Fall | 2011

Experiment 4

RC and RLC Circuits
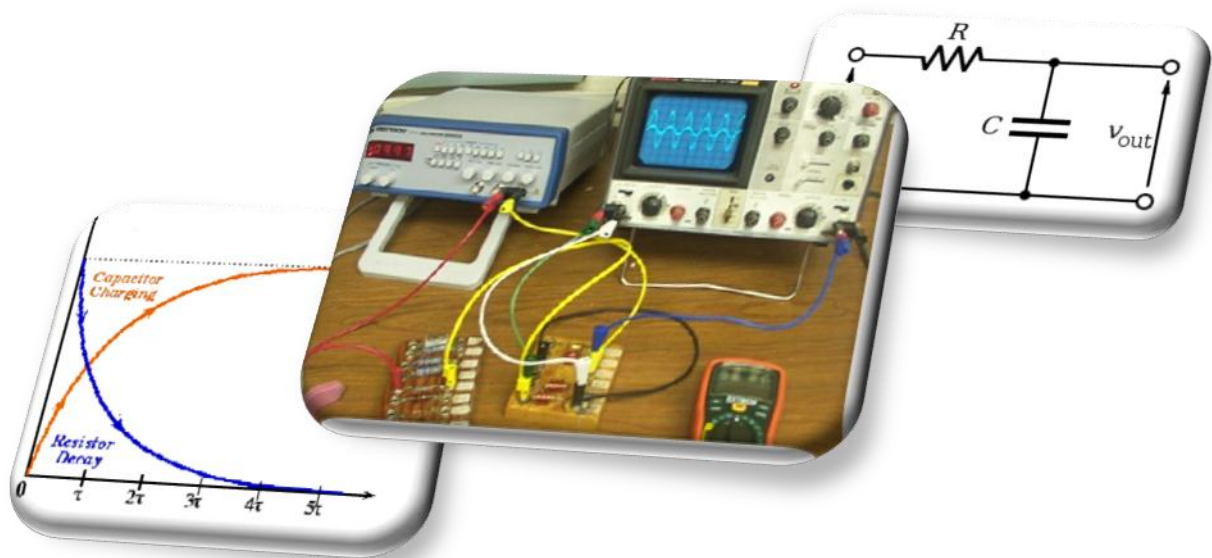
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I.   OBJECTIVES

In this experiment you will learn how to:

- Investigate the frequency response and time response of RC circuits.
- Investigate the frequency response of series RLC circuits.
- Use the oscilloscope to do frequency, time, and phase measurements.

II.   MATERIAL AND PROCEDURE

A.   PHASE SHIFT MEASUREMENTS

Procedure

A1.   PHASE SHIFT CALCULATION

The phase angle can be calculated using the formula:
$$\tan \varphi = \frac{X_C}{R}$$
where
$$X_C = \frac{1}{\omega C} = \frac{1}{2\pi f C}$$

- Calculate the phase angle with f = 5 KHz, C = 1 nF and R = 20 KΩ.

Exercise • Refer to in-lab 4 exercise A-1

A2.   PHASE DIFFERENCE MEASUREMENTS

There are two standard ways to measure the phase difference between two sinusoids of the same frequency by using an oscilloscope. The first method, which requires a dual trace scope, exploits the fact that a phase difference is equivalent to a time shift. The second one relies on the well-known *Lissajous* pattern using the X-Y mode of the oscilloscope.

