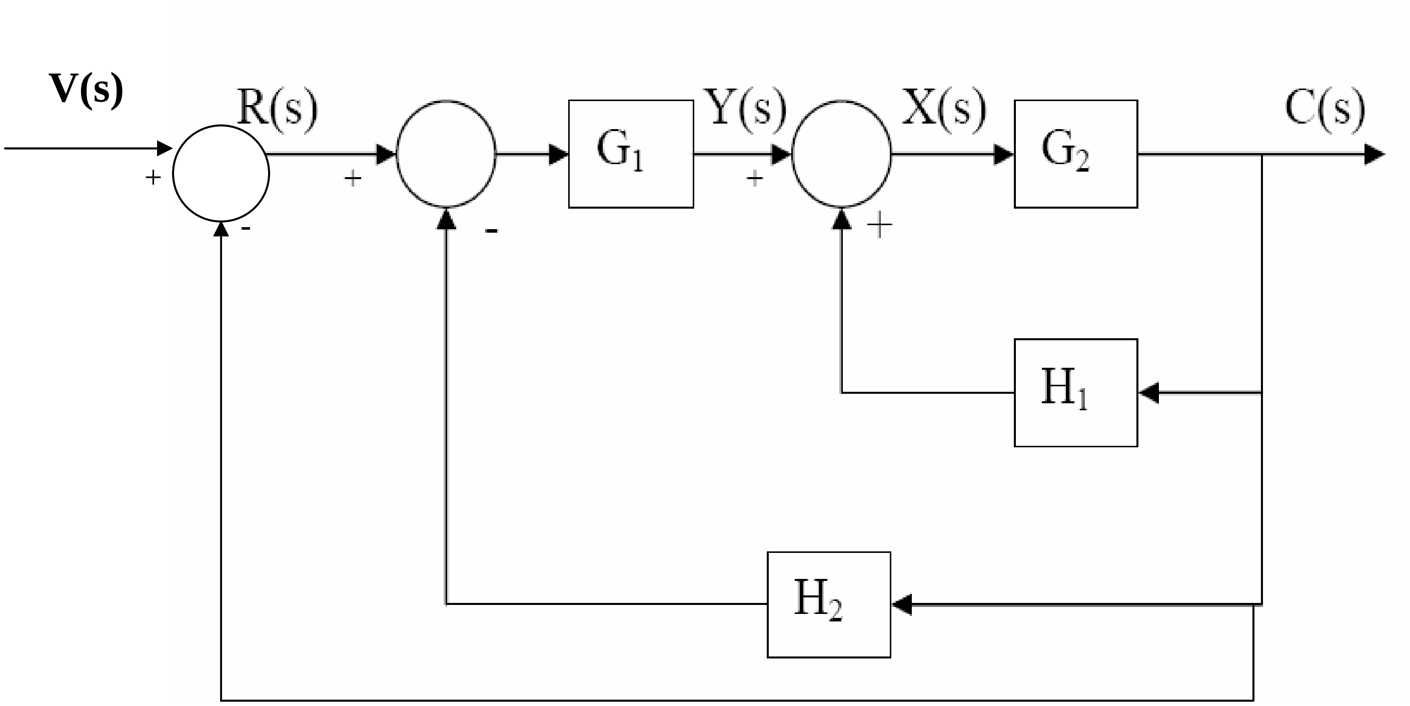
$$F_1(s)\cdot F_2(s) = \frac{e^{+s}}{s^3} - 2\frac{1}{s^3} + 2\frac{e^{-2s}}{s^3} - \frac{e^{-3s}}{s^3} \Rightarrow$$

$$f_1(t) * f_2(t) = \frac{1}{2}(t+1)^2 u(t+1) - t^2 u(t) + (t-2)^2 u(t-2) - \frac{1}{2}(t-3)^2 u(t-3)$$



Problem 2

Determine the transfer function C(s)/V(s) of the System below



$$X(s) = Y(s) + H_1C(s)$$

$$X(s) = G_1 \{ R(s) - H_2C(s) \} + H_1C(s)$$

$$C(s) = G_1G_2R(s) - G_1G_2H_2C(s) + G_2H_1C(s)$$

$$\frac{C(s)}{R(s)} = \frac{G_1G_2}{1 + G_1G_2H_2 - G_2H_1}$$

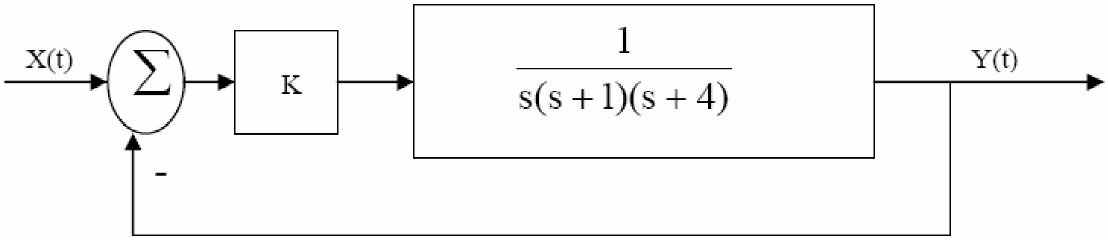
$$R(s) = V(s) - C(s)$$

$$let: G(s) = \frac{C(s)}{R(s)} \Rightarrow$$

$$\frac{C(s)}{V(s)} = \frac{G(s)}{1+G(s)} = \frac{\dfrac{G_1G_2}{1+G_1G_2H_2-G_2H_1}}{1+\dfrac{G_1G_2}{1+G_1G_2H_2-G_2H_1}} = \frac{G_1G_2}{1+G_1G_2H_2-G_2H_1+G_1G_2}$$

Problem 3

Consider the unit feedback system shown below:



a) Determine the error signal E(s).

$$E(s) = \frac{X(s)}{1 + G(s)} = \frac{X(s)[s(s+1)(s+4)]}{s(s+1)(s+4) + K}$$

b) Determine the range of K for the system to be stable.

$$\frac{X(s)}{Y(s)} = \frac{K}{s^3 + 5s^2 + 4s + K}$$

RH table

S^4	1	5
S^3	5	K
S	$\frac{20-K}{5}$	
S^0	K	

For stability, $K>0$ and $20-K>0$, implies $0<K<20$

Problem 4

The transfer function of a linear control system is given by:

$$\frac{C(s)}{R(s)} = \frac{1}{s^2 + 3s + 2}$$

a) Determine the state equation for this system.

$$\begin{bmatrix} X_1'(t) \\ X_2'(t) \end{bmatrix} = \begin{bmatrix} -3 & 1 \\ -2 & 0 \end{bmatrix} \begin{bmatrix} X_1(t) \\ X_2(t) \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} r(t)$$

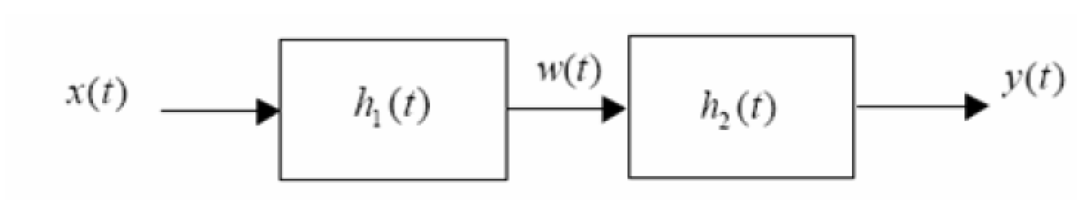
b) Determine the corresponding output equation.

$$y(t) = \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} X_1(t) \\ X_2(t) \end{bmatrix} + [0] r(t)$$

Problem 5

The input signal of the LTI system shown below (initially at rest) is

x(t) = u(t)-u(t-3)



The impulse responses of the subsystems are:

$$h_1(t) = \left(e^{-t} + e^{-2t}\right) u(t)$$

$$h_2(t) = e^{-3t} u(t)$$

a) Compute the impulse response h(t) of the overall system.

$$H_1(s) = \frac{1}{s+1} + \frac{1}{s+2}$$

$$H_2(s) = \frac{1}{s+3}$$

$$\Rightarrow$$

$$H(s) = H_1(s) \cdot H_2(s) = \left(\frac{1}{s+1} + \frac{1}{s+2} \right) \cdot \frac{1}{s+3}$$

$$= \frac{1}{(s+1)(s+3)} + \frac{1}{(s+2)(s+3)}$$

$$= \frac{A}{s+1} + \frac{B}{s+2} + \frac{C}{s+3}$$

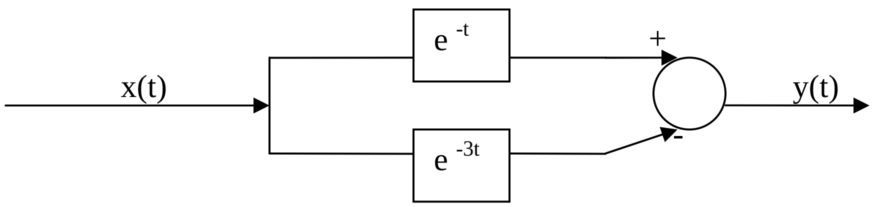
Solving for A, B, and C we find:

A = 1; B = 0; C = -1 →

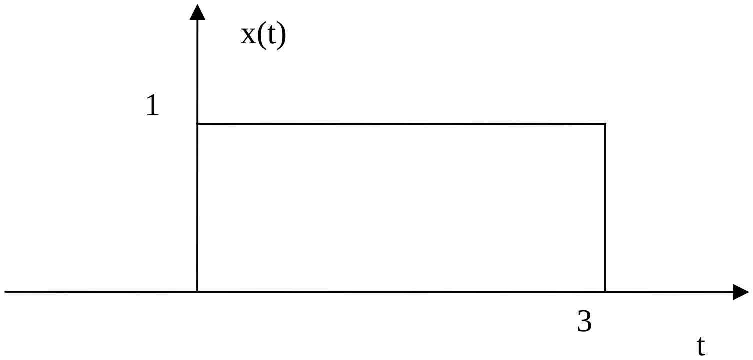
$$H(s) = \frac{1}{s+1} - \frac{1}{s+3} \Rightarrow$$

$$h(t) = \left(e^{-t} - e^{-3t} \right) u(t)$$

b) Find an equivalent system (same impulse response) configured as a parallel interconnection of two LTI subsystems.



c) Sketch the input signal x(t). Compute the output signal y(t).



$$X(s) = \frac{1}{s} - \frac{e^{-3s}}{s}$$

$$H(s) = \left(\frac{1}{s+1} - \frac{1}{s+3} \right)$$

$$Y(s) = H(s)X(s)$$

$$= \frac{1}{s(s+1)} - \frac{e^{-3s}}{s(s+1)} - \frac{1}{s(s+3)} + \frac{e^{-3s}}{s(s+3)}$$

$$= \left(\frac{1}{s} - \frac{1}{s+1} \right) - \left(\frac{e^{-3s}}{s} - \frac{e^{-3s}}{s+1} \right) - \frac{1}{3}\left(\frac{1}{s} - \frac{1}{s+3} \right) + \frac{e^{-3s}}{3}\left(\frac{1}{s} - \frac{1}{s+3} \right)$$

