Problem #1

$$f(t) = e^{t}u(t)$$

$$E = \int_0^\infty e^{2t}dt = \frac{1}{2}e^{2t}\Big|_0^\infty = -\frac{1}{2} + \frac{1}{2}e^{\infty} \longrightarrow \infty$$

$$P = \lim_{T\to\infty}\frac{1}{T}\int_{-T/2}^{T/2} e^{2t}u(t)dt = \lim_{T\to\infty}\frac{1}{T}\int_0^{T/2} e^{2t}dt$$

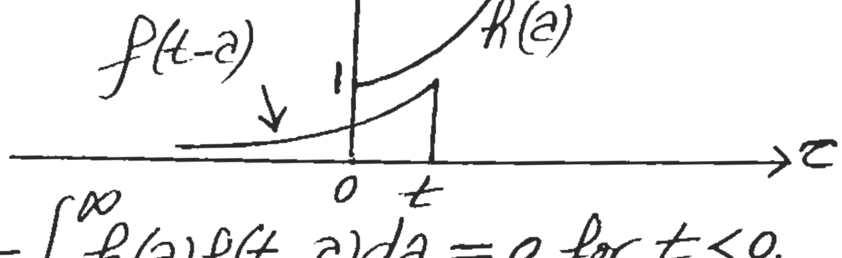
$$= \lim_{T\to\infty}\frac{1}{T}\frac{1}{2}e^{2t}\Big|_0^{T/2} = \lim_{T\to\infty}\frac{e^{T}-1}{2T}$$

$$= \lim_{T\to\infty}\frac{e^{T}}{2T} \longrightarrow \infty.$$

Hence, $f(t)$ is neither an energy nor a power signal.

Problem #2

$$h(t) = e^{t}u(t)\,;\ f(t) = e^{-2t}u(t).$$



$$g(t) = \int_{-\infty}^{\infty} h(\tau)f(t-\tau)d\tau = 0 \text{ for } t<0.$$

$$\text{For } t>0,\ g(t) = \int_0^t e^{-2(t-\tau)}e^{\tau}d\tau = e^{-2t}\int_0 e^{3\tau}d\tau$$

$$= \frac{1}{3}e^{-2t}e^{3\tau}\Big|_0^t = \frac{1}{3}\left(e^{t} - e^{-2t}\right)$$

Hence, $g(t) = \frac{1}{3}(e^{-t} - e^{-2t})\,u(t)$

$$= \underbrace{\tfrac{1}{3}e^{-t}u(t)}_{\text{Transient response}} - \underbrace{\tfrac{1}{3}e^{-2t}u(t)}_{\text{steady state response}}.$$

Actually, the transient response has the form of the system impulse response. It also has the form of the signal given in Problem 1. $g(t)$ is, therefore, neither an energy nor a power signal.

This result can also be checked by evaluating the energy and average power of $g(t)$. Both E and P will turn out to be tending towards infinity.

<u>Problem #3</u>

$$h(t) = e^{-t}u(t)$$
$$f(t) = e^{-2t}u(t)$$

$g(t) = 0$, for $t < 0$.

For $t > 0$, $g(t) = \int_0^t e^{-2(t-\lambda)}e^{-\lambda}\,d\lambda = e^{-2t}\int_0^t e^{\lambda}\,d\lambda$

$$= e^{-2t}(e^{t} - 1) = e^{-t} - e^{-2t}$$

Hence, $g(t) = \underbrace{e^{-t}u(t)}_{\text{Transient response}} - \underbrace{e^{-2t}u(t)}_{\text{steady state response}}.$

Obviously, $g(t)$ is an energy signal. Since,

$$E = \int_{-\infty}^{\infty} g^2(t)\,dt = \int_0^{\infty} \left(e^{-t} - e^{-2t}\right)^2 dt$$

$$= \int_0^{\infty} \left(e^{-2t} + e^{-4t} - 2e^{-3t}\right) dt$$

$$= -\frac{1}{2} e^{-2t}\Big|_0^{\infty} - \frac{1}{4} e^{-4t}\Big|_0^{\infty} + \frac{2}{3} e^{-3t}\Big|_0^{\infty}$$

$$= \frac{1}{2} + \frac{1}{4} - \frac{2}{3} = \frac{1}{12}.$$

<u>Problem # 4</u> The considered system is:

