AMERICAN UNIVERSITY OF BEIRUT
ELECTRICAL AND COMPUTER ENGINEERING DEPARTMENT

EECE 440	SIGNALS AND SYSTEMS	Spring 2004-2005
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Problem 1 (3 pts)

Let s(t) be the SSB-SC wave obtained by transmitting only the upper sideband, $\hat{s}(t)$ its Hilbert transform, and m(t) the message signal.

a. Express m(t) as a function of s(t) and its Hilbert transform (1 pt)

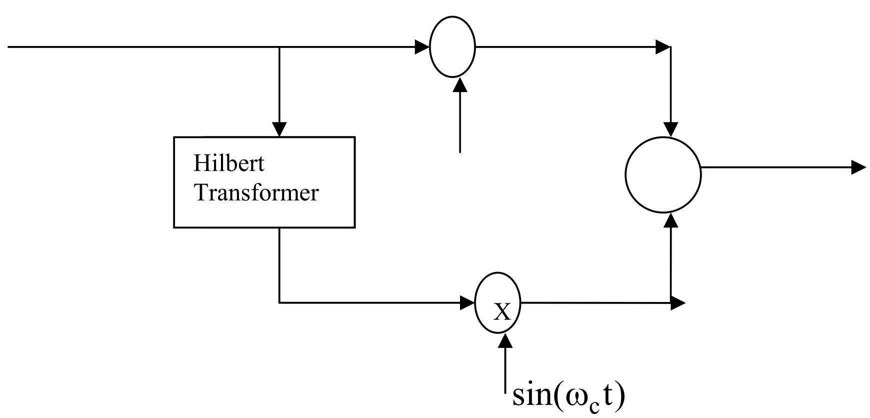
$$s(t) = \frac{A_c}{2} m(t)\cos(\omega_c t) - \frac{A_c}{2}\hat{m}(t)\sin(\omega_c t)$$

$$\hat{s}(t) = \frac{A_c}{2} m(t)\sin(\omega_c t) + \frac{A_c}{2}\hat{m}(t)\cos(\omega_c t)$$ **(0.5 pt)**

from the above, we conclude

$$m(t) = \frac{2}{A_c} s(t)\cos(\omega_c t) + \frac{2}{A_c}\hat{s}(t)\sin(\omega_c t)$$ **(0.5 pt)**

b. Using this result obtained in (a) to set up the block diagram of a receiver for demodulating SSB-SC wave. (2pts)



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Problem 2 (2 pts)
Find the instantaneous frequency of the FM signal

$$s(t) = 10\left[\cos(10t)\cos(30t^2) - \sin(10t)\sin(30t^2)\right]$$

Solution

$$s(t) = 10\cos(10t + 30t^2) = 10\cos(\theta_i(t)) \qquad 0.5 \text{ pt}$$

$$\omega_i(t) = \frac{d\theta_i(t)}{dt} = 10 + 60t \ \text{ rad/s} \qquad 1.5 \text{ pts}$$

Problem 3 (3 pts)
A sinusoidal carrier is frequency modulated by a 4 KHz sinusoid wave resulting in an FM signal having a maximum frequency of 107.41 MHz and a minimum frequency of 107.196 MHz.

a. Determine the carrier frequency. (1 pt)

$$f_c = \frac{f_{Max} + f_{Min}}{2} = \frac{107.41 + 107.196}{2} = 107.303 \ \text{ MHz}$$

b. Determine the number of impulses in the spectrum of s(t) within its bandwidth. (2 pts)

Number of impulses on each side of the carrier= 107/4 = 26.75
Total number of impulses+ 26+26+1 = 53 impulses

Problem 4 (3 pts)
An FM modulator has a carrier frequency of 1 MHz and a frequency sensitivity $k_f = 5\,(\text{Vsec})^{-1}$. The modulator has input $m(t) = 4\cos(20\pi t)$ Volts. What is the modulation index?

Solution

$$\beta = \frac{k_f a_m}{f_m} = 2 \quad \text{using different scale, -1 pt}$$

Problem 5 (3 pts)
Consider the FM signal obtained from

$$m(t) = \begin{cases} 1\text{ V} & 0 \le t \le T = 100\mu s \\ 0 & \text{elsewhere} \end{cases}$$

Let the bandwidth of m(t) be defined by the first crossing of M(ω) with the ω axis. Determine the frequency sensitivity of the FM modulator if the transmission bandwidth is set at 180Khz. The bandwidth computation is to be based on the Carson's rule.

