EECE 310L: Electric Circuits Laboratory

Experiment 4: RC and RLC Circuits

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Group 10

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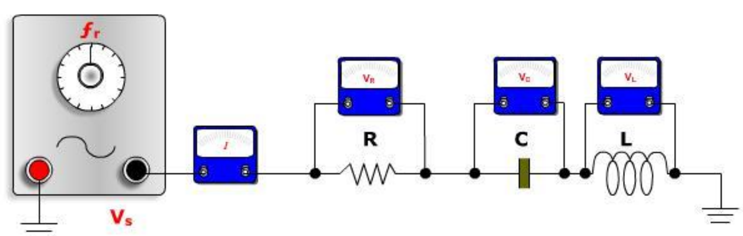

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Table of Contents

List of figures and tables

I. Objectives:

- Investigate the frequency response and time response of RC circuits.
- Investigate the frequency response of series RLC circuits.
- Use the oscilloscope to do frequency, time, and phase measurements.

II. Lab Equipment Used:

- Function Generator (Agilent 33120A)
- Oscilloscope (Tektronix TDS220)
- Breadboard

III. Lab Tools Used:

No tools were basically used since the circuits were built as testing circuits on the breadboard.

IV. Components Used:

Resistors:

Theoretical Value (Ω)	Measured Value (Ω)	% Error
56	55.6	0.71
100	101	1
1000	996	0.4
20000	20400	2

Inductors:

Theoretical Value (µH)	Measured Value	% Error
220	NA	NA
470	NA	NA

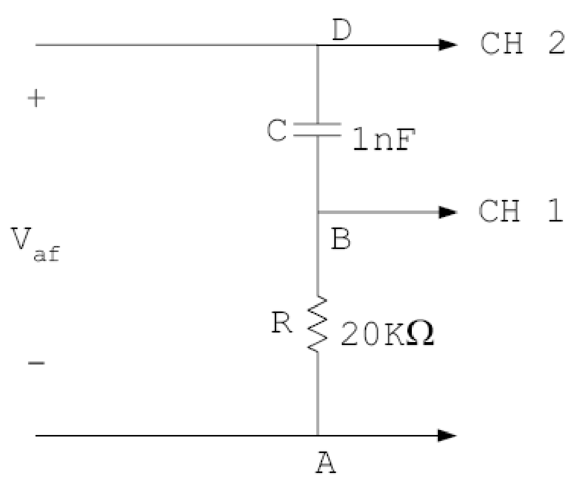
Capacitors:

Theoretical Value (nF)	Measured Value (nF)	% Error
1000	NA	NA
100	NA	NA
1	NA	NA

VI. Experimental Procedure and Discussion

A. Phase Shift Measurements

A1. Circuit Diagrams:



A2. Detailed Experimental Procedure:

We apply a sinusoidal voltage signal V_{af} using the function generator to the circuit shown in figure F1 with the following settings:

- Frequency: 5KHz
- Peak to Peak Voltage: 6V

Then we connect the resistor to channel 1 (i.e. V_{BA}) of the oscilloscope and V_{af} to channel 2.

A2.I. Time Shift Method

This method utilizes the fact that a phase difference between two sinusoidal signals is equivalent to a shift in time domain.

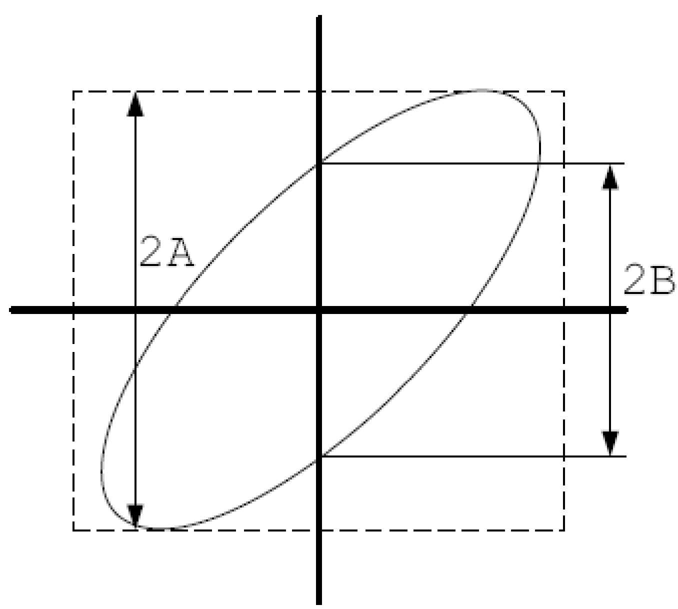
We set the oscilloscope to the Y-T mode and adjust both the vertical and horizontal sensitivities of both channels to the same values. Using the cursors we get the time difference between V_{af} and V_{BA} (We set cursor 1 to an arbitrary point on the first signal and cursor 2 to the equivalent point on the second signal).

Since every 1 period in time domain is equivalent to 360^0 in angles domain then:

$\phi = \frac{\Delta t \ x \ 360^o}{T}$ where ϕ is the phase difference. T is the period of any of the signals (since they are equal). Δt is the time difference measured using the cursors in the oscilloscope.

A2.II Lissajous Figure Method

We set the oscilloscope to the X-Y mode and keep the same connections as the time shift method. An ellipse will appear due to the superposition of 2 orthogonal sinusoids; however, it should be centered using the horizontal and vertical position knobs to center the ellipse with respect to the origin. The centered ellipse will have a shape as follows:



The phase difference ϕ in a Lissajous figure is $\phi = \sin^{-1}(\frac{B}{A})$.

A3. Measurements and Results

From the time shift method we got:

Δt = 32x10^{-6} s and T=1/f = 200x10^{-6}. After applying the formula in section A2.I,

ϕ = 57.6^{o}.

From Lissajous figure we got:

B=2.5 units and A=3 units. Thus ϕ =56.44^{o} (By applying the formula in section A2.II)

From circuit theory, the theoretical value of the phase angle in an RC circuit is

governed by $\tan\phi = \frac{Xc}{R}$. Thus theoretically ϕ should be 57.858^{o}.

% error (Time shift) = 0.44 %

% error (Lissajous) = 2.45 %

Both errors are acceptable and they can be categorized into visual and reading errors, in addition to the 5% range in the accuracy of the ratings of the resistor and capacitor.

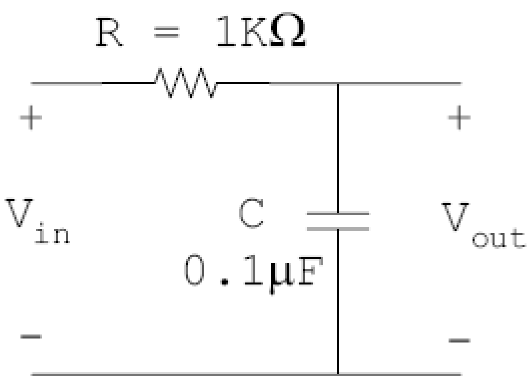
A4. Discussions

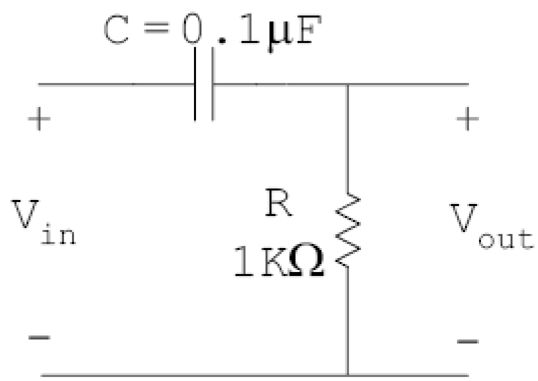
After fluctuating the frequency of the voltage signal from very low frequencies to relatively high frequencies we observed that:

- At low frequencies the Lissajous figure was resembled by a vertical straight line since the capacitor acts an open circuit; so V across R would be zero.
- At high frequencies it appeared to be a straight line, and this is due to the capacitor acting as a short circuit; thus V across the resistor would be directly proportional to the current (V=RI). So the phase difference would be zero and that's represented by a linear relationship in Lissajous (i.e. straight line).
- We were not able to observe a circle using the RC circuit.

