AMERICAN UNIVERSITY OF BEIRUT
Department of Electrical and Computer Engineering
EECE440 Signals and Systems

Lecturer: Prof. Fadi N Karameh

Sample problems 3

PROBLEM 1 (12%) Consider a DT LTI system with the impulse response

$$h[n] = \left(\frac{1}{2}\right)^n u[n]$$

Determine, using DTFTs, the response to each of the following inputs:

(i) $x[n] = \left(\frac{3}{4}\right)^n u[n]$

(ii) $x[n] = (n+1)\left(\frac{1}{4}\right)^n u[n]$

(iii) $x[n] = (-1)^n u[n]$

PROBLEM 2 Consider a system consisting of the cascade of two DT LTI systems with the frequency responses

$$H_1(e^{j\Omega}) = \frac{2 - e^{-j\Omega}}{1 + \frac{1}{2}e^{-j\Omega}}$$

and

$$H_2(e^{j\Omega}) = \frac{1}{1 - \frac{1}{2}e^{-j\Omega} + \frac{1}{4}e^{-j2\Omega}}$$

a) Find the difference equation describing the overall system.

b) Determine the impulse response of the overall system.

PROBLEM 3 $y_1[n]$ and $y_2[n]$ are two real 8-pt sequences. You are given only the first five points of their DFTs as

$$\tilde{Y}_1[k] = [2 \quad 1 \quad 0 \quad 1 \quad 2]$$
$$\tilde{Y}_2[k] = [0 \quad 1 \quad 2 \quad 0 \quad 0]$$

b) Find the DTFT $\tilde{M}[k]$ for $k = 1$ and $k = 4$ where $m[n]$ is

$$m[n] = y_1[n].y_2[n]$$

Hint: Exploit knowledge of DFT properties for real sequences.

PROBLEM 4 Find the DTFT of the following:

a) $x[n] = \left(\frac{1}{2}\right)^n u[n+2]$

b) $x[n] = (n-2)(u[n+4] - u[n-5])$

PROBLEM 5 Find the signal $x[n]$ which has the following DTFT

a) $X(e^{j\Omega}) = j\sin(4\Omega) - 2$

b) $\left[e^{-j2\Omega}\frac{\sin(15\Omega/2)}{\sin(\Omega/2)}\right] * \frac{\sin(7\Omega/2)}{\sin(\Omega/2)}$.

PROBLEM 6 (10 %)

Consider the analog-to-discrete-to-analog system shown in figure 1. The CT signal $x_a(t)$ is sampled at a frequency of $F_s = 2000$ Hz ($T_s = 0.5$ msec). The resulting impulse train is then converted to a discrete time sequence $x_d[n]$. The Lowpass DT filter $H_d(e^{j\Omega})$ is subsequently used to filter $x_d[n]$ giving $y_d[n]$. Finally, a CT version of the output $y_a(t)$ is created, using an ideal DT-to-CT converter (at the same sampling frequency $F_s = 2000$ Hz).

Note : $H_d(e^{j\Omega})$, which is obviously periodic, is shown for only one period.

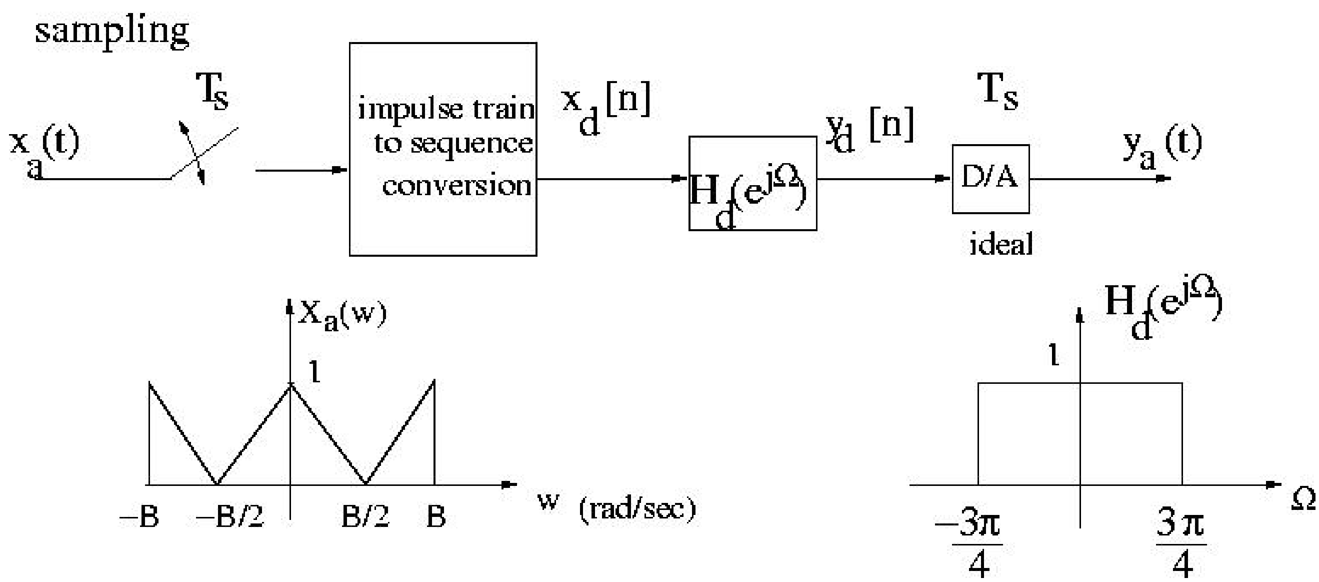

Figure 1: Problem 6

a) For the CTFT of $x_a(t)$ given by $X_a(w)$ in the figure with $B = 2000\pi$ rad/sec, sketch the $X_d(e^{j\Omega})$, the DTFT of the DT sequence $x_d[n]$.

b) Sketch $Y_a(w)$, the CTFT of the CT signal $y_a(t)$. Again, assume that we are using the same frequency as that of sampling $F_s = 2000$ Hz.

c) (BONUS 5%) Describe, using Matlab code, how would you use *Matlab* to obtain $X(w)$, the CTFT of a CT signal $x(t)$. Assume that I sample $x(t)$ at 200 Hz and I have 1 second recording of it. Clearly describe the various input and output vectors you obtain, and label the coordinates of your axes. Hint: you have done this twice in the course so far.