$$g(t) = T[f(t)] = 4\int_{-\infty}^{t} f(\tau)\,d\tau + u(t)$$

1) Linearity:

$$T\left[a_1 f_1(t) + a_2 f_2(t)\right] = 4\int_{-\infty}^{t} \left[a_1 f_1(\tau) + a_2 f_2(\tau)\right] d\tau + u(t)$$

$$= 4\int_{-\infty}^{t} a_1 f_1(\tau)\,d\tau + 4\int_{-\infty}^{t} a_2 f_2(\tau)\,d\tau + u(t)$$

$$\neq a_1 T[f_1(t)] + a_2 T[f_2(t)] = a_1 g_1(t) + a_2 g_2(t).$$

Hence, the system is non-linear.

2) Causality: $g(t_0) = 4\int_{\infty}^{t_0} f(\tau)\,d\tau + u(t_0)$

Hence, $g(t_0)$ depends on values of the input $f(t)$ for $t \leq t_0$. The system is causal.

3) Time invariance:

$$g(t-t_0) = 4\int_{-\infty}^{t-t_0} f(\tau)\,d\tau + u(t-t_0).$$

$$T[f(t-t_0)] = 4\int_{-\infty}^{t} f(\tau-t_0)\,d\tau + u(t)$$

$$= 4\int_{-\infty}^{t-t_0} f(x)\,dx + u(t).$$

Hence, $g(t-t_0) \neq T[f(t-t_0)]$ and the system is time-varying.

Problem # 5

In Technique #1, the aliasing effect in the spectrum of the sampled signal is removed entirely when we sample at a rate larger than or equal to the Nyquist rate. Hence, the signal obtained at the output of LPF used for band-limiting can be retrieved as is from the spectrum of the sampled signal.

In Technique #2, the aliasing effect cannot be removed entirely. Hence, the LPF will retrieve the band-limited version of the original signal + portions from the replicas of the spectrum of this original signal. These portions constitute distortion which makes the retrieved signal deviate further from the original one.

Therefore, Technique 1 leads to the retrieval of a signal closer to the original low-pass one than the signal retrieved from the spectrum of the sampled signal using technique 2.

Problem # 6

$$S_r(t) = A_r f(t) \cos(\omega_c t - \phi_1)$$

$$S_r(t) c(t) = A_r A_c f(t) \cos(\omega_c t - \phi_1) \cos(\omega_c t - \phi_2)$$
$$= \frac{1}{2} A_r A_c f(t) \left[\cos(2\omega_c t - \phi_1 - \phi_2) + \cos(\phi_1 - \phi_2)\right]$$

The LPF removes the first term in the above expression and it will output

$\frac{1}{2} A_r A_c f(t) \cos(\phi_1 - \phi_2)$ within its bandwidth.

Hence, if $(\phi_1 - \phi_2) = (2k+1)\pi/2$, $k = 0, \pm 1, \pm 2, \ldots$, then $\cos(\phi_1 - \phi_2) = 0$ and the LPF output will zero. That is no signal is obtained at the output of the LPF.

The above effect is called the quadrature null effect.

Problem # 7

$$S_r(t) = A_r f(t) \cos(\omega_c t - \phi_1)$$

Since ϕ_1 is the phase shift due to travel-time delay between transmitter and receiver, then we can write the following:

$$S_r(t) = A_r f(t) \cos\left[\omega_c \left(t - \frac{\phi_1}{\omega_c}\right)\right]$$

Hence, the time delay or travel-time is:

$$t_d = \phi_1/\omega_c = \phi_1/2\pi f_c = \frac{200\pi}{2\pi \times 1000\,KHz}$$

$$= 100 \times 10^{-6} = 10^{-4}\,sec. = 0.1\,ms.$$

Note $t_d = \dfrac{\phi_1}{2\pi f_c} = \dfrac{d}{v}$, where d is the distance between the transmitter and receiver and v is the velocity of the traveling signal as an electromagnetic wave. V is usually taken as the speed of light.

