American University of Beirut
الجامعة الأميركية في بيروت

EXPERIMENT 4:
RC & RLC CIRCUITS

EECE 310L

Group1 – Section5

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OBJECTIVES

- Investigate the frequency and time responses of RC circuits.
- Investigate the frequency response of series RLC circuits.
- Use the oscilloscope to do frequency, time, and phase measurements.

LAB EQUIPMENT USED

- Breadboard
- Function generator (Agilent 33120A)
- Oscilloscope (Tektronix TDS220)
- Digital Multimeter (DMM)
- Cables/wires

LAB TOOLS USED

- Wire stripper and Cutter.

COMPONENTS USED

Table 1: Components Used

Resistor	Theoretical Value	Measured Value	% error
	56 Ω	74.75 Ω	33.48 %
	100 Ω	119.4 Ω	19.4 %
	1 k Ω	0.9934 k Ω	0.66 %
	20 k Ω	19.73 k Ω	1.35 %
Capacitor	1 nF	NA	NA
	100 nF	NA	NA
	1000 nF	NA	NA
Inductor	220 µH	NA	NA
	470 µH	NA	NA

EXPERIMENTAL PROCEDURE AND DISCUSSION

A. Phase shift measurements

1. CIRCUIT DIAGRAMS

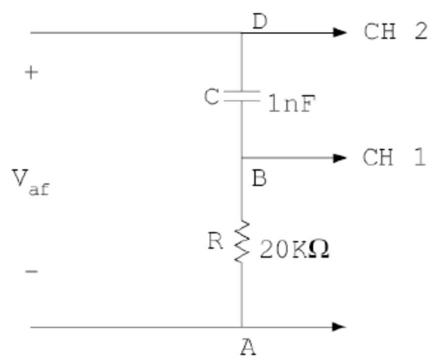


Figure 1: Phase shift circuit diagram

2. DETAILED EXPERIMENTAL PROCEDURE

- Select the correct resistor by calculating the resistance using color band. Choose a 20-kΩ resistor and a 1nF capacitor from the given package.
- Build the given circuit (Figure1) on the Breadboard. Connect the components in series. Connect the frequency generator in series to the positive and ground lines of the breadboard and connect it in series with the components.
- Set the frequency generator to: Type of Signal Sinusoidal , DC Offset (Amplitude) = 6 V peak-peak , Frequency :5kHz.
- Oscilloscope Settings: Connect both grounds of the oscilloscope's channels to node A. Then connect Channel 1 of the oscilloscope to node B (measuring the voltage across the resistor) and connect Channel 2 of the oscilloscope to node D (measuring the voltage across the frequency generator).

  - In order to calculate the phase shift we have two methods:

1. Phase Shift Measurements Using Time Shift:

- This method takes advantage of the fact that a phase difference between two sinusoids, which is equivalent to a time shift.

- To set the oscilloscope to Y-T mode:
  - Superimpose the two traces (V_{BA} , V_{DA}) in order to obtain the same horizontal axis.
  - Adjust the *Volts/div* and *Sec/div* to get stable traces.
  - Using the two cursors, we measure the time difference of the traces (ΔT).
  - Calculate the phase difference using: $\phi = (\Delta T \times 360)/T$ (knowing that a period is 360°).

2. Phase Shift Measurements Using Lissajous Figure:

- This method uses Lissajous pattern via the X-Y mode of the oscilloscope.

- To set the oscilloscope to Y-T mode:

- Keep the previous settings but change the sweep rate to X-Y by pressing on the *Display* found on the oscilloscopes.
- An ellipse appears on the oscilloscope screen caused by the superposition of the sinusoidal signals VBA and VDA. (see Figure 2)
- Center the ellipse symmetrically by adjusting the *Volts/div* of X and Y and adjust the horizontal and vertical *Position* knobs.

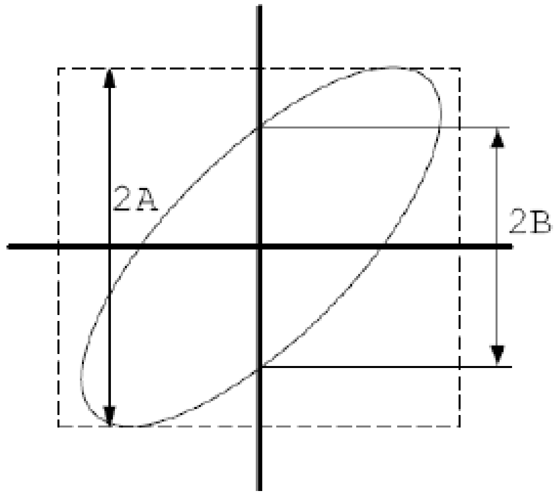


Figure 2: Lissajous figure

- To calculate the phase difference: Count the number of divisions from the min value to the max value of the ellipse, and then we count the number of divisions between the intersection points of the ellipse with the Y-axis, which correspond to 2A, and 2B respectively.

The phase difference:

$$\sin\varphi = \frac{B}{A} \Rightarrow \varphi = \sin^{-1}\frac{B}{A}$$

- We assumed that the wires have no resistance and we ignored the percentage of error on the resistor, capacitor and inductor.

