AMERICAN UNIVERSITY OF BEIRUT
EECE440 –
Sample problems 3 (Karameh)

Problem 1

(i) We have

$$X(e^{j\omega}) = \frac{1}{1-\frac{3}{4}e^{-j\omega}}.$$

Therefore,

$$\begin{aligned} Y(e^{j\omega}) &= \left[\frac{1}{1-\frac{3}{4}e^{-j\omega}}\right]\left[\frac{1}{1-\frac{1}{2}e^{-j\omega}}\right] \\ &= \frac{-2}{1-\frac{1}{2}e^{-j\omega}} + \frac{3}{1-\frac{3}{4}e^{-j\omega}} \end{aligned}$$

Taking the inverse Fourier transform, we obtain

$$y[n] = 3\left(\frac{3}{4}\right)^n u[n] - 2\left(\frac{1}{2}\right)^n u[n].$$

(ii) We have

$$X(e^{j\omega}) = \frac{1}{(1-\frac{1}{4}e^{-j\omega})^2}.$$

Therefore,

$$\begin{aligned} Y(e^{j\omega}) &= \left[\frac{1}{(1-\frac{1}{4}e^{-j\omega})^2}\right]\left[\frac{1}{1-\frac{1}{2}e^{-j\omega}}\right] \\ &= \frac{4}{1-\frac{1}{2}e^{-j\omega}} - \frac{2}{1-\frac{1}{4}e^{-j\omega}} - \frac{3}{(1-\frac{1}{4}e^{-j\omega})^2} \end{aligned}$$

Taking the inverse Fourier transform, we obtain

$$y[n] = 4\left(\frac{1}{2}\right)^n u[n] - 2\left(\frac{1}{4}\right)^n u[n] - 3(n+1)\left(\frac{1}{4}\right)^n u[n].$$

(iii) We have

$$X(e^{j\omega}) = 2\pi \sum_{k=-\infty}^{\infty} \delta(\omega - (2k+1)\pi).$$

Therefore,

$$\begin{aligned} Y(e^{j\omega}) &= \left[2\pi \sum_{k=-\infty}^{\infty} \delta(\omega-(2k+1)\pi)\right]\left[\frac{1}{1-\frac{1}{2}e^{-j\omega}}\right] \\ &= \frac{4\pi}{3}\sum_{k=-\infty}^{\infty}\delta(\omega-(2k+1)\pi) \end{aligned}$$

Taking the inverse Fourier transform, we obtain

$$x[n] = \frac{2}{3}(-1)^n.$$

Problem 2

(a) Since the two systems are cascaded, the frequency response of the overall system is

$$H(e^{j\omega}) = H_1(e^{j\omega})H_2(e^{j\omega}) = \frac{2 - e^{-j\omega}}{1 + \frac{1}{8}e^{-j3\omega}}$$

Therefore, the Fourier transforms of the input and output of the overall system are related by

$$\frac{Y(e^{j\omega})}{X(e^{j\omega})} = \frac{2 - e^{-j\omega}}{1 + \frac{1}{8}e^{-j3\omega}}.$$

Cross-multiplying and taking the inverse Fourier transform, we get

$$y[n] + \frac{1}{8}y[n-3] = 2x[n] - x[n-1].$$

b) We may rewrite the overall frequency response as

$$H(e^{j\omega}) = \frac{4/3}{1 + \frac{1}{2}e^{-j\omega}} + \frac{(1 + j\sqrt{3})/3}{1 - \frac{1}{2}e^{j120}e^{-j\omega}} + \frac{(1 - j\sqrt{3})/3}{1 - \frac{1}{2}e^{-j120}e^{-j\omega}}.$$

Taking the inverse Fourier transform we get

$$h[n] = \frac{4}{3}\left(-\frac{1}{2}\right)^n u[n] + \frac{1 + j\sqrt{3}}{3}\left(\frac{1}{2}e^{j120}\right)^n u[n] + \frac{1 - j\sqrt{3}}{3}\left(\frac{1}{2}e^{-j120}\right)^n u[n].$$

Problem 3

$\check{Y}_1(k) = [2\ 1\ 0\ 1\ 2]$; $\tilde{Y}_2(k) = [0\ 1\ 2\ 0\ 0]$

$z(n) = y_1(n) \circledast_8 y_2(n)$

Since we have 8-pt DFT & $y_1(n)$, $y_2(n)$ are real

Then DFT is symmetric @ 4 pts. $\Rightarrow \tilde{Y}_1(k) = [2\ 1\ 0\ 1\ 2\ 1\ 0\ 1]$

$\check{Y}_2(k) = [0\ 1\ 2\ 0\ 0\ 0\ 2\ 1]$

$m[n] = y_1(n) \cdot y_2(n)$

$\Rightarrow \tilde{M}[k] = \frac{1}{8}\left(\tilde{Y}_1(k) \circledast_8 \tilde{Y}_2(k)\right) = \frac{1}{8} \sum \tilde{Y}_1(\lambda)\, \tilde{Y}_2((k-\lambda)_8)$

$\tilde{Y}_1(\lambda) = (2\ 1\ 0\ 1\ 2\ 1\ 0\ 1)$

$\tilde{Y}_2((0-n)_8) = [0\ 1\ 2\ 0\ 0\ 0\ 2\ 1]$

$\tilde{Y}_2((1-n)_8) = [1\ 0\ 1\ 2\ 0\ 0\ 0\ 2]$

$\Rightarrow \tilde{M}[1] = \frac{1}{8}(2+2+2) = \frac{6}{8} = \frac{3}{4}$

also $\tilde{Y}_2((4-n)_8) = [0\ 0\ 2\ 1\ 0\ 1\ 2\ 0]$

$\Rightarrow \tilde{M}[4] = \frac{1+1}{8} = \frac{2}{8} = \frac{1}{4}$

Problem 4

(a) $x[n] = \left(\frac{1}{3}\right)^n u[n+2]$

$$\begin{aligned} x[n] &= (\frac{1}{3})^n u[n+2] \\ &= (\frac{1}{3})^{-2}(\frac{1}{3})^{n+2}u[n+2] \\ (\frac{1}{3})^n u[n] &\xleftrightarrow{DTFT} \frac{1}{1-\frac{1}{3}e^{-j\Omega}} \\ s[n+2] &\xleftrightarrow{DTFT} e^{j2\Omega}S(e^{j\Omega}) \\ X(e^{j\Omega}) &= \frac{9e^{j2\Omega}}{1-\frac{1}{3}e^{-j\Omega}} \end{aligned}$$

(b) $x[n] = (n-2)(u[n+4]-u[n-5])$

$$\begin{aligned} u[n+4]-u[n-5] &\xleftrightarrow{DTFT} \frac{\sin(\frac{9\Omega}{2})}{\sin(\frac{\Omega}{2})} \\ ns[n] &\xleftrightarrow{DTFT} j\frac{d}{d\Omega}S(e^{j\Omega}) \\ x[n] &= j\frac{d}{d\Omega}\frac{\sin(\frac{9\Omega}{2})}{\sin(\frac{\Omega}{2})} - 2\frac{\sin(\frac{9\Omega}{2})}{\sin(\frac{\Omega}{2})} \end{aligned}$$

Problem 5

(a)

$$\begin{aligned} X(e^{j\Omega}) &= j\sin(4\Omega) - 2 \\ &= \frac{1}{2}e^{j4\Omega} - \frac{1}{2}e^{-j4\Omega} - 2 \\ x[n] &= \frac{1}{2}\delta[n+4] - \frac{1}{2}\delta[n-4] - 2\delta[n] \end{aligned}$$

(b) $X(e^{j\Omega}) = \left[e^{-j2\Omega}\frac{\sin(\frac{15}{2}\Omega)}{\sin(\frac{\Omega}{2})}\right] \circledast \left[\frac{\sin(\frac{7}{2}\Omega)}{\sin(\frac{\Omega}{2})}\right]$

Let the first part be $A((e^{j\Omega}))$, and the second be $B(e^{j\Omega})$.

$$\begin{aligned}
a[n] &= \begin{cases} 1 & |n-2| \le 7 \\ 0, & \text{otherwise} \end{cases} \\
b[n] &= \begin{cases} 1 & |n| \le 3 \\ 0, & \text{otherwise} \end{cases} \\
X((e^{j\Omega})) &= A((e^{j\Omega})) \circledast B(e^{j\Omega}) \xleftrightarrow{DTFT} x[n] = 2\pi a[n] b[n] \\
x[n] &= \begin{cases} 2\pi & |n| \le 3 \\ 0, & \text{otherwise} \end{cases}
\end{aligned}$$