Using the function generator, apply a sinusoidal voltage (V_{af} = 6 V peak-to-peak) of frequency 5 kHz to the input of the circuit shown in Fig. A-1. Apply V_{BA} to CH 1 of the oscilloscope and V_{DA} to CH 2.
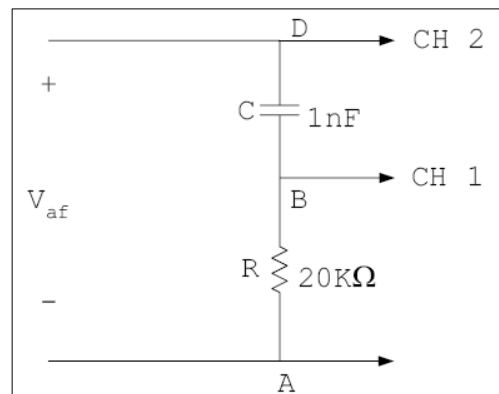
Figure A-1: Circuit Diagram

A.2.I PHASE SHIFT MEASUREMENTS USING TIME SHIFT

Superpose the two traces of VBA and VDA to have the same horizontal axis and adjust the VOLT/DIV and SEC/DIV settings to get stable traces.
Measure the phase difference φ on the oscilloscope.

Note   *The phase difference can be measured from the time instants at which the waveforms cross the time axis. Consider V_{af} to be of the form $3\sin(\omega t)$ V and V_{BA} to be of the form $V_m \sin(\omega t + \varphi)$ V.*

Exercise • Refer to in-lab 4 exercise A-2.I

A.2.II   PHASE SHIFT MEASUREMENTS USING LISSAJOUS FIGURE

Leaving the connections the same as in Part A.2., set the sweep rate to X-Y mode. V_{BA} and V_{DA} will be connected to the X and Y channels of the oscilloscope. An ellipse (called the **Lissajous figure**) will be observed on the oscilloscope screen resulting from the superposition of two perpendicular sinusoidal signals V_{BA} and V_{DA}. Adjust the VOLTS/DIV controls of X and Y and use the vertical and horizontal POSITION knobs to center the ellipse symmetrically as shown in Fig. 2.
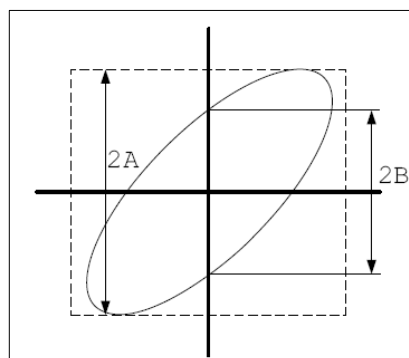

Figure A-2: Lissajous figure

It can be shown that

$$\sin \varphi = \frac{B}{A}$$

- Measure 2B and 2A and calculate φ. Compare to the value calculated using the formula of Part A.1.

Exercise • Refer to in-lab 4 exercise A-2.II

A.2.III

Change the frequency of the input and observe how the shape of the ellipse changes with frequency.

- For what range of frequencies does the ellipse look like a full circle?
- For what range of frequencies does the ellipse look like a straight line?

B. LEAD AND LAG NETWORKS

Procedure

B1. LAG NETWORK SINUSOIDAL RESPONSE CALCULATION

Calculate the output voltage of the lag network in figure B-1 if a 1 V_{Pk-Pk} sinusoidal signal is applied to the input. Record the results in table B.1 with frequencies of 100Hz, 1 KHz and 10 KHz.
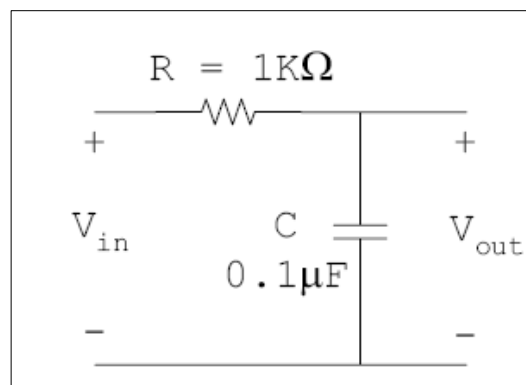

Figure B-1: Lag Network

Exercise • Refer to in-lab 4 exercise B-1

B2. LAG NETWROK SINUSOIDAL RESPONSE MEASUREMENTS

 Starting with a frequency of 100 Hz on the function generator, apply a sinusoidal wave input of amplitude 1 V to the lag network shown in Fig. B-1. Observe the input and output waveforms on the oscilloscope and record the results. Repeat for sinusoidal waves with frequencies of 1 KHz and 10 KHz.
Capture the figure of input and output wave.