3. MEASUREMENTS AND RESULTS

Method 1: Time Shift (Y-T format)

Table 2: Phase Shift Measurements

Phase shift measurements			
Y-T	T=0.000032 sec	T=0.0002 sec	Ө=57.6̂

Method 2: Lissajous (X-T format)

Table 3: Phase shift measurements

Phase shift measurements			
Lissajous figure	2B=5	2A=6	Ɵ=56.44°

Theoretical value: $\Theta = \tan^{-1}(X_c/R)$ ➔ Ɵ=57.86°

Table 4: Error Calculation

	Measured Ɵ in degrees	Error Percentage
Method 1	57.6°	0.5%
Method 2	56.44°	2.45%

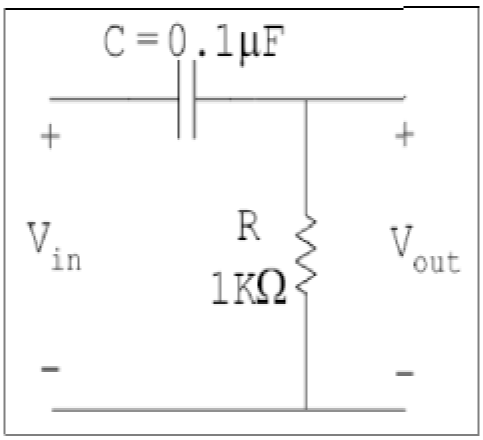
4. DISCUSSIONS

- Comparing the values from (X-Y) mode and (X-T) mode, we find a small difference in Ɵ measured (57.6° & 56.44°).
- At low frequencies (approaching zero), the capacitor acts like an open circuit. The ellipse formed on the oscilloscope's screen is now a vertical straight line.
- At high frequencies (tending to infinity), the capacitor acts like a short circuit. Vout=Vin. The ellipse formed on the oscilloscope's screen is a straight line (X=Y : bisector).

B. Lead and Lag Networks:

I. Lead Networks:

1. CIRCUIT DIAGRAMS:



Lead Network Circuit

Figure 3: Lead Network Circuit

2. DETAILED EXPERIMENTAL PROCEDURE:

Process:

- Connect a 1kΩ resistor at the output voltage, a 0.1 µF capacitor on the breadboard in series.
- Connect the frequency generator in series to the positive and ground lines of the breadboard. Then connect the inputs of the circuits to positive and ground nodes.
- Set the frequency generator to: Type of Signal Sinusoidal , DC Offset (Amplitude) = 1 V peak-peak , Frequency : 100Hz, 1kHz, 10Khz (Change the frequency following the different parts of the measurements).
- Oscilloscope Settings: Connect Channel 1 of the oscilloscope to the nodes of the function generator (Vin) . Connect Channel 2 of the oscilloscope to output to the resistor (Vout).

- Set the oscilloscope mode to (Y-T) and then Press Auto-Set.

- Repeat the same process, setting the frequency generator to Square Wave.

  - We assumed that the wires have no resistance and we ignored the percentage of error on the resistor, capacitor and inductor.

3. MEASUREMENTS AND RESULTS:

- Press “Measure” on the oscilloscope, to get the different measurements of the output voltage and input signal (Peak-Peak value, Frequency).

  i. Sinusoidal Input:

  (a) 100 Hz:

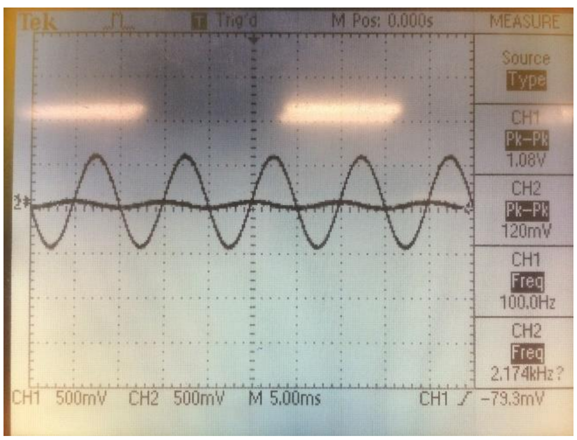


Figure 4: Lead;100Hz

(b) <u>1kHz:</u>

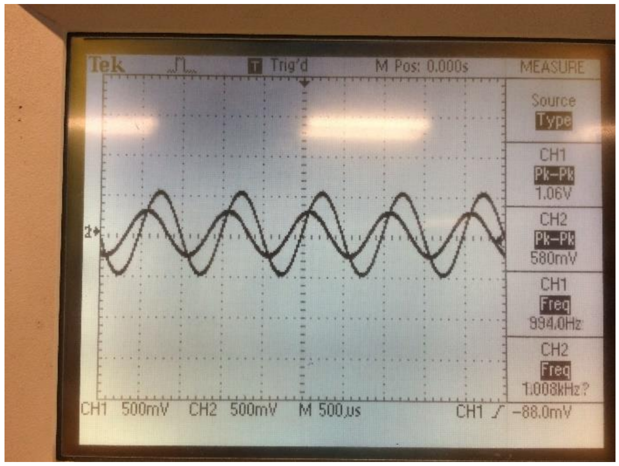


Figure 5: Lead;1kHz

(c) <u>100kHz:</u>

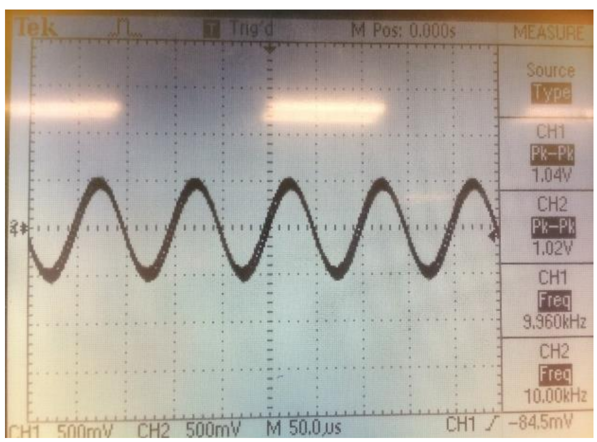


Figure 6: Lead;100kHz

- Table with measured output voltages (Lead Network; Sinusoidal):

