

**American University of Beirut**  
**Department of Electrical and Computer Engineering**

EECE 440 Signals and Systems  
Homework 3: Due Friday March 30, 2007

**Problem 1**

- a. If  $\text{FT}\{g(t)\} = G(\omega)$ , show that  $\text{FT}\{g(t-a)\} = \exp(-jka) G(\omega)$ .
- b. Calculate the Fourier Transform of  $A \cos(k_0 t + a)$ , for  $a/k_0 = \{0, \pi/4, \pi/2, \pi\}$ . Plot the result.
- c. If  $\text{FT}\{g(t)\} = G(\omega)$ , show that  $\text{FT}\{g(at)\} = (1/a) G(\omega/a)$ .

**Problem 2**

- a. Find the Fourier transform of the following signals:

1)

$$x_1(t) = \begin{cases} \sin \omega_0 t, & -\frac{2\pi}{\omega_0} \leq t \leq \frac{2\pi}{\omega_0} \\ 0, & \text{otherwise} \end{cases}$$

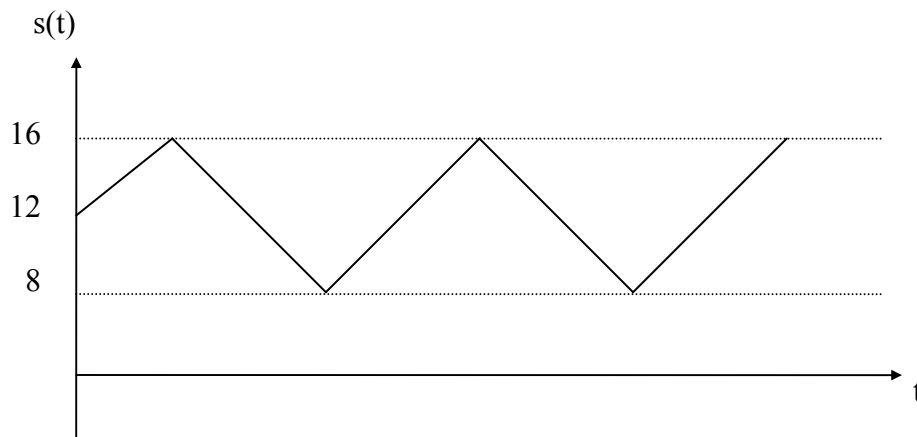
2)  $x(t) = e^{-3(t-1)} \cos[2(t-1)] u(t-1)$

- b. Find the time domain signals corresponding to the following Fourier transforms:

1)  $X(j\omega) = \frac{\sin(\omega)}{\omega}$ ; 2)  $X(j\omega) = \frac{j\omega + 2}{j\omega(j\omega + 5)} + \frac{2\pi}{5} \delta(\omega)$

**Problem 3**

The envelope of the output of an AM modulator is shown below



Determine the following:

- The modulation index.
- The carrier amplitude.

#### **Problem 4**

An amplitude modulated wave-form has the form

$$s(t) = 15[1 + 0.6 \cos(300\pi t) + 0.6 \cos(600\pi t)] \cos(3000\pi t)$$

- Find the total power.
- Find the total side-band power.

#### **Problem 5**

Consider a DSB-LC signal where the message  $m(t)$  is given by:

$$m(t) = \cos(5\pi t) + \cos(25\pi t) + \cos(45\pi t) + \cos(90\pi t)$$

The DSB-LC signal is inputted to an ideal band-pass filter followed by an envelope detector. The ideal band-pass filter is tuned to the frequency of the carrier with a bandwidth that passes 3 Hz on either side of this frequency. Find the output of the envelope detector.

#### **Problem 6**

An FM modulator is used to transmit a tone message with amplitude of 3 volts and frequency of 15 Hz. The frequency deviation constant for the modulator is 25Hz/V, and the carrier wave has amplitude 8 Volts and frequency 2000Hz.

- What's the power of the FM modulated signal?
- What's the approximate bandwidth of the FM modulated signal using Carson's rule?
- How should your answer to part b) change if the amplitude of the message is doubled?
- How should your answer to part b) change if the frequency of the carrier is doubled?

#### **Problem 7**

A sinusoidal carrier is frequency modulated by a 6 KHz sinusoid wave resulting in an FM signal having a maximum frequency of 213.41 MHz and a minimum frequency of 213.196 MHz.

- Determine the carrier frequency.
- Determine the number of impulses in the spectrum of  $s(t)$  within its bandwidth.

#### **Problem 8**

Find the instantaneous frequency of the FM signal:

$$s(t) = 10 \left[ \cos(20t) \cos(50t^3) - \sin(20t) \sin(50t^3) \right]$$

### **Problem 9**

Consider a single tone FM signal with amplitude 10 Volts, frequency 5 KHz, and modulation index  $\beta = 2$ . Determine the ratio of the average power of the frequency components of this FM signal contained within its bandwidth to the total signal average power. Use Carson's rule for bandwidth computation.

## **Matlab**

### **Problem 1**

Find the Fourier Transforms of the following time domain functions using Matlab:

a)  $x_1(t) = 12[1 + 0.6 \cos(250\pi t) + 0.32 \cos(600\pi t)] \cos(2100\pi t)$

b)  $x_2(t) = \cos(-t) \frac{\sin(t)}{3t}$

### **Problem 2**

Find the Inverse Fourier Transforms of the following signals using Matlab:

a)  $X_1(\omega) = \frac{j\omega + 1}{-\omega^2 + j2\sqrt{2}\omega + 4}$

b)  $X_1(\omega) = \frac{j\omega + 3}{-\omega^2 + j4\omega + 4}$

### **Problem 3**

Write an M file that modulates the signal  $v(t) = \cos(3t)$  using AM modulation with a carrier of amplitude  $A_c = 10$  V, frequency  $f_c$  such that  $\omega_c = 2\pi f_c = 300$  rad/s, and a modulation index  $\mu = 0.5$ . (Consider a time interval from 0s to 10s in steps of 0.01s).

### **Problem 4**

Consider the signal  $0.3t^2$ . We want to modulate this signal using FM modulation with a carrier frequency of 2 rad/s.

- What is the instantaneous frequency?
- Write the matlab code required to modulate this signal and plot it along with the unmodulated carrier.

### **Problem 5**

Consider the signal  $\{-\sin(t)\}$ . We want to modulate this signal using FM modulation with a carrier frequency of 2 rad/s.

- What is the instantaneous frequency?
- Write the matlab code required to modulate this signal and plot it along with the unmodulated carrier.