B. Lead and Lag Networks

B1. Circuit Diagrams:





B2. Detailed Experimental Procedure:

We setup the circuits shown in figures F3 and F4, and connect both Vin and Vout to the channels of the oscilloscope. Figure F3 represents a lag network, while figure F4 represents a lead network. For each of the circuits we apply square and sinusoidal signals, and we fluctuate the frequency between 3 values : 100Hz (low frequency), 1000Hz (Medium frequency), and 10KHz (High frequency). For each of the signals we measure the peak to peak value of V_{out}.

For the lag network, the magnitude of the transfer function is:

$|H(jw)| = \frac{1}{\sqrt{1+\omega^2 R^2 C^2}}$ so at low frequencies we expect the output signal to be an approximate replica of the input one. At higher frequencies, the magnitude (i.e. peal to peak value) is expected to decrease gradually.

For the lead network, the magnitude of the transfer function is:

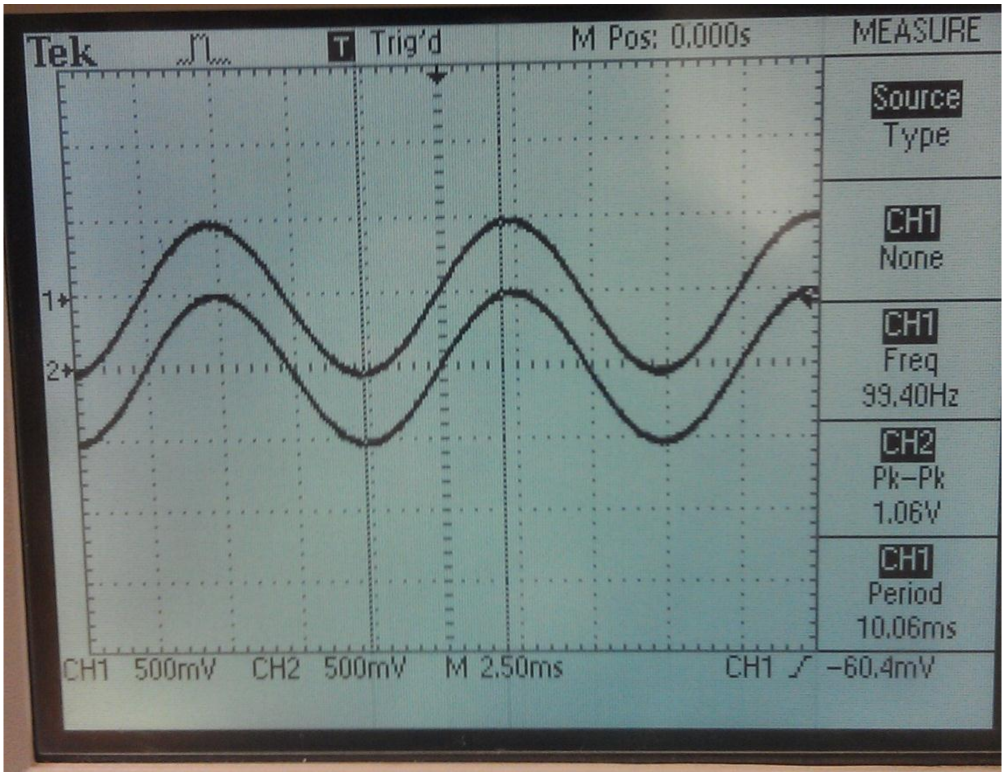
$|H(jw)| = \frac{\omega RC}{\sqrt{1+\omega^2 R^2 C^2}}$ so at low frequencies the signal is attenuated, while high frequencies are approximate replicas of the input signal.