Exercise • Refer to in-lab 4 exercise B-2

B3. LAG NETWORK WITH SQUARE WAVE

Apply a square waveform of 100 Hz frequency and 1 V peak-to-peak amplitude to the lag network and measure the amplitude of the output voltage on the oscilloscope. Repeat for frequencies of 1 KHz and 10 KHz.

Capture the figure of input and output wave.

Exercise • Refer to in-lab 4 exercise B-3

B4. LEAD NETWORK SINUSOIDAL RESPONSE CALCULATION

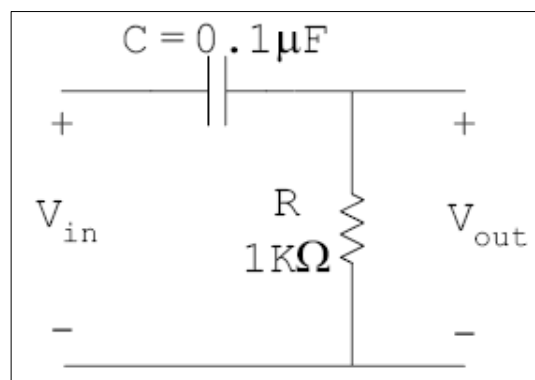Repeat Parts B.1 for the lead network shown in figure B-4. Record the results in table B.4.

Figure B-4: Lead Network

Exercise • Refer to in-lab 4 exercise B-4

B5. LEAD NETWROK SINUSOIDAL RESPONSE MEASUREMENTS

Repeat Parts B.2 for the lead network shown in figure B-4. Record the results in table B.5.
Capture the figure of input and output wave.

Exercise • Refer to in-lab 4 exercise B-5

B6. LEAD NETWORK WITH SQUARE WAVE

Repeat Parts B.3 for the lead network shown in figure B-4. Record the results in table B.6. Capture the figure of input and output wave.

Exercise • Refer to in-lab 4 exercise B-6

Discussion on Part B

- Explain the shape of the output waveforms of the lag and lead networks to square wave inputs of various frequencies, with particular reference to the fundamental property of a capacitor not changing its voltage instantaneously.

- What should be the relationship between the RC time constant and the frequency of the square wave so that:

  - The lag network does not appreciably distort the square wave.
  - The lag network acts as an integrator.
  - The lead network does not appreciably distort the square wave.
  - The lead-network acts as a differentiator.

Discussion on Part B

- What should be the relationship between the RC time constant and the frequency of the sinusoidal input so that:

    - The lag network does not introduce appreciable attenuation.
    - The lead network does not introduce appreciable attenuation.
    - How do these relationships compare with those for the square wave? What is the relationship between a periodic waveform (such as the square wave) and sinusoids? (Refer to Fourier's Theorem).

- The lag and lead networks are also referred to as low-pass and high-pass filters, respectively. Explain what these terms mean and indicate the cutoff frequency in each case. **Note:** *The cutoff frequency is defined as the frequency at which the output amplitude is 1/√2 times its maximum value.*

- Considering one of the RC elements to be a source impedance, and the other to be a load impedance, explain the integrating and differentiating action of these networks on the basis of the relationship between source and load impedances in the s domain.

- If the input voltage to either network has an average value of VDC, what will be the average value of the voltage across the resistor and the capacitor? What will be the relationship between these three voltage values?