Hence, $\phi_1 = \dfrac{2\pi f_c d}{v}$

$$= \frac{2\pi d}{v/f_c}$$

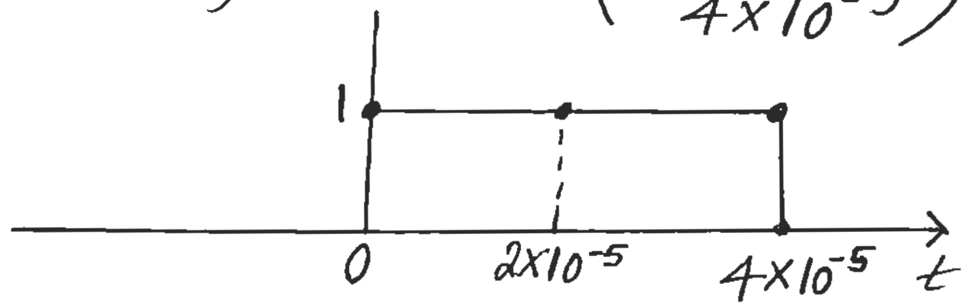
$$= \frac{2\pi d}{v \cdot T_c}$$

$$= \frac{2\pi d}{\lambda},$$

Where λ, called the wavelength of the traveling electromagnetic wave, is the distance traveled by the EM wave during one period.

Problem # 8

$$f(t) = rect\left(\frac{t - 2\times10^{-5}}{4\times10^{-5}}\right)$$



$\frac{1}{T} = 50{,}000$ samples/sec $\Rightarrow T = 2\times10^{-5}$ secs.

$$\Rightarrow f(n) = \begin{cases} 1, & n = 0, 1, 2 \\ 0, & \text{elsewhere} \end{cases}$$

$$F(\omega) = \sum_{n=0} f(n) e^{-j\omega nT} = \sum_{n=0}^{2} 1.\left(e^{-j\omega T}\right)^{n}$$

$$= \frac{1 - \left(e^{-j\omega T}\right)^{3}}{1 - e^{-j\omega T}} = \frac{1 - e^{-j3\omega T}}{1 - e^{-j\omega T}}$$

$$= \frac{e^{-j\frac{3\omega T}{2}}\left(e^{j\frac{3\omega T}{2}} - e^{-j\frac{3\omega T}{2}}\right)}{e^{-j\frac{\omega T}{2}}\left(e^{j\frac{\omega T}{2}} - e^{-j\frac{\omega T}{2}}\right)}$$

$$= e^{-j\omega T} \frac{\sin\left(\frac{3\omega T}{2}\right)}{\sin\left(\frac{\omega T}{2}\right)}$$

$$|F(\omega)| = \left|\frac{\sin\left(\frac{3\omega T}{2}\right)}{\sin\left(\frac{\omega T}{2}\right)}\right|$$

Problem # 9

$$h(t) = e^{(j\omega_c - 2)t}\,\mu(t)$$

Applying the one-sided Laplace transform to $h(t)$ we obtain: $H(S) = \dfrac{1}{S+2-j\omega_c}$.

Hence, the pole of $H(S)$ is at $S = -2 + j\omega_c$.

Problem #10

$$h(t) = 4e^{-2t}\cos(\omega_c t)\,\mu(t).$$

$h(t)$ can be written as:

$$h(t) = 2e^{-2t}\left(e^{j\omega_c t} + e^{-j\omega_c t}\right)\mu(t)$$

$$= \left\{2e^{-(2-j\omega_c)t} + 2e^{-(2+j\omega_c)t}\right\}\mu(t)$$

Hence, $H(S) = \dfrac{2}{S+2-j\omega_c} + \dfrac{2}{S+2+j\omega_c}$

$$= \frac{4(S+2)}{(S+2+j\omega_c)(S+2-j\omega_c)} = \frac{4(S+2)}{(S+2)^2+\omega^2}$$

The poles of $H(S)$ are at:

$S_1 = -2 - j\omega_c$

$S_2 = -2 + j\omega_c$

Problem #11

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$$h(t)=\begin{cases}4e^{-2t}\cos(\omega_c t), & t>0\\ 4e^{2t}\cos(\omega_c t), & t<0\end{cases}$$

$$H(s)=2\int_{-\infty}^{0} e^{-(s-2-j\omega_c)t}\,dt+2\int_{-\infty}^{0} e^{-(s-2+j\omega_c)t}\,dt$$

$$+2\int_{0}^{\infty} e^{-(s+2-j\omega_c)t}\,dt+2\int_{0}^{\infty} e^{-(s+2+j\omega_c)t}\,dt$$

$$=-\frac{2e^{-(s-2-j\omega_c)t}}{(s-2-j\omega_c)}\Bigg|_{-\infty}^{0}-\frac{2e^{-(s-2+j\omega_c)t}}{(s-2+j\omega_c)}\Bigg|_{-\infty}^{0}$$

$$-\frac{2e^{-(s+2-j\omega_c)t}}{(s+2-j\omega_c)}\Bigg|_{0}^{\infty}-\frac{2e^{-(s+2+j\omega_c)t}}{(s+2+j\omega_c)}\Bigg|_{0}^{\infty}$$

$$=\frac{-2}{(s-2-j\omega_c)}-\frac{2}{(s-2+j\omega_c)}\;;\;\sigma-2<0 \text{ or } \sigma<2$$

$$+\frac{2}{(s+2-j\omega_c)}+\frac{2}{(s+2+j\omega_c)}\;;\;\sigma+2>0 \text{ or } \sigma>-2$$

$$=-\frac{4(s-2)}{(s-2)^2+\omega_c^2}+\frac{4(s+2)}{(s+2)^2+\omega_c^2}\;;\;-2<\sigma<2$$

Hence, the ROC of $H(s)$ is the vertical strip in the s-plane with $-2<\sigma<2$.

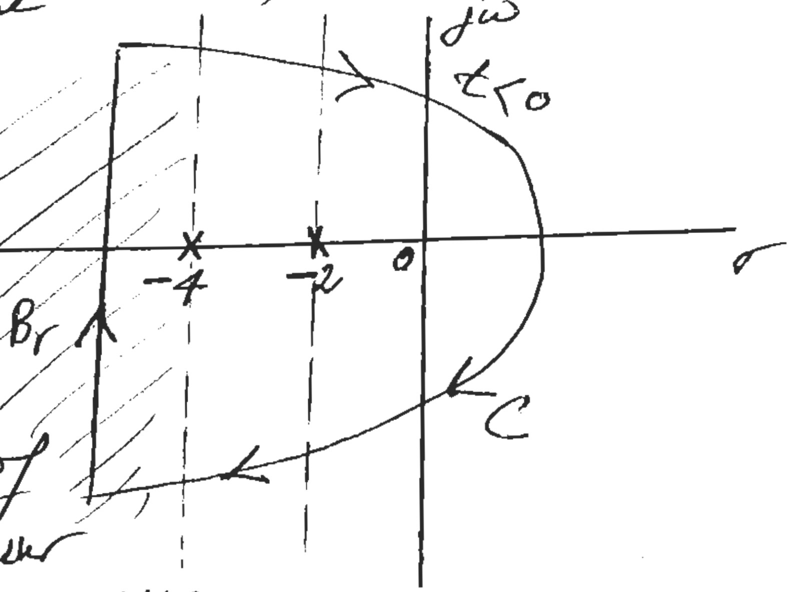
The ROC contains the $j\omega$ axis and the system is stable.

Problem #12 10/10

$$H(s) = \frac{2}{(s+4)(s+2)}$$

The vertical strips in the s-plane such that,

1) $\sigma < -4$
2) $-4 < \sigma < -2$
3) $\sigma > -2$



are possible regions of convergence for whatever system having the above $H(s)$.

In our problem, $h(t) = 0$, for $t > 0$ and it is unstable and non-causal.

Since system is unstable $\Rightarrow$ ROC $\not\ni$ $j\omega$ axis.

" is non-causal $\Rightarrow$ ROC is not a right-half s-plane.

$\Rightarrow \sigma > -2$ is excluded.

the system is such $h(t) = 0$, for $t > 0$, hence $-4 < \sigma < -2$ is also exclude.

$\Rightarrow$ ROC is $\sigma < -4$.

$$h(t) = \frac{1}{2\pi j}\int_{B_r} H(s) e^{st} ds = -\text{Residue at } s=-4 \; - \text{residue at } s=-2$$

$$= \frac{-2e^{st}}{s+2}\Big|_{s=-4} + \frac{-2e^{st}}{s+4}\Big|_{s=-2}$$

$$= -e^{-2t} + e^{-4t}; \text{ for } t<0 \text{ or } h(t) = (e^{-4t} - e^{-2t})u(-t)$$