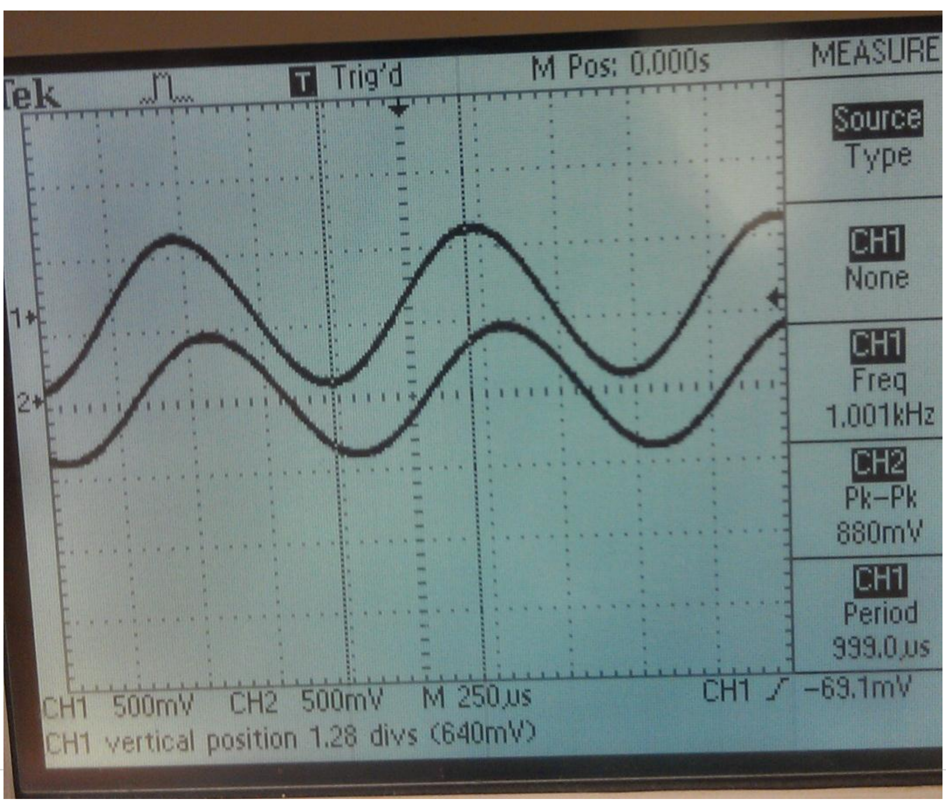
B3. Measurements and Results

Lag Network (Calculated)		
Frequency	Input Voltage	Output Voltage V_{pk-pk}
100 Hz	1 V_{pk-pk}	998 mV_{pk-pk}
1 KHz	1 V_{pk-pk}	846 mV_{pk-pk}
10 KHz	1 V_{pk-pk}	157 mV_{pk-pk}

For sinusoids:

Lag Network (Measured)		
Frequency	Input Voltage	Output Voltage V_{pk-pk}
100 Hz	1 V_{pk-pk}	Vpp= 1060 mV_{pk-pk}
1 KHz	1 V_{pk-pk}	Vpp= 880 mV_{pk-pk}
10 KHz	1 V_{pk-pk}	Vpp= 160 mV_{pk-pk}