C. SERIES RLC CIRCUITS

Procedure

C1. RLC RESONANT AND FREQUENCY CALCULATION

For the circuit of Fig. C-1, and with the values given in table C-1, calculate the resonant frequency and the bandwidth.
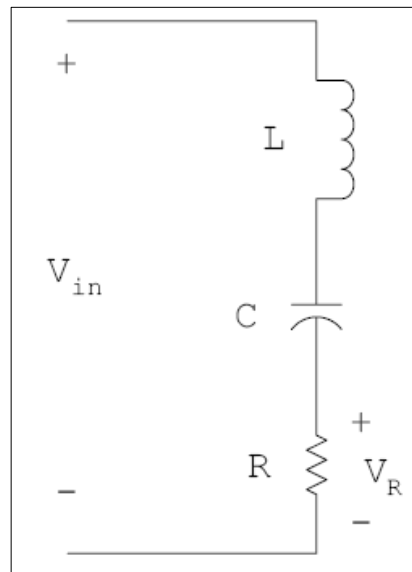

Figure C-1: RLC Circuit

Resistance	Inductor	Capacitor	Resonant Frequency	Bandwidth
100 Ω	220 uH	1uF		
56 Ω	220 uH	1uF		
100 Ω	470 uH	1uF		
100 Ω	470 uH	0.1uF		

Table C-1: Resonant and Bandwidth Calculation

Exercise • Refer to in-lab 4 exercise C-1

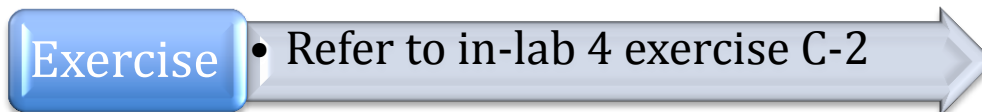#### C2. RLC MAGNITUDE AND PHASE ANGLE MEASUREMENTS

For the circuit of Fig. C-1., and with R = 100, L = 220 uH, and C = 1 uF, measure the magnitude of the input and output and phase angle (Vin peak-to-peak, VR peak-to-peak and ΔT). Plot them versus frequency on semi-log paper for the range of values of frequency shown in Table C-2. Use a 1 V peak-to-peak sinusoidal voltage for V_{in}.

f kHz	R = 100 Ω L = 220 µH C = 1µF			R = 56 Ω L = 220 µH C = 1µF			R = 100 Ω L = 470 µH C = 1µF			R = 100 Ω L = 470 µH C = 0.1µF		
	V_{in} pk-pk	V_R pk-pk	ΔT	V_{in} pk-pk	V_R pk-pk	ΔT	V_{in} pk-pk	V_R pk-pk	ΔT	V_{in} pk-pk	V_R pk-pk	ΔT
1												
1.4												
2												
2.8												
4												
5.4												
7.5												
10												
14												
20												
28												
40												
54												
75												
100												

Table C-2: Magnitude and phase shift measurements

Exercise • Refer to in-lab 4 exercise C-2

C3. RLC MAGNITUDE AND PHASE ANGLE MEASUREMENTS

Repeat Part C.2. with R = 56 ohm. (L and C are unchanged).

C4. RLC MAGNITUDE AND PHASE ANGLE MEASUREMENTS

Repeat Part C.2 with R = 100, L = 470 uH, and C = 1 uF.

C5. RLC MAGNITUDE AND PHASE ANGLE MEASUREMENTS

Repeat Part C.2 with R = 100, L = 470 uH, and C = 0.1 uF.

Discussion on Part C

- Calculate the resonance frequencies for the four cases in Table C.2 and compare them with the measured values.

- Measure the bandwidth from your plots for R = 56 Ω and R = 100 Ω

    - How does bandwidth vary with R?

    - Deduce the value of the effective series resistance for the two R values. Why does this value differ from R? What is the DC resistance of the coil?

    - *Note Bandwidth is defined as f2 - f1, where f2 and f1 (f2 > f1) are the frequencies where the magnitude of the transfer function is 1/√2 times its maximum value.*

- Compare your plots with the theoretical values of resonance frequency and bandwidth and explain any discrepancies. How does the bandwidth vary with varying L and with varying C? Explain.

III. OUTCOMES

By the end of Experiment IV, students:
- Should know the different ways of calculating and measuring the phase shift between two sinusoidal signals
- Should know the differences between RC lead and lag networks and their applications.
- Should be familiar with frequency response of RLC circuits and their characteristics.