AMERICAN UNIVERSITY OF BEIRUT
ELECTRICAL AND COMPUTER ENGINEERING DEPARTMENT

EECE 440 SIGNALS AND SYSTEMS Spring 2006-2007

Quiz I- Solution

Problem 1 (3 pts)

Determine whether the following signal is periodic. If it is periodic, find its period.

$$x(t) = \sin\left(\frac{5}{13}\pi^2 t\right)$$

x(t) is periodic with a period T=26/5π seconds

Problem 2 (4 pts)

a. Consider the everlasting signal $X(t) = e^{-at}$. Is X(t) an energy signal? (2 pts)

$E = \int_{-\infty}^{\infty} x^2(t)dt = \infty$, therefore X(t) is not an energy signal.

b. For which values of “a” X(t) is a power signal? Determine its average power. (2 pts)

X(t) is a power signal if a is a complex quantity. In this case Pav=1Watt.

Problem 3 (6 pts)

The input-output relationship of a system is given by:

$$y(t) = \int_{-\infty}^{+\infty} (-1)^t e^{\tau} x(\tau) d\tau$$

a. Is this system stable? Justify your answer. (2 pts)

$$|y(t)| \le \int_{-\infty}^{+\infty} \left|(-1)^t e^{\tau} x(\tau)\right| d\tau \le M \int_{-\infty}^{\infty} e^{\tau} d\tau = \infty$$

Not stable

b. Is this system linear? Justify your answer. (2 pts)

Yes, apply the definition

c. Is this system Time invariant? Justify your answer. (2 pts)

No, the system is a time varying system

Problem 4 (6 pts)

The input-output relationship of a system is given by:

$$\frac{dy}{dt}+3y+2\int_{-\infty}^{t} y(t)\,dt = x(t)$$

a. Is this system linear? Justify your answer (2 pts)

$$\frac{d^2y(t)}{dt^2}+3\frac{dy(t)}{dt}+2y(t)=\frac{dx(t)}{dt}$$

Yes, it represents a Linear differential equation with constant coefficient

b. Is this system stable? Justify your answer. (2 pts)

$$\frac{Y(s)}{X(s)}=\frac{S}{S^2+3S+2}$$

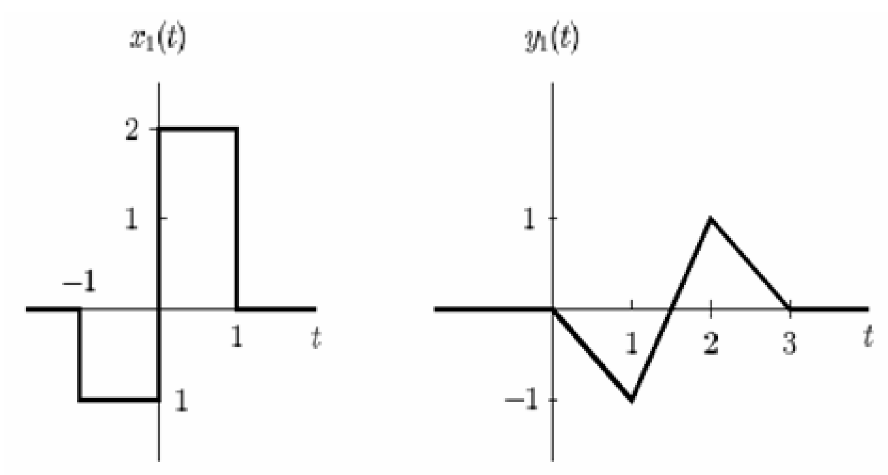
Stable as all poles are in the rhp, s=-1, and s=-2

c. Is this system causal? justify your answer. (2 pts)

Yes, h(t)=0 for t<0.

Problem 5 (5 pts)

The signal $x_1(t)$, shown below, is the input of an LTI system whose impulse response $y_1(t)$ is also shown below. Determine the output signal.



$$X_1(s)=\frac{1}{s}\left[-e^{s}+3-2e^{-s}\right] \quad 1\text{ pt}$$

$$Y_1(s)=\frac{1}{s^2}\left[-1+3e^{-s}-3e^{-2s}+e^{-3s}\right] \quad 1\text{ pt}$$

$$Y(s)=X_1(s)Y_1(s)=\frac{1}{s^3}\left[e^{s}-6+14e^{-s}-16e^{-2s}+9e^{-3s}-2e^{-4s}\right] \quad 1\text{ pt}$$

$$y(t)=\frac{1}{2}\begin{bmatrix}(t+1)^2u(t+1)-6t^2u(t)+14(t-1)^2u(t-1)-16(t-2)^2u(t-2)\\+9(t-3)^2u(t-3)-2(t-4)^2u(t-4)\end{bmatrix}$$ 2 pts

Problem 6 (6 pts)

Let h(t) be the impulse response of a LTI system and its Laplace transform is given by:

$$H(s)=\frac{10(-s+1)}{(s+10)(s+1)}$$

a. Find the differential equation describing the system (2 pts)

$$\frac{d^2y(t)}{dt^2}+11\frac{dy(t)}{dt}+10y(t)=-10\frac{dr(t)}{dt}+10r(t)$$

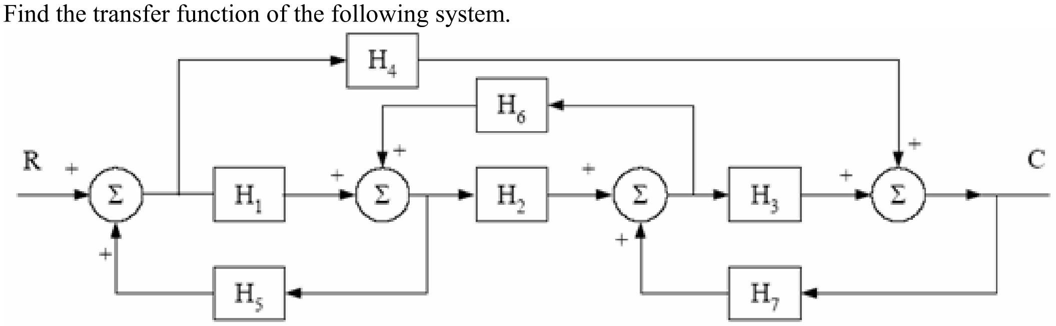
b. Represent this system in a state-space form by writing the state equations and the output equation . (4 pts)

$$\begin{bmatrix}X_1'(t)\\X_2'(t)\end{bmatrix}=\begin{bmatrix}-11 & 1\\-10 & 0\end{bmatrix}\begin{bmatrix}X_1(t)\\X_2(t)\end{bmatrix}+\begin{bmatrix}-10\\10\end{bmatrix}r(t)$$

$$y(t)=\begin{bmatrix}1 & 0\end{bmatrix}\begin{bmatrix}X_1(t)\\X_2(t)\end{bmatrix}$$

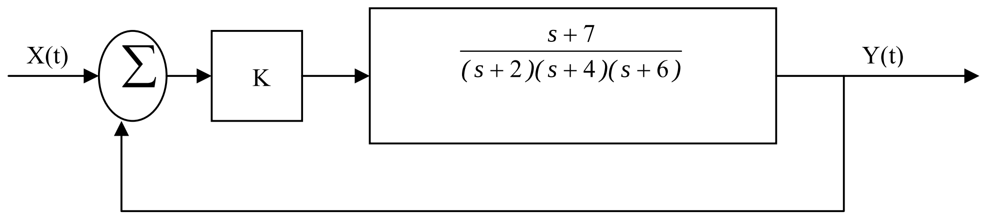
Problem 7 (4 pts)

Find the transfer function of the following system.



$$TF=\frac{H_1H_2H_3+H_4[1-H_2H_6]}{1-[H_1H_5+H_3H_7+H_2H_6+H_4H_5H_6H_7]+[H_1H_3H_5H_7]}$$

Problem 8 (6 pts)
Consider the unit feedback system shown below



a. Determine the error signal E(s). (3 pts)

$$E(s)=\frac{X(s)}{1+G(s)}=\frac{K(s+2)(s+4)(s+6)X(s)}{(s+2)(s+4)(s+6)+K(s+7)}$$

b. Determine the range of K for the system to be stable (3 pts)

$$\frac{Y(s)}{X(s)}=\frac{K(s+7)}{(s+2)(s+4)(s+6)+K(s+7)}=\frac{K(s+7)}{s^3+12s^2+(44+K)s+48+7K}$$

s^3	1	44+K
s^2	12	48+7K
s^1	$\frac{528+5K}{12}$	
s^0	48+7K	

System is stable for all K>0.