Tek
Trig'd
M Pos: 0.000s
MEASURE
Source
Type
CH1
None
CH1
Freq
99.40Hz
CH2
Pk-Pk
1.06V
CH1
Period
10.06ms
CH1 500mV CH2 500mV M 2.50ms CH1 -60.4mV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CH1
Freq
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CH2
Pk-Pk
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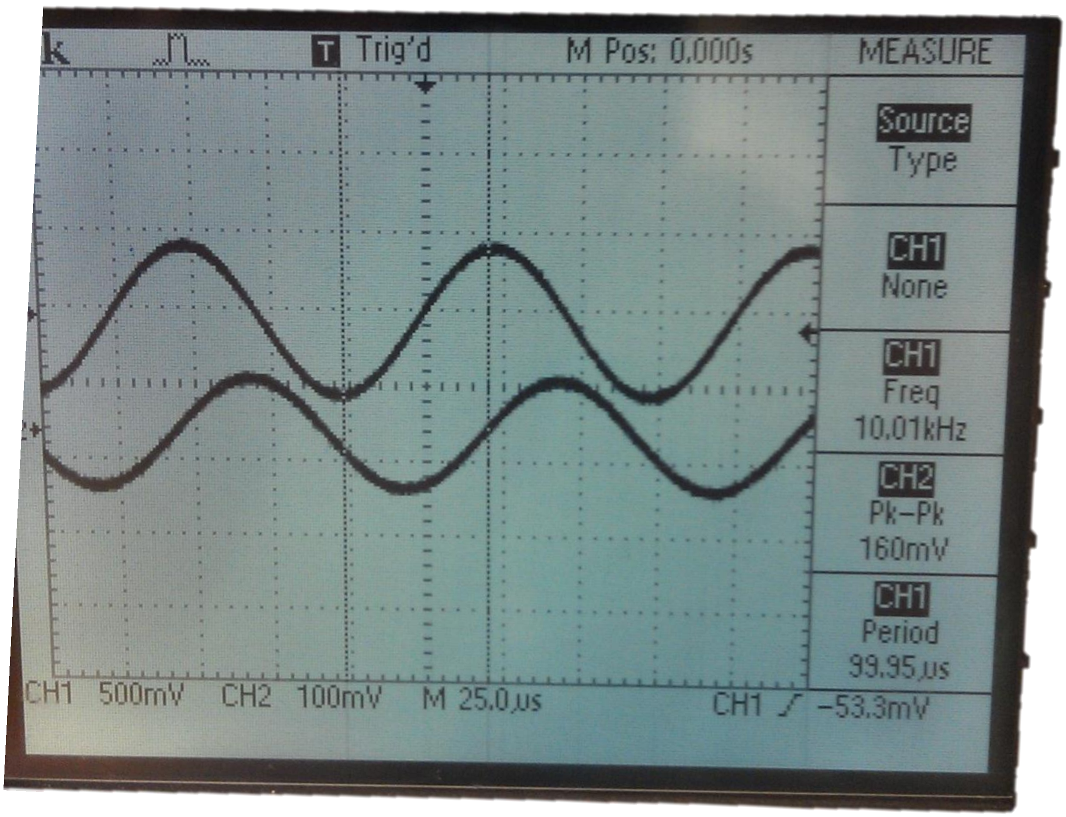
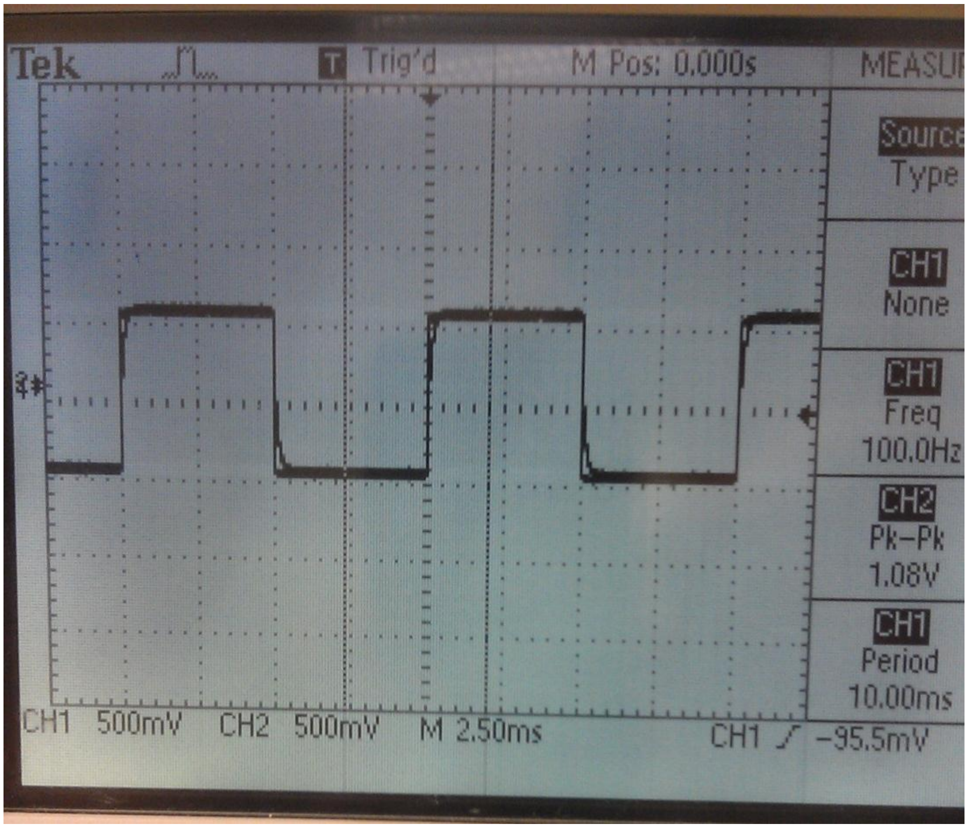
CH1
Period
999.0µs
CH1 500mV CH2 500mV M 250µs CH1 -69.1mV
CH1 vertical position 1.28 divs (640mV)