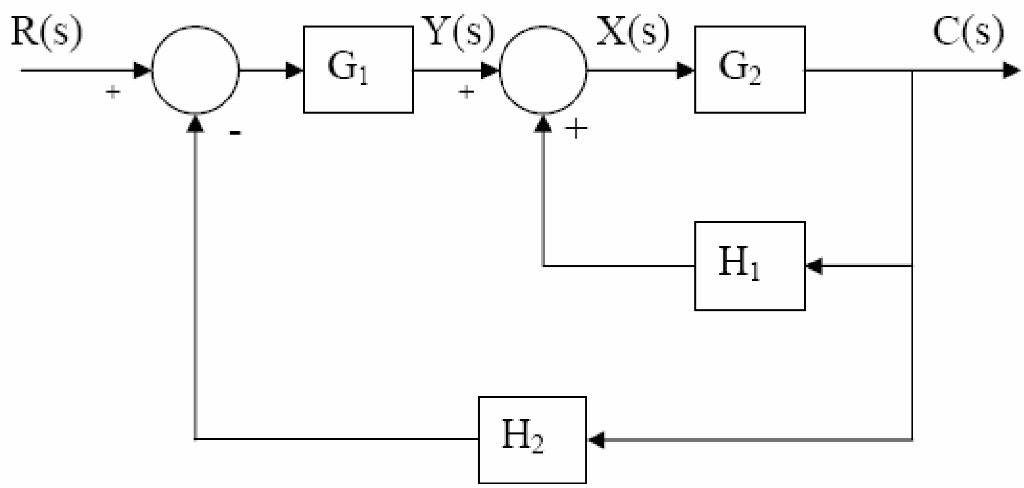
$\Rightarrow$

$$y(t) = u(t) - e^{-t}u(t) - \left(u(t-3) - e^{-(t-3)}u(t-3) \right) - \frac{1}{3}\left(u(t) - e^{-3t}u(t) \right) + \frac{1}{3}\left(u(t-3) - e^{-3(t-3)}u(t-3) \right)$$

$$= \frac{2}{3}u(t) - e^{-t}u(t) + \frac{1}{3}e^{-3t}u(t) - \frac{2}{3}u(t-3) + e^{-(t-3)}u(t-3) - \frac{1}{3}e^{-3(t-3)}u(t-3)$$

MATLAB Solution

Problem 1:



$$X(s) = Y(s) + H_1C(s)$$

$$X(s) = G_1 \{ R(s) - H_2C(s) \} + H_1C(s)$$

$$C(s) = G_1G_2R(s) - G_1G_2H_2C(s) + G_2H_1C(s)$$

$$\frac{C(s)}{R(s)} = \frac{G_1G_2}{1 + G_1G_2H_2 - G_2H_1}$$

Given that: $G_1 = \frac{s^2 + 3s + 5}{s^3 + 2s + 1}$; $G_2 = 4$; $H_1 = \frac{1}{s+4}$; and $H_2 = \frac{s+1}{s^2 + 2s + 2}$.

% Matlab Solution

```
G1 = tf([1 3 5],[1 0 2 1]);
G2 = tf([4],[1]);
H1 = tf([1],[1 4]);
H2 = tf([1 1],[1 2 2]);

H = G1*G2/(1+G1*G2*H2-G2*H1);
```

Problem 2

Use Matlab to determine whether the system with the transfer function shown below is stable. Get a state space representation of the system.

$$H(s) = \frac{s^2 + 3s + 4}{s^6 - 2s^5 + 7s^3 - 3s^2 + s + 1}$$

```
% Enter the coefficients of the denominator
A = [1 -2 0 7 -3 1 1];

% Compute the poles of the system. If not all of them lie in the left half
%plane then the system is not stable
roots(A)
if real(roots(A) < 0)
        display 'The system is stable'
else
        display 'The system is not stable'
end

% Get the State Space Representation
[a, b, c, d] = tf2ss([1 3 4], [1 -2 0 7 -3 1 1])
```

```
ans =

   1.6003 + 1.3159i
   1.6003 - 1.3159i
  -1.5872
   0.3678 + 0.5341i
   0.3678 - 0.5341i
  -0.3490

The system is not stable
```

a =

2	0	-7	3	-1	-1
1	0	0	0	0	0
0	1	0	0	0	0
0	0	1	0	0	0
0	0	0	1	0	0
0	0	0	0	1	0

b =

1
0
0
0
0
0

c =

0	0	0	1	3	4

d =

0