Solution

$$M(\omega) = \frac{-1}{j\omega}\left[e^{j\omega 10^{-4}} - 1\right]$$

The first crossing takes place when $10^{-4}\omega = 2\pi$, that is
The bandwidth of the signal m(t) = 10 KHz (2 pts)

$$B_T = 2(B + \Delta f) \Rightarrow 2\Delta f = 80\text{KHz}$$
$\Delta f = a_m k_f \Rightarrow k_f = 80$ KHz/V because $a_m = 1$ Volt.

Problem 6 (3 pts)
Consider a single tone FM signal with amplitude 10 Volts, frequency 5 KHz, and modulation index $\beta = 2$. Determine the ratio of the average power of the frequency components of this FM signal contained within its bandwidth to the total signal average power. Use Carson's rule for bandwidth computation.

1. Total Power $= \frac{A_c^2}{2}$ Watts (1 pt)
2. The transmission bandwidth of this FM signal$= 2f_m(1+\beta) = 30$ KHz 1 pt
3. Power with the bandwidth$= \frac{A_c^2}{2}\left[J_0^2(2) + 2J_1^2(2) + 2J_2^2(2) + 2J_3^2(2)\right]$
   1 pt
4. Ratio ≈ 1

Problem 7 (3 pts)

Consider the signal $x[n] = \cos(0.125\pi n)$. Determine whether or not x[n] is periodic. If so, determine the number of samples per fundamental frequency.

x[n] is periodic with a period N=16

Problem 8 (4 pts)

The input-output of a discrete-time system is given by:

$$y[n] = \cos(x[n+2])$$

a. Is this system linear? Justify your answer
   No, because $\cos(ax[n+2]) \neq a\cos(x[n+2])$
b. Is this system time invariant? Justify your answer.
   Yes, because $x[n-n_0] \rightarrow y[n-n_0]$

Problem 9 (4 pts)

The input-output of a discrete-time system is given by:

$$y[n]=0.8\, x[n+1] + 0.5\, x[n]$$

a. Determine the system impulse response.
   $h[n] = 0.8\delta[n+1] + 0.5\delta[n]$
b. Is this system causal? Justify your answer.
   No, because $h[n] \neq 0$ for n<0.

Problem 10 (4 pts)

The signal $x(t) = e^{-2t}u(t)$ is sampled every 0.5 seconds.

a. Represent the sampled signal as a sequence x[n]. (2 pts)
   $x[n] = e^{-n}u[n]$

b. Find the z-transform X(z) of x[n]. (2 pts)
   $X(z) = \frac{1}{1-e^{-1}z^{-1}}$ with ROC $|z| > \frac{1}{e}$ (-1 for no ROC)

Problem 11 (3 pts)

Assume that the response of an LTI system to the input x[n]=u[n] (discrete-time unit step) is given by:

$y[n] = \delta[n] + 2\delta[n-1] - \delta[n-2]$

Derive the impulse response h[n] for this system.

$$
\begin{aligned}
h[n] &= y[n] - y[n-1] \\
&= \delta[n] + 2\delta[n-1] - \delta[n-2] - \delta[n-1] - 2\delta[n-2] + \delta[n-3] \\
&= \delta[n] + \delta[n-1] - 3\delta[n-2] + \delta[n-3]
\end{aligned}
$$

Problem 12 (4 pts)

Assume an LTI system of the following form:

$$y[n] = \sum_{k=0}^{M} b_k x[n-k]$$

with impulse response,

$$h[n] = 3\delta[n] + 7\delta[n-1] + 13\delta[n-2] + 9\delta[n-3] + 5\delta[n-4]$$

a. Give numeric values for *M* and b_k

M=4

$b_0 = 3, b_1 = 7, b_2 = 13, b_3 = 9, b_4 = 5$

b. Compute y[n] for an input x[n] given by:

$$x[n] = \begin{cases} 0 & \text{n is even} \\ 1 & \text{n is odd} \end{cases}$$

$$y[n] = \begin{cases} 16 & \text{n is even} \\ 21 & \text{n is odd} \end{cases}$$

Problem 13 (3 pts)

Determine the inverse *z*-transform of:

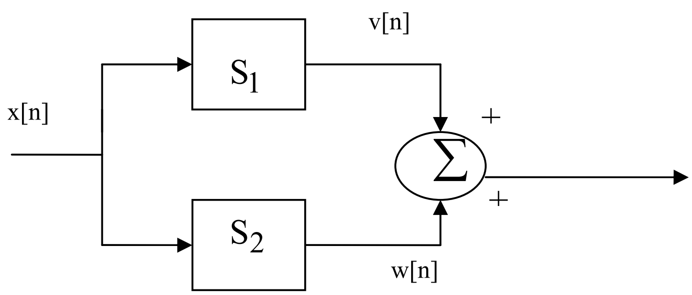
$$F(z) = \frac{z(z+1)}{(z-1)(z^2 - z + 1/4)} \quad \text{with ROC } \frac{1}{2} < |z| < 1$$

$$\mathrm{F(z)} = \frac{8}{(1 - z^{-1})} - \frac{6}{\left(1 - 0.5z^{-1}\right)^2} - \frac{2}{\left(1 - 0.5z^{-1}\right)}$$

$$\mathrm{f[n]} = -6(n-1)\left(\frac{1}{2}\right)^{n-1} u[n-1] - 2\left(\frac{1}{2}\right)^{n} u[n] - 8(1)^n u[-n-1]$$

Problem 14 (3 pts)

For discrete-time system shown,



The input-output relationship for subsystems S_1 and S_2 are given by the difference equations

$$v[n] - \frac{1}{2} v[n-1] = x[n] + x[n-1] \qquad v[n]=0,\ n<0$$

$$w[n] - \frac{1}{2} w[n-1] = x[n] - x[n-1] \qquad w[n]=0,\ n<0$$

Determine the system impulse response.

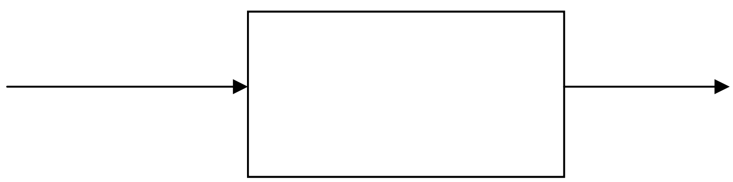
$$H_1(z) = \frac{1 + z^{-1}}{1 - 0.5z^{-1}} \quad \text{or} \quad H_2(z) = \frac{1 + z^{-1}}{1 - 0.5z^{-1}}$$

$$H(z) = H_1(z) + H_2(z) = \frac{2}{1 - 0.5z^{-1}}, \quad u[n] = 2\left(\frac{1}{2}\right)^n u[n]$$

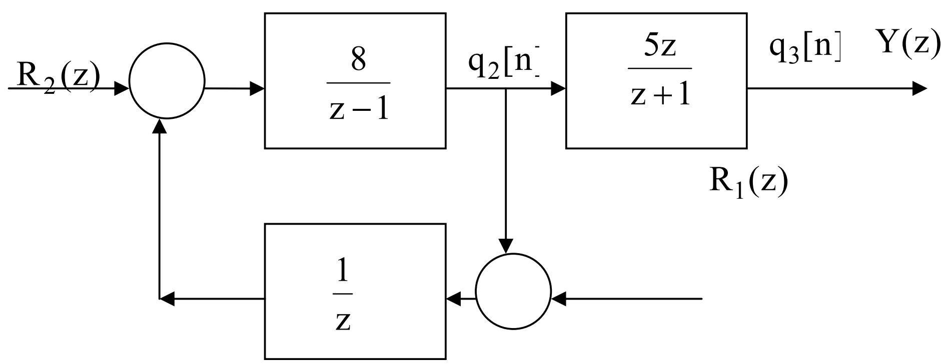
Problem 15 (2 pts)
Find a **block diagram** representation for the following difference equation

$$y[n]+2y[n-1]=5x[n]+2x[n-1]$$

$$Y(z)=\frac{5+2z^{-1}}{1+2z^{-1}}X(z)$$



Problem 16 (3 pts)
Given the following block diagram for a discrete-time system, find a state variable representation for the system.



$$q_1[n+1]=q_2[n]+r_2[n]$$
$$q_2[n+1]=8q_1[n]+q_2[n]+8r_2[n]$$
$$q_3[n+1]=40q_1[n]+5q_2[n]-q_3[n]+40r_2[n]$$

$$y[n]=q_3[n]$$