For square signals:

Lag Network (Measured)		
Frequency	Input Voltage	Output Voltage V_{Pk-Pk}
100 Hz	1 V_{Pk-Pk}	Vpp= 1080 mV_{Pk-Pk}
1 KHz	1 V_{Pk-Pk}	Vpp= 1040 mV_{Pk-Pk}
10 KHz	1 V_{Pk-Pk}	Vpp= 240 mV_{Pk-Pk}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CH1
Freq
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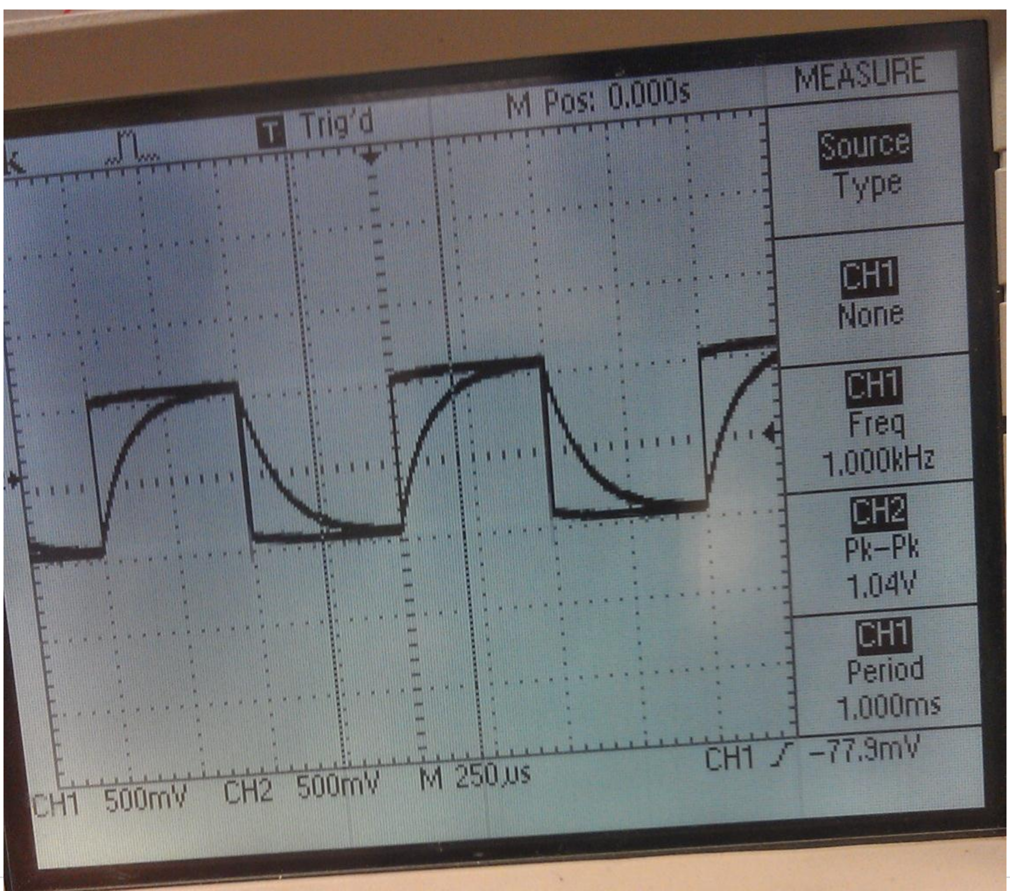
CH2
Pk-Pk
1.08V
CH1
Period
10.00ms
CH1 500mV CH2 500mV M 2.50ms
CH1 -95.5mV