Lead Network (measured)		
Frequency	Input Voltage	Output Voltage V_{Pk-Pk}
100 Hz	1 V_{Pk-Pk}	72 mV_{Pk-Pk}
1 KHz	1 V_{Pk-Pk}	552 mV_{Pk-Pk}
10 KHz	1 V_{Pk-Pk}	1020 mV_{Pk-Pk}

Table 5: Lead Measurements

- Table with calculated output voltages (Lead Network; Sinusoidal):

The measured magnitude of the output voltage can be calculated using:

$$|H(jw)| = \frac{\omega RC}{\sqrt{1+\omega^2 R^2 C^2}}$$

Lead Network (Calculated)		
Frequency	Input Voltage	Output Voltage V_{Pk-Pk}
100 Hz	1 V_{Pk-Pk}	62.8 mV_{Pk-Pk}
1 KHz	1 V_{Pk-Pk}	532 mV_{Pk-Pk}
10 KHz	1 V_{Pk-Pk}	987 mV_{Pk-Pk}

Table 6: Lead Calculations

- Table with measurements error (Lead Network; Sinusoidal):

Freqeuncy (Hz)	Calculated $V_{peak-peak}$ (in mv)	Measured $V_{peak-peak}$ (in mv)	Error Percentage
100 Hz	62.8	72	14.64%
1 kHz	532	552	3.76%
10kHz	987	1020	3.34%

Table 7: Error Calculations

ii. Square Wave:

(a) 100Hz:

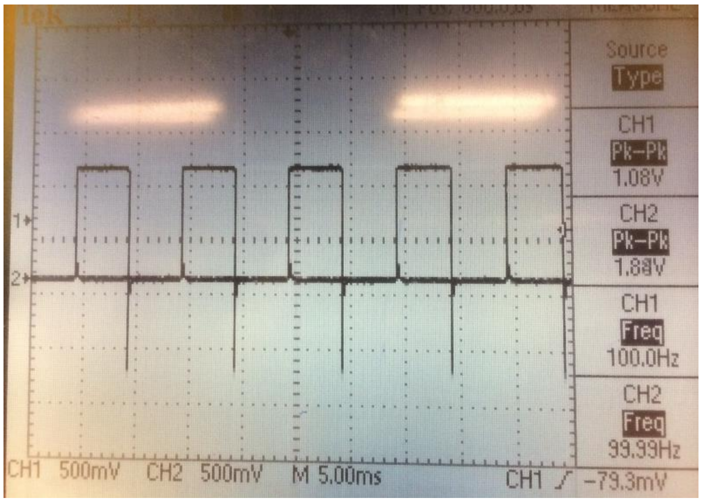


Figure 7: Lead; Square;100Hz

(b) 1kHz:

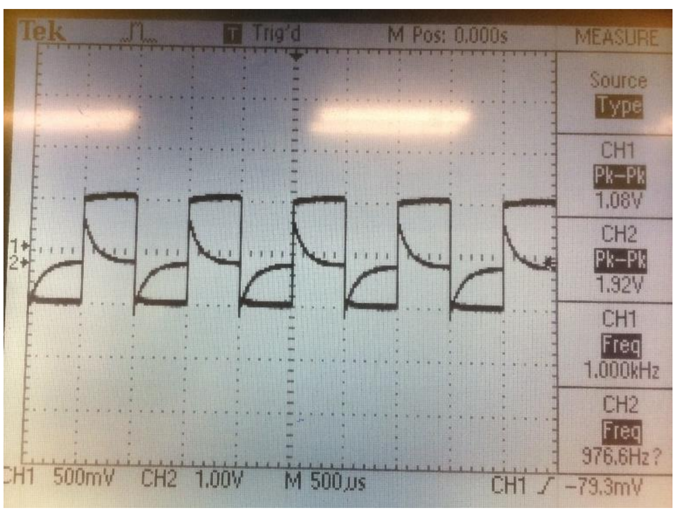


Figure 8: Lead;Square;1kHz

(c) 10kHz:

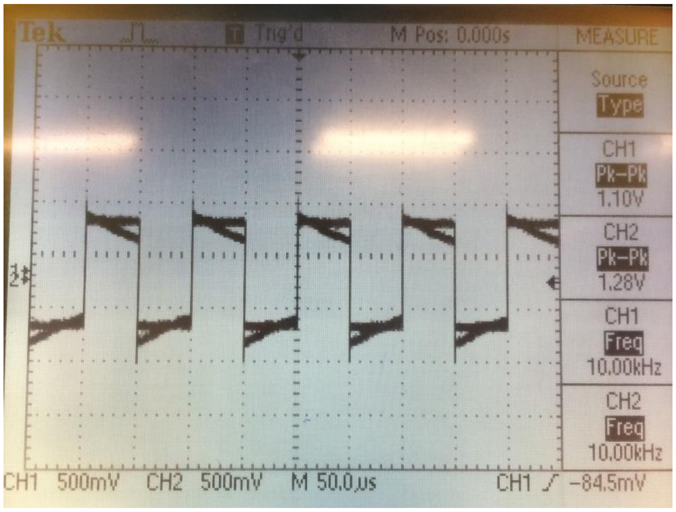


Figure 9: Lead; Square; 10kHz

- Table with measured output voltages (Lead Network; Square):

Lead Network (Measured)		
Frequency	Input Voltage	Output Voltage V_{Pk-Pk}
100 Hz	1 V_{Pk-Pk}	Vpp= 1760 V
1 KHz	1 V_{Pk-Pk}	Vpp= 1920 V
10 KHz	1 V_{Pk-Pk}	Vpp= 1360 V

Table 8: Lead Square Measurements

II. Lag Networks:

4. CIRCUIT DIAGRAMS:

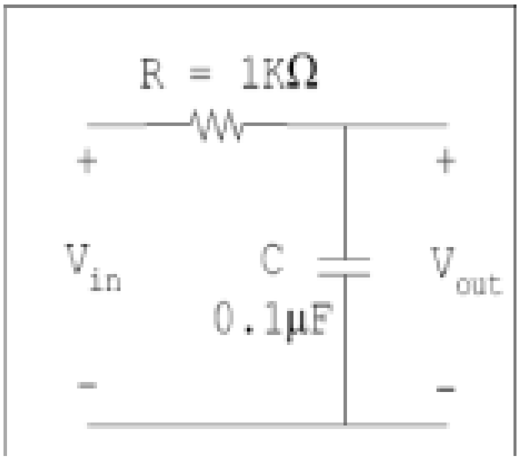


Figure 10: Lag Network Circuit

Lag Network Circuit

5. DETAILED EXPERIMENTAL PROCEDURE:

Process:

- Connect a 1 µF capacitor at the output voltage, a 1kΩ resistor on the breadboard in series.
- Connect the frequency generator in series to the positive and ground lines of the breadboard. Then connect the inputs of the circuits to positive and ground nodes.
- Set the frequency generator to: Type of Signal Sinusoidal , DC Offset (Amplitude) = 1 V peak-peak , Frequency : 100Hz, 1kHz, 10Khz (Change the frequency following the different parts of the measurements).
- Oscilloscope Settings: Connect Channel 1 of the oscilloscope to the nodes of the function generator (Vin) . Connect Channel 2 of the oscilloscope to output to the capacitor (Vout).
- Set the oscilloscope mode to (Y-T) and then Press Auto-Set.

- Repeat the same process, setting the frequency generator to Square Wave.

  - We assumed that the wires have no resistance and we ignored the percentage of error on the resistor, capacitor and inductor.

6. MEASUREMENTS AND RESULTS:

- Press "Measure" on the oscilloscope, to get the different measurements of the output voltage and input signal (Peak-Peak value, Frequency).

  i. Sinusoidal Input:

  (a) 100 Hz:

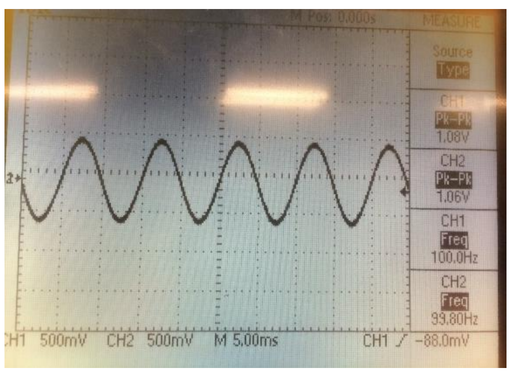


Figure 11: Lag 100Hz

  (b) 1kHz:

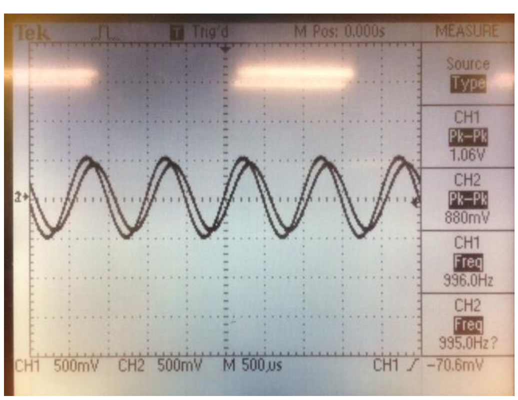


Figure 12: Lag;1kHz

(c) 100kHz:

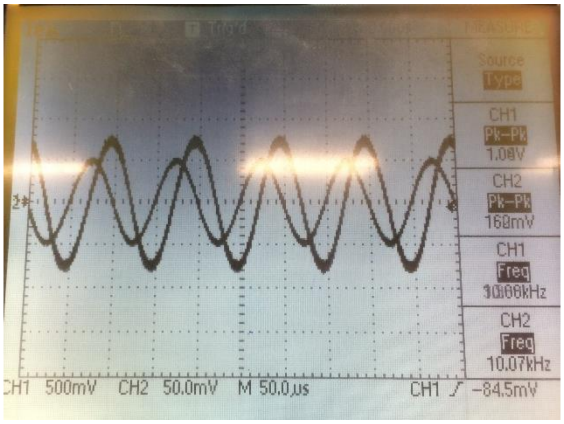


Figure 13: 100kHz

- Table with measured output voltages (Lag Network; Sinusoidal):

Lag Network (Measured)		
Frequency	Input Voltage	Output Voltage $V_{Pk\text{-}Pk}$
100 Hz	1 $V_{Pk\text{-}Pk}$	Vpp= 1080 mV $_{pk\text{-}pk}$
1 KHz	1 $V_{Pk\text{-}Pk}$	Vpp= 880 mV $_{pk\text{-}pk}$
10 KHz	1 $V_{Pk\text{-}Pk}$	Vpp= 160 mV $_{pk\text{-}pk}$

Table 9: Lag Measurements

- Table with calculated output voltages (Lag Network; Sinusoidal):

The measured magnitude of the output voltage can be calculated using:

$$\left|H(jw)\right| = \frac{1}{\sqrt{1+\omega^2 R^2 C^2}}$$

Lag Network (Calculated)		
Frequency	Input Voltage	Output Voltage $V_{Pk\text{-}Pk}$
100 Hz	1 $V_{Pk\text{-}Pk}$	998 $mV_{Pk\text{-}Pk}$
1 KHz	1 $V_{Pk\text{-}Pk}$	846 $mV_{Pk\text{-}Pk}$
10 KHz	1 $V_{Pk\text{-}Pk}$	157 $mV_{Pk\text{-}Pk}$

Table 10: Lag Calculations

- Table with measurements error (Lag Network; Sinusoidal):

Freqeuncy (Hz)	Calculated $V_{peak\text{-}peak}$ (in mv)	Measured $V_{peak\text{-}peak}$ (in mv)	Error Percentage
100 Hz	998	1080	8.21%
1 kHz	846	880	4.01%
10kHz	157	160	1.91%

Table 11: Error Calculations

ii. Square Wave:

(d) <u>100Hz:</u>

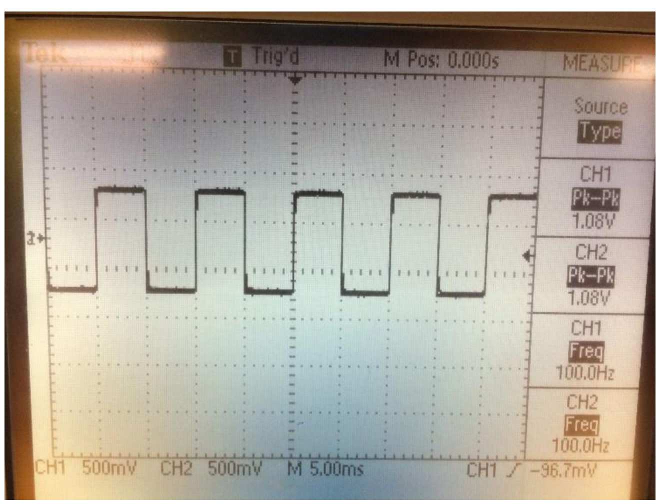


Figure 14: Lag;Square;100Hz

(e) 1kHz:

Figure 15:Lag;Square;1kHz

(f) 10kHz:

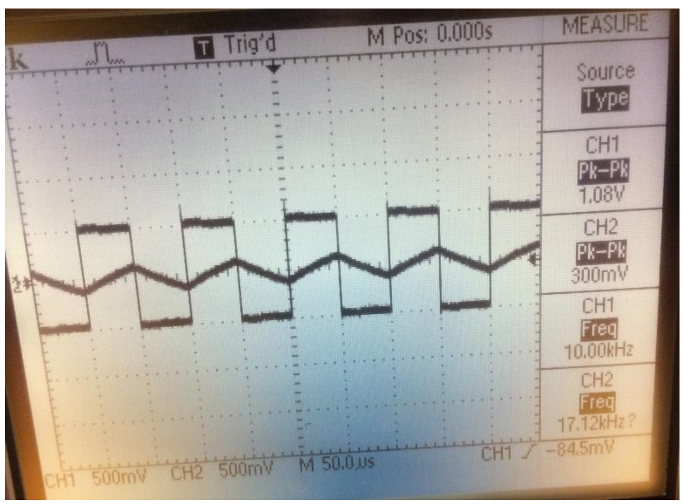


Figure 16: Lag;Square;10kHz

- Table with measured output voltages (Lag Network; Square):

Lag Network (Measured)		
Frequency	Input Voltage	Output Voltage V_{Pk-Pk}
100 Hz	1 V_{Pk-Pk}	Vpp= 1080 mV_{Pk-Pk}
1 KHz	1 V_{Pk-Pk}	Vpp= 1060 mV_{Pk-Pk}
10 KHz	1 V_{Pk-Pk}	Vpp= 300 mV_{Pk-Pk}

Table 12: Lag Square Measurements

7. DISCUSSIONS:

Shape of output with Square wave inputs:

- For the lag network, the shape of the output voltage deviates more from the input square wave, when the frequency increases.
- For the lead network, the shape of the output voltage deviates more from the input square wave, when the frequency decreases.
- The capacitor connected does not change its voltage instantaneously. It keeps the same voltage in transition state while charging and discharging.

The relationship between the RC time constant and the frequency of the square wave so that:

- The lag network does not appreciably distort the square wave: $\frac{1}{F}$ >>> RC
- The lag network acts as an integrator: $\frac{1}{F}$ < RC
- The lead network does not appreciably distort the square wave: $\frac{1}{F}$ < RC
- The lead-network acts as a differentiator: $\frac{1}{F}$ >>> RC

What should be the relationship between the RC time constant and the frequency of the sinusoidal input so that:

- The lag network does not introduce appreciable attenuation: they must both be small.
- The lead network does not introduce appreciable attenuation: they must both be large.

- The above relations are similar to those for the square wave.

  By referring to Fourier's Theorem, a periodic function is the sum of sine's and cosine's.

Low-pass and High-pass filters:

- A low-pass filter is one that allows low frequency, but blocks signals with high frequency, such as frequencies that are above the cutoff frequency. $\omega_c = \dfrac{1}{2\pi RC}$
- A high-pass filter is one that allows high frequency, but blocks signals with low frequency, such as frequency that are below the cutoff frequency. $\omega_c = \dfrac{1}{2\pi RC}$

C. Series RLC Circuits:

1. CIRCUIT DIAGRAMS:

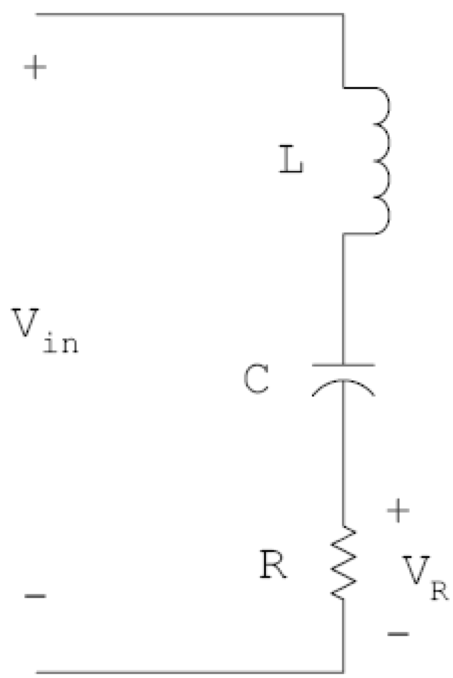


Figure 17: RLC Circuit

2. DETAILED EXPERIMENTAL PROCEDURE:

- Process:
  - Connect a resistor in series with a 1µF capacitor and an inductor.
  - Connect the frequency generator in series to the positive and ground lines of the breadboard. Then connect the inputs of the circuits to positive and ground nodes.
  - Set the frequency generator to: DC Offset (Amplitude) = 1 V peak-peak , Change the frequency following the different parts of the measurements.
  - Use two values different values for each of the resistor, inductor and capacitor: 100Ω and 56Ω for the resistor, 220µH and 470µH for the inductor, 1µF and 0.1µF for the capacitor.

- We assumed that the wires have no resistance and we ignored the percentage of error on the resistor, capacitor and inductor.

3. MEASUREMENTS AND RESULTS:

- **Resonance frequency & Bandwidth calculations:**

- Angular resonant frequency of a RLC: $\omega_0 = 1 / (\sqrt{(LC)})$ in rad/s.
- Resonant frequency in Hz: $f = 1 / (2\pi\sqrt{(LC)})$.
- Bandwidth in radians for the series RLC circuit is given by the formula $BW = \omega_0 / Q$.
- Bandwidth in Hz simplifies to $BW = R / 2\pi L$.

1. (Case 1) : For R = 100Ω, L = 220µH, C = 1µF

   $f = 1/(2\pi\sqrt{(220 * 10^{-6} * 10^{-6})}) = 10730$ Hz = 10.73 kHz.

   BW= R/L = 100 / (2π*220*10^{-6}) = 72.343 kHz.

2. (Case 2): For R = 56Ω, L = 220µH, C = 1µF

   $f = 1/(2\pi\sqrt{(220 * 10^{-6} * 10^{-6})}) = 10730$ Hz = 10.73 kHz.

   BW= R/L = 56 / (2π*220*10^{-6}) = 40.512 kHz.

3. (Case 3): For R = 100Ω, L = 470μH, C = 0.1μF

$f = 1/(2\pi\sqrt{(470 * 10^{-6} * 0.1 * 10^{-6})}) = 23215$ Hz = 23.215 kHz.

$BW = R/L = 100 / (2\pi * 470 * 10^{-6}) = 33.862$ kHz.

Resistance	Inductor	Capacitor	Resonant Frequency	Bandwidth
100 Ω	220 µH	1 µf	10.7 KHz	72.343 KHz
56 Ω	220 µH	1 µf	10.7 KHz	40.512 kHz
100 Ω	470 µH	0.1 µf	23.2 KHz	33.862 kHz

Table 13: Values

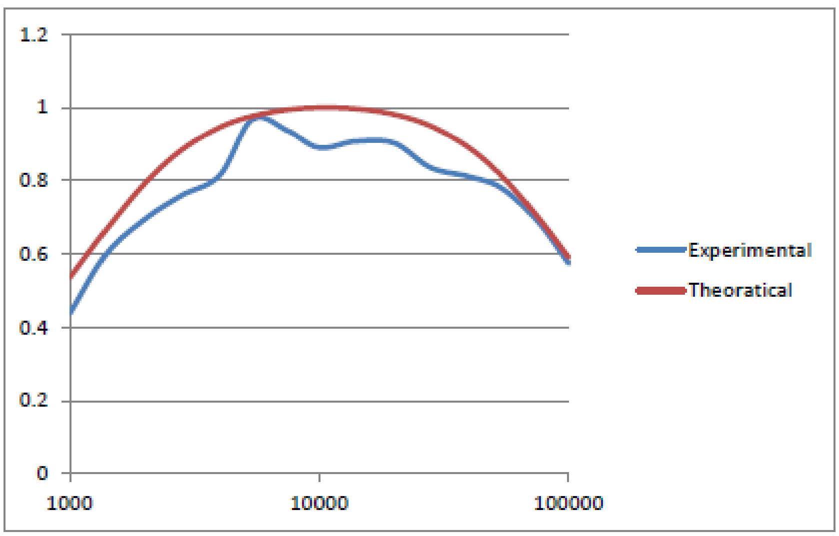
Magnitude and phase measurements:

1. For R = 100Ω, L = 220 µH, and C = 1 µF

Frequency(Hz)	Vout (V)	Vin (V)	Vout/Vin	delta T (seconds)	Theoretical	Output vs input
1000	0.35	0.8	0.4375	0.00014	0.535	leads
1400	0.432	0.72	0.6	0.000092	0.667	leads
2000	0.456	0.656	0.695122	0.000048	0.793	leads
2800	0.504	0.664	0.759036	0.000028	0.884	leads
4000	0.424	0.52	0.815385	0.000013	0.946	leads
5400	0.488	0.504	0.968254	0.000005	0.977	leads
7500	0.472	0.504	0.936508	0.000002	0.994	leads
10000	0.55	0.584	0.941781	0	0.999	in phase
14000	0.552	0.608	0.907895	0	0.997	In phase

20000	0.52	0.576	0.902778	0.0000011	0.981	lags
28000	0.488	0.584	0.835616	0.0000015	0.950	lags
40000	0.48	0.592	0.810811	0.0000017	0.890	lags
54000	0.48	0.616	0.779221	0.0000015	0.813	lags
75000	0.48	0.696	0.689655	0.00000168	0.702	lags
100000	0.432	0.752	0.574468	0.00000148	0.590	lags

Table 14: Case 1



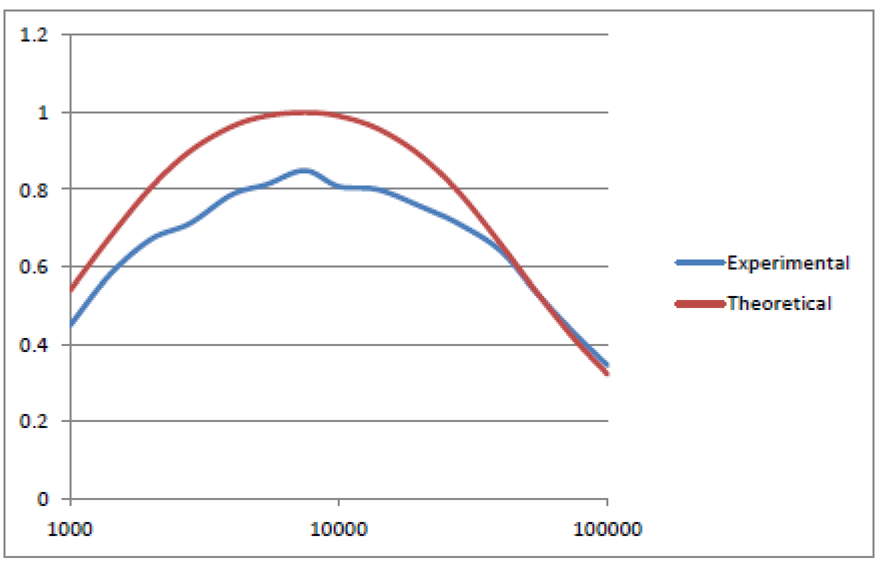
Plot 1: Case 1

2. For R = 100Ω, L = 470 μH, and C = 1 μF

Frequency(Hz)	Vout (V)	Vin (V)	Vout/Vin	delta T (seconds)	Theoretical	Output vs input
1000	0.368	0.82	0.44878	0.000172	0.539	R leads
1400	0.416	0.72	0.577778	0.00009	0.674	R leads
2000	0.44	0.656	0.670732	0.00004	0.805	R leads

2800	0.456	0.64	0.7125	0.000024	0.899	R leads
4000	0.472	0.6	0.786667	0.000012	0.963	R leads
5400	0.488	0.6	0.813333	0.000005	0.991	R leads
7500	0.496	0.584	0.849315	0.000003	0.999	R leads
10000	0.482	0.584	0.825342	0	0.991	In phase
14000	0.48	0.6	0.8	0.000002	0.958	R lags
20000	0.488	0.644	0.757764	0.0000024	0.890	R lags
28000	0.472	0.664	0.710843	0.0000032	0.792	R lags
40000	0.472	0.736	0.641304	0.0000028	0.659	R lags
54000	0.416	0.776	0.536082	0.0000029	0.538	R lags
75000	0.368	0.856	0.429907	0.0000024	0.415	R lags
100000	0.312	0.904	0.345133	0.0000021	0.322	R lags

Table 15: Case 2

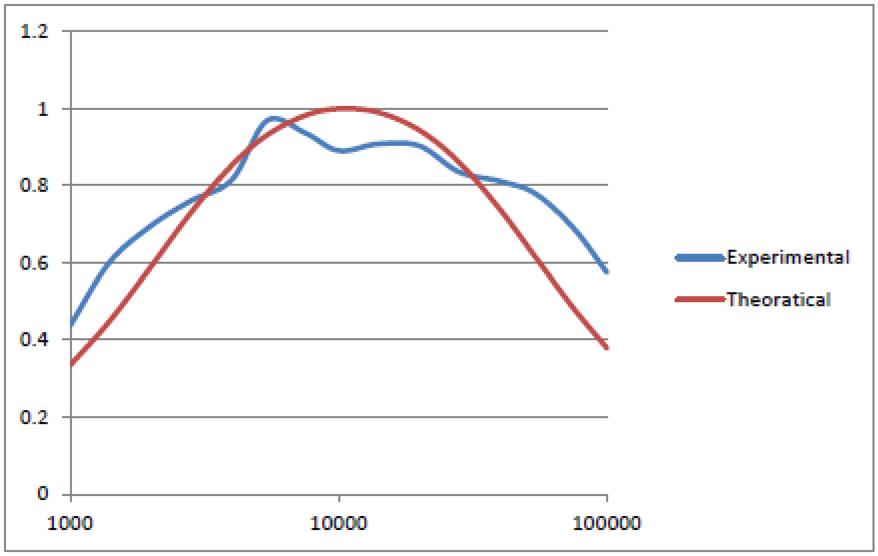


Plot 2:Case 2

3. For R = 56Ω, L = 220 μH, and C = 1 μF

Frequency(Hz)	Vout (V)	Vin (V)	Vout/Vin	delta T (seconds)	Theoretical	Output vs input
1000	0.24	0.8	0.3	0.000188	0.334	R leads
1400	0.252	0.74	0.340541	0.00011	0.448	R leads
2000	0.296	0.592	0.5	0.000064	0.589	R leads
2800	0.316	0.52	0.607692	0.00004	0.726	R leads
4000	0.332	0.48	0.691667	0.000026	0.853	R leads
5400	0.324	0.45	0.72	0.000011	0.930	R leads
7500	0.344	0.432	0.796296	0.000007	0.982	R leads
10000	0.34	0.42	0.809524	0	0.999	In phase
14000	0.34	0.42	0.809524	0	0.990	In phase
20000	0.336	0.432	0.777778	0.000003	0.981	R lags
28000	0.332	0.456	0.72807	0.0000035	0.861	R lags
40000	0.328	0.488	0.672131	0.0000028	0.737	R lags
54000	0.316	0.552	0.572464	0.0000026	0.616	R lags
75000	0.296	0.632	0.468354	0.000002	0.483	R lags
100000	0.268	0.712	0.376404	0.0000018	0.379	R lags

Table 16: Case 3



Plot 3: Case3

4. DISCUSSIONS:

- Magnitude of the transfer function of the RLC circuit: (Output is taken across the resistor (band-pass filter))

$$\left|H(jw)\right| = \frac{R}{\sqrt{R^2 + (\omega L - \frac{1}{\omega C})^2}}$$

(Theoretical values are included above in the tables)

Using the plots, we find the maximal value of f, where the magnitude of the transfer function is maximum and the output voltage = input voltage (the maximum value on the plots, approximately 1).

Bandwidth BW= f2 - f1 (f2 > f1). f2 and f1 are the frequencies for which the magnitude of the transfer function is (1/√ 2). From the plots, we take the approximate values of the BW.

- Case 1: f = 10000 Hz, BW = f2- f1 = 75000 – 2000 = 73000 Hz.
- Case 2: f = 7500 Hz, BW = 40000 - 2000 = 38000 Hz.
- Case 3: f = 10000 Hz, BW = 45000 - 3500 = 41500 Hz.

- We take into account experimental errors for the capacitor, inductor and the approximations for the plots values.

	Bandwidth (kHz)			Resonance Frequency (kHz)		
	Measured value	Calculated Value	Error %	Measured value	Calculated Value	Error %
Case1	73	72.34	0.91%	10	10.73	6.80%
Case2	38	40.51	6.19%	7.5	10.73	30.1%
Case3	41.5	33.86	22.56%	10	23.215	56.9%

Table 17: Frequency & Bandwidth

- *BW= R /2π L :*

The bandwidth is proportional to R. as R increases, the BW increases.

- *BW= R /2π L :*

The bandwidth is inversely proportional to L. As L increases, the BW will decrease.

- BW is independent of C; any variation will not affect it.

- Comparison of the three circuits:

Resonance frequency (case 2)> Resonance frequency (case 1) = (case3).

REFERENCES

- **EECE 310L Experiment 4**
  http://moodle.aub.edu.lb/pluginfile.php/227622/mod_resource/content/2/LAB_04_RC%20and%20RLC%20Circuits.pdf
- **EECE 310L Lab Report Format**
  http://moodle.aub.edu.lb/pluginfile.php/227540/mod_resource/content/4/lab%20report%20format.pdf
- **Moodle in lab page (Experiment 4)**
  http://moodle.aub.edu.lb

MISTAKES AND PROBLEMS FACED

No major difficulties during the experiment, however we had a slight problem reading the values of the DMM where the values kept fluctuating.

"I HAVE NEITHER GIVEN NOR RECEIVED AID ON THIS REPORT NOR HAVE I CONCEALED ANY VIOLATION OF THE AUB STUDENT CODE OF CONDUCT."

Signature: