AMERICAN UNIVERSITY OF BEIRUT

Department of Electrical and Computer Engineering

EECE440 Signals and Systems - Spring 2006

Lecturer: Prof. Fadi N Karameh

Quiz 2, May 17, 2006

Directions:

- You will have 1.5 hrs for this quiz.
- Write down your initials *in ink* on all the pages. DO IT NOW!
- Enter ALL your work and your answers on the answer booklet. You can use the back of these pages for scratch. I will ONLY grade the work you neatly transfer to the assigned spaces on the booklet.
- Answers must be explained or derived. DO NOT just write down an answer, unless otherwise indicated.
- It is a good idea to read the whole test before you begin. Problems are divided into several parts with percentages indicated. *You might be able to solve different parts independently.*
- DO NOT talk to any of your colleagues under any circumstances. You will be penalized without warning.

YOUR NAME HERE:

Please space below empty for graders to use.

Problem 1

Problem 2

Problem 3

Problem 4

Problem 5

Problem 6

Total ____________

PROBLEM 1 (24%)

a) Consider the following differential equation of a CT LTI system:

$$\frac{d^2y(t)}{dt^2} - 3\frac{dy(t)}{dt} - 4y(t) = x(t)$$

Find the impulse response of the corresponding system if it is known to be stable.

b) Find the unit sample responses of all *stable* DT LTI systems which can result in the following difference equation

$$y[n-1] - \frac{5}{2}y[n] + y[n+1] = x[n]$$

c) The CT pulse $x(t)$ shown in figure 1 is applied to a CT LTI system to give the output $y(t)$ shown in the same figure. Find the impulse response of this system $h(t)$.

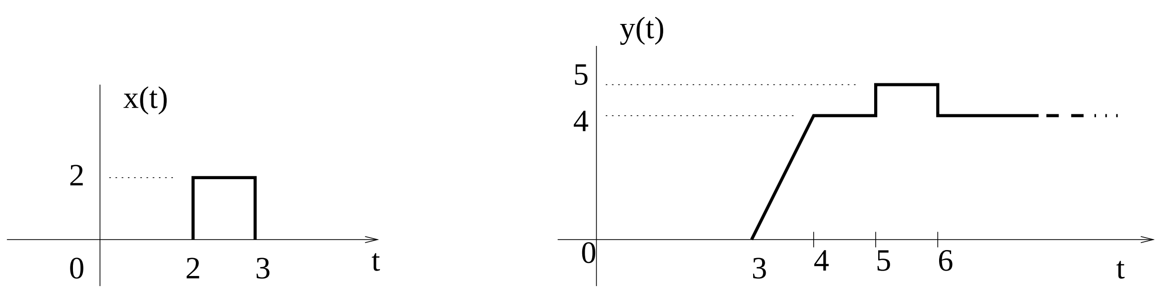

Figure 1:

d) Find the output of a DT LTI system when the input sequence $x[n]$ and the sample response $h[n]$ are as shown in figure 2.

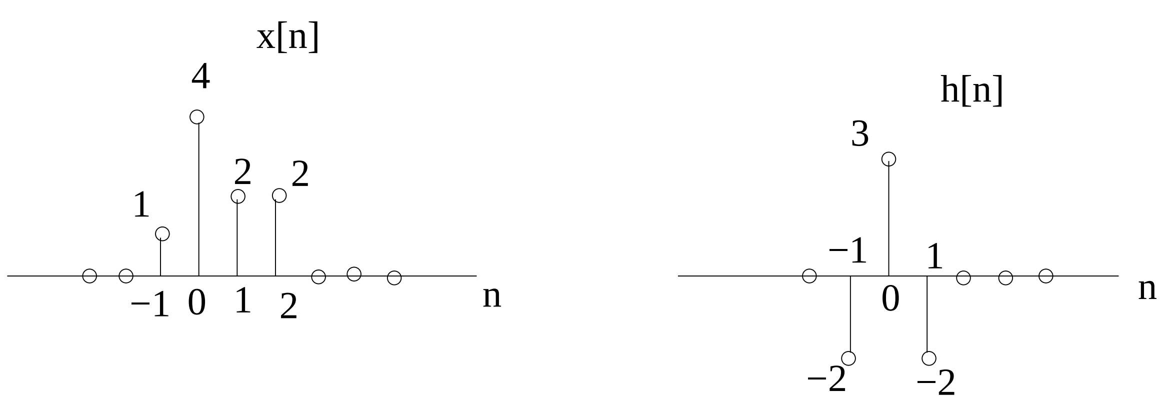

Figure 2:

PROBLEM 2 (16%)

Consider the linear difference equation describing a DT LTI system:

$$y[n] + \frac{2}{9}y[n-2] - y[n-1] = x[n] - 2x[n-1]$$

a) Find a block diagram realization of this system using *a minimum number* of delays.

b) Find the zero state response of this system if $x[n] = u[n] - u[n-2]$.

c) Find the output of this system if the input is $x[n] = (-2)^n$, $\forall n$.

PROBLEM 3 (12%)

Consider the system shown in figure 3. Given the input is

$$x(t) = \frac{2\pi}{w_m}\left(\frac{\sin\frac{w_m t}{2}}{\pi t}\right)^2$$

and $s(t)$ is the ideal picket fence, given by

$$s(t) = \sum_{k=-\infty}^{\infty} \delta(t - kT)$$

and

$$\frac{2\pi}{T} = \frac{3}{2}w_m$$

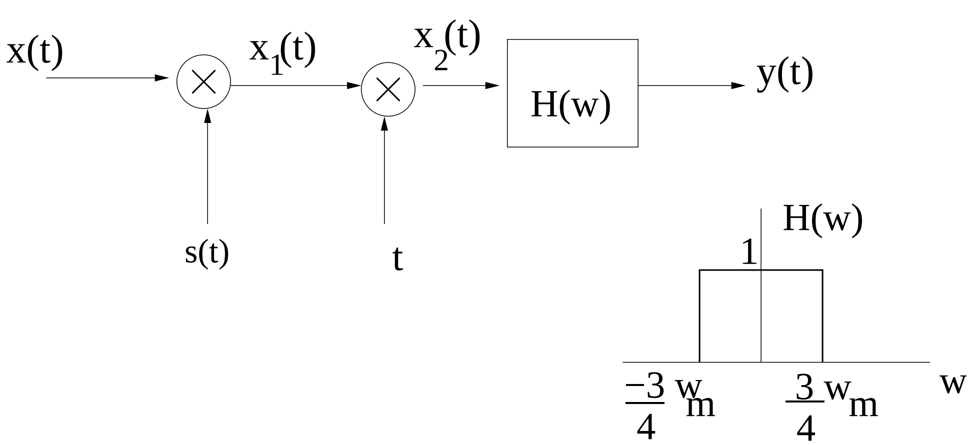

Figure 3:

(a) Find the output signal in the frequency domain $Y(w)$ and in the time domain $y(t)$.

PROBLEM 4 (15%)

Consider a stable CT LTI system whose frequency response $H(w) = \int_{-\infty}^{\infty} h(t)e^{-jwt}dt$ is as shown in figure 4.

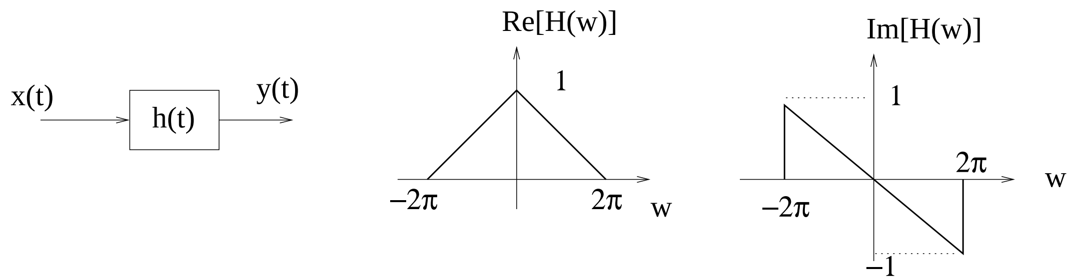

Figure 4:

a) Determine the output $y(t)$ if the input to this system is $x(t) = 3\sin(\pi t)$.

b) If the output of this system is $y(t) = e^{-3t}u(t)$, find the corresponding input to $h(t)$ that could have resulted in $y(t)$. Explain your reasoning.

c) Find the impulse response $h(t)$ of this system. (Hint: use simple Fourier transform pairs.)

PROBLEM 5 (21%)

In this problem we will consider one type of AM modulators known as the *switching modulator*. This is given in figure 5

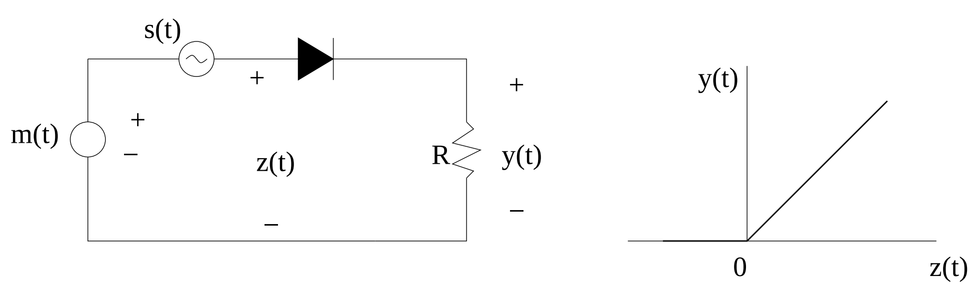

Figure 5:

In the above figure, $m(t)$ is the message signal which is bandlimited to W_m, that is $X(w) = 0$, $|w| > w_m$. $s(t)$ is the carrier signal $s(t) = A\cos w_c t$ where $w_c >> w_m$.

The diode is assumed to be an ideal switch with the input-output characteristics given in the figure. That is,

$$y(t) = \begin{cases} z(t), & z(t) > 0; \\ 0, & \text{else.} \end{cases}$$

a) To understand this modulator, we need to approximate the operation of the diode. To do so, assume that $A >> |m(t)|$. In other words, as far as the diode is concerned, we can assume that $z(t) = m(t) + A\cos w_c t \approx A\cos w_c t$.

Sketch a time function $d(t)$ which *approximates* the effect of the diode. That is, find $d(t)$ which allows us to write

$$y(t) \approx (m(t) + A\cos w_c t).d(t)$$

b) Using your knowledge of Fourier transform, and what you obtained in part (a) about $d(t)$, sketch $y(t)$ in the frequency domain $Y(w)$.

c) Which are the undesired components of $Y(w)$? suggest a scheme whereby only the desired AM signal is retained.

PROBLEM 6 (12%)

Consider the sampling scheme shown in figure 6. Assume that $x(t)$ is bandlimited to w_m ($X(w) = 0$, $|w| > w_m$). The sampling function $s(t)$ is a periodic square wave of period T as shown in figure 6.

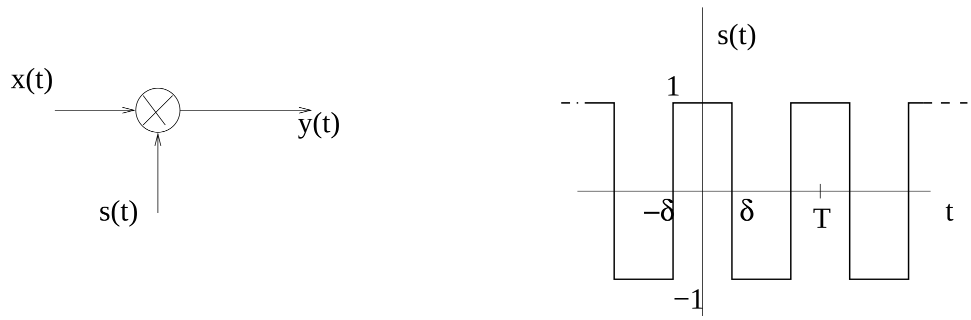

Figure 6:

Assume that $\delta = \frac{T}{4}$, determine the *minimum* frequency of $f_s = \frac{1}{T}$ so that no aliasing occurs in $y(t)$.