Trig'd
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Source
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CH1
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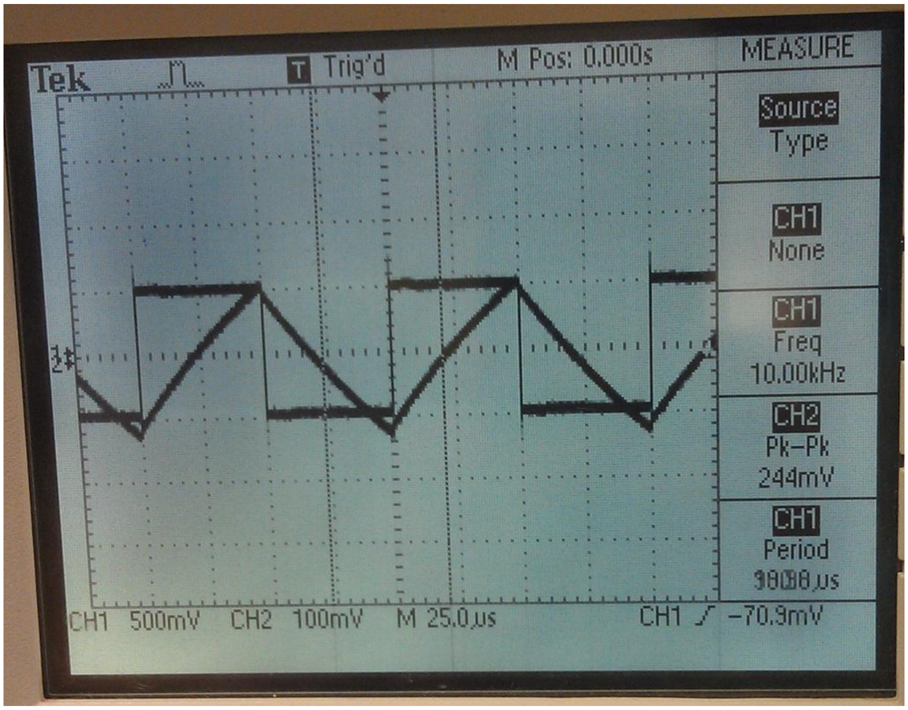
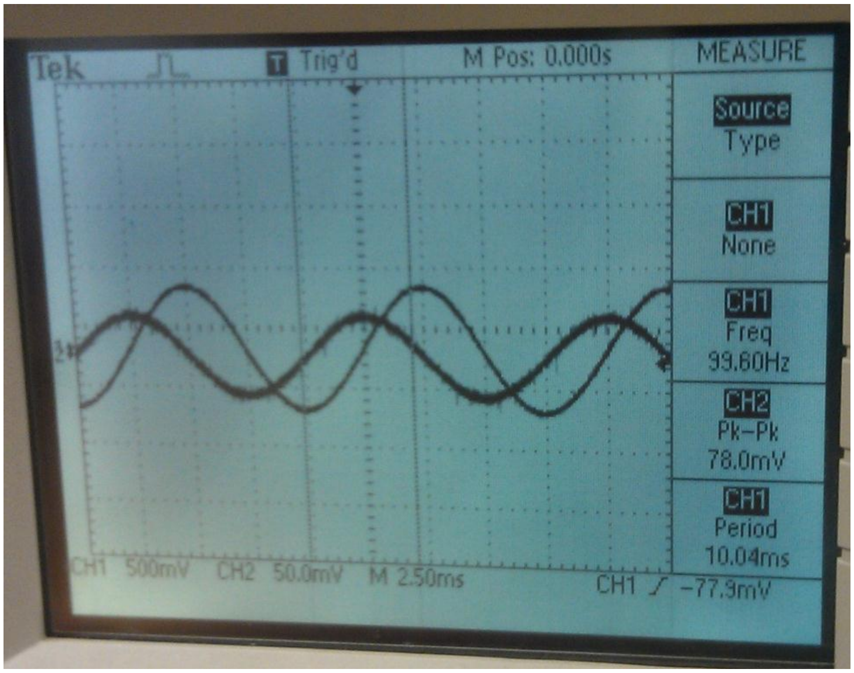
CH1
Freq
1.000kHz
CH2
Pk-Pk
1.04V
CH1
Period
1.000ms
CH1 -77.9mV
CH1 500mV CH2 500mV M 250µs

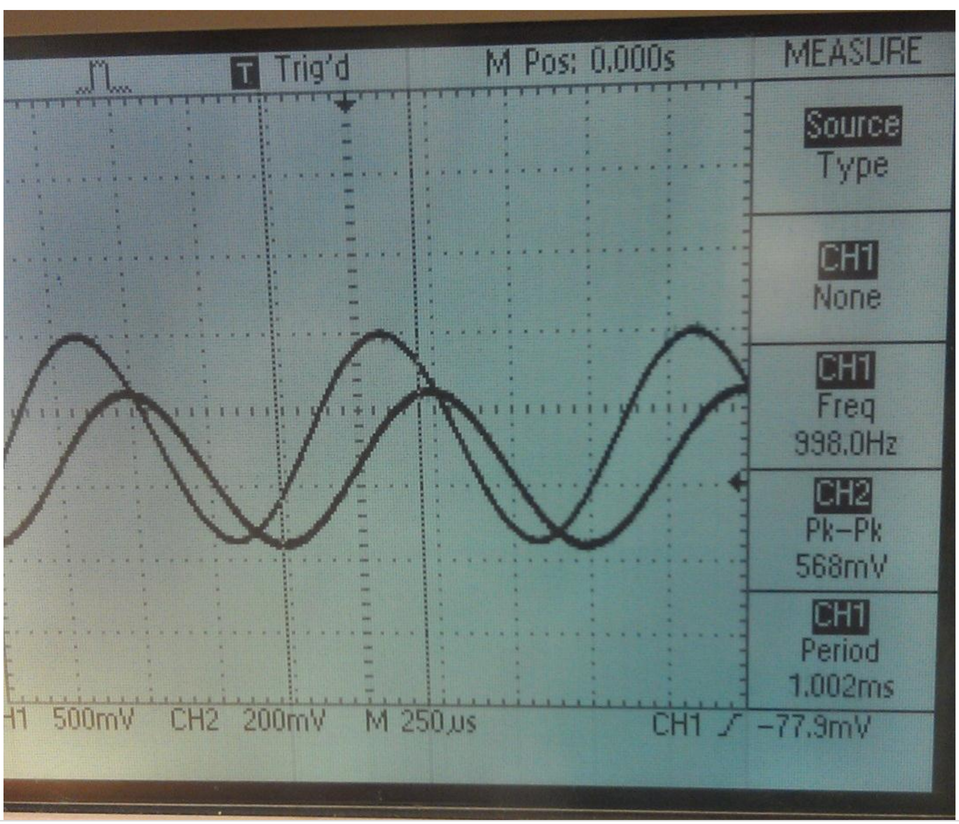


Lead Network (Calculated)		
Frequency	Input Voltage	Output Voltage V_{Pk-Pk}
100 Hz	1 V_{Pk-Pk}	62.7 mV_{Pk-Pk}
1 KHz	1 V_{Pk-Pk}	532.0 mV_{Pk-Pk}
10 KHz	1 V_{Pk-Pk}	987.5 mV_{Pk-Pk}

For sinusoids (Lead):

Lead Network (measured)		
Frequency	Input Voltage	Output Voltage V_{Pk-Pk}
100 Hz	1 V_{Pk-Pk}	78 mV_{Pk-Pk}
1 KHz	1 V_{Pk-Pk}	568 mV_{Pk-Pk}
10 KHz	1 V_{Pk-Pk}	1020 mV_{Pk-Pk}


Tek
Trig'd
M Pos: 0.000s
MEASURE
Source
Type
CH1
None
CH1
Freq
99.60Hz
CH2
Pk-Pk
78.0mV
CH1
Period
10.04ms
CH1 500mV CH2 50.0mV M 2.50ms
CH1 -77.9mV


Trig'd
M Pos: 0.000s
MEASURE
Source
Type
CH1
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