

(60%) Part I: Circle the correct answer on the table provided on page one

(Multiple choices, with 2 points for each correct answer and -0.25 point penalty on each wrong answer)

1. The two solutions of the quadratic equation $2x^2 - 6x = 0$ are :

a) 3 and 0 b) 2 and 0 c) -3 and 0 d) -2 and 0

2. The solution of the logarithmic equation $\ln(3x-5) = 0$ is

a) -3 b) -2 c) 2 d) 3

3. The coordinates of the vertex of the parabola $y = x^2 + 8x$ are :

a) (-5, -16) b) (-4, -25) c) (-4, -16) d) (-5, -25)

4. $\lim_{x \rightarrow 1} \frac{3x-2}{6x} =$

a) $-\frac{1}{6}$ b) $\frac{1}{6}$ c) $\frac{1}{5}$ d) $-\frac{1}{5}$

5. The equation of the vertical line through (-2, 3) is:

a) $x = -2$ b) $y = -2$ c) $x = 3$ d) $y = 3$

6. The solution of the inequality $(x-2)(x^2+3) \geq 0$ is

a) $[3, +\infty)$ b) $(-\infty, -3) \cup (2, +\infty)$ c) $(-\infty, -2) \cup (3, +\infty)$ d) $[2, +\infty)$

7. $\frac{a^2c^3}{b} \times \left(\frac{b^2c^{-1}}{a}\right)^2 =$

a) ca^3 b) b^2c^{-1} c) b^3c^{-5} d) cb^3

8. Given the logarithmic equation $\log_z n = m$. The equivalent exponential equation is:

a) $n^m = z$ b) $z^m = n$ c) $m^z = n$ d) $z^n = m$

9. Let $y = f(x) = \frac{2}{3}x - 5$ and $g(x) = \frac{3}{\sqrt{x}}$ then $f(g(4)) =$

a) -4 b) -2 c) -3 d) -5

10. $\log_a 1 + \log_a 6 - \log_a 3 =$

a) $\log_a 3$ b) $\log_a \frac{1}{2}$ c) $\log_a 2$ d) $\log_a \frac{1}{3}$

11. $\lim_{x \rightarrow 0} \frac{x^2 + 1}{2x^2 - 3x} =$

a) $\frac{1}{2}$ b) 0 c) ∞ d) $-\frac{1}{2}$

12. $\lim_{x \rightarrow 0} \frac{5x - 1}{x + 2}$

a) $\frac{1}{2}$ b) 0 c) ∞ d) $-\frac{1}{2}$

13. If $g(x) = 5^{-2x}$ and $f(x) = x^2 - 1$ then $g(0) - 2f(1) =$

- a) 2 b) 1 c) 0 d) -1
-

14. Consider the system $\begin{cases} x - y = 2 \\ -x + 3y = -4 \end{cases}$ then the unique solution is:

- a) $(1, -1)$ b) $(2, -1)$ c) $(-1, 1)$ d) $(2, 1)$
-

15. The domain of definition of $f(x) = \frac{\sqrt{x^2 - 3x}}{x^2 + 1}$ is

- a) $x \neq -1, x \neq 1$ b) $-3 \leq x \leq 3$ c) $x \leq 0, x \geq 3$ d) $0 \leq x \leq 3$
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16. The system $\begin{cases} 2x - y = 0 \\ x + y = -6 \\ x + 2y = 5 \end{cases}$ has

a) no solution b) a unique solution
 c) infinitely many solutions d) exactly two solutions

17. The solution set of the equation : $\frac{3x + 2}{x^2 + 1} = 0$ is :

- a) $\{3\}$ b) $\left\{-\frac{2}{3}\right\}$ c) $\{2\}$ d) $\left\{-\frac{3}{2}\right\}$
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18. Given $f(x, y, z) = 3e^x - y^2 - (\ln z + 2)^2$ then $f(0, -1, 1) =$

- a) -3 b) 3 c) -2 d) 2
-

19. $\sum_{i=1}^3 e^{\ln(i+1)} =$

a) 12 b) 9 c) 11 d) 5

20. Given $f(x) = \begin{cases} x + 3 & \text{if } x < 1 \\ 2x + 1 & \text{if } x \geq 1 \end{cases}$ then $\lim_{x \rightarrow 1^-} f(x) =$

- a) 2 b) 4 c) 3 d) 1

21. If $f(x) = x^2 - 1$, then $f(2a+1) =$

- a) $3a^2(3a+2)$ b) $3a(3a+2)$ c) $4a^2(a+1)$ d) $4a(a+1)$

22. $\frac{x^2 + 5x + 6}{(x+3)(x+1)} \div \frac{x+2}{x^2 - 1} =$

- a) $(x-2)$ b) $(x+2)$ c) $(x+1)$ d) $(x-1)$

23. $\log_3 \sqrt{3^5} =$

- a) $\frac{2}{5}$ b) $\frac{5}{2}$ c) $\frac{3}{2}$ d) $\frac{2}{3}$

24. The value of a truck is estimated by the function: $V = f(t) = 45,000 - 3,000t$ where V equals the value stated in dollars and t equals the age of the truck expressed in years, then the value of the truck will be equal to zero after:

- a) 15 years b) 16 years c) 22 years d) 12 years

25. The y-intercept of the function $y = 2 \ln(x^2 + e) - e^{3x} + 5$ is

- a) 5 b) 6 c) 8 d) 7

26. A step of (3x3) Gaussian elimination method has produced the following array:

$$\left(\begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ 0 & 1 & -2 & 0 \end{array} \right), \text{ then the solution set is:}$$

- a) $\{(x_1, x_2, x_3) / x_1 = x_2 = x_3 = 0\}$ b) $\{(x_1, x_2, x_3) / x_1 = 0, x_2 = -2x_3, x_3 \in R\}$
c) $\{(x_1, x_2, x_3) / x_1 = 0, x_2 = 2x_3, x_3 \in R\}$ d) $\{(x_1, x_2, x_3) / x_1 = 0, x_3 = -2x_2, x_2 \in R\}$

If $f(x) = 3^x + 2$, then:

27. Domain $f =$

- a) \mathbb{R} b) $(2, +\infty)$ c) $(-2, +\infty)$ d) $(3, +\infty)$

28. Range $f =$

- a) $(3, +\infty)$ b) $(2, +\infty)$ c) $(-2, +\infty)$ d) \mathbb{R}

29. The equation of the asymptote is:

- a) $y = 2$ b) $y = 3$ c) $y = -3$ d) $y = -2$

30. The profit function for a firm is: $P(q) = -10q^2 + 3600q - 45000$

Where q equals the number of units sold and P equals annual profit in dollars.
If 150 units were sold then the profit expected is equal to:

- a) \$ 207 000 b) \$ 249 000 c) \$ 239 000 d) \$ 270 000

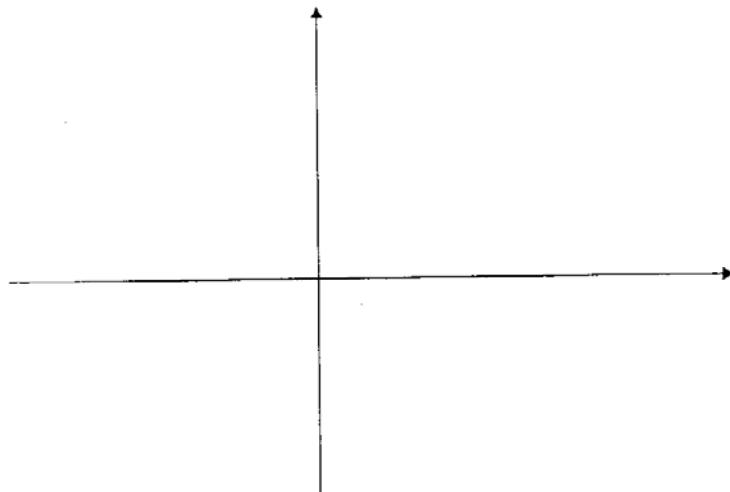
(40%)Part II (Answer each of the following questions in the space provided. Justify and show your work)

- 1) Given $\left(\begin{array}{ccc|c} 1 & 0 & -7 & -20 \\ 0 & -2 & -12 & 18 \\ 0 & 0 & 5 & 15 \end{array} \right)$, complete the remaining steps of the Gaussian method and determine the solution (x_1, x_2, x_3) .

(8%)

- 2) Sketch function $f(x) = \begin{cases} x+2 & \text{if } x \leq 0 \\ 2^x & \text{if } x > 0 \end{cases}$

(4%)



3) Solve the following equations:

a) $x^2 \ln x - 9 \ln x = 0$

(3%)

b) $(e^{2x} - 4)(e^x - 1) = 0$

(3%)

c) $|2x + 3| = x - 4$

(3%)

4) For each of the following find the limit (if it exists):

a- $\lim_{x \rightarrow 3} \frac{x-3}{x+1} =$

(2%)

b- $\lim_{x \rightarrow 2} \frac{x^2 + 5x - 14}{x^2 - 4}$

(3%)

c- $f(x) = \begin{cases} \frac{x^2}{x^2 + 2x} & \text{if } x \leq 1, x \neq 0 \\ \frac{2-x}{3} & \text{if } x > 1 \end{cases}$

$$\lim_{x \rightarrow 1^-} f(x) =$$

(3%)

$$\lim_{x \rightarrow 1^+} f(x) =$$

$$\lim_{x \rightarrow 1} f(x) =$$

Is $f(x)$ continuous at $x = 1$? Justify your answer.

(3%)

- 5) For each of the following functions determine the derivative of $y = f(x)$ using the rules of differentiation.
(Don't simplify your answer):

a) $y = 3x^2 - 5$

(2%)

b) $y = (x^3 - 2x)e^x$

(3%)

c) $y = \frac{\ln x}{x^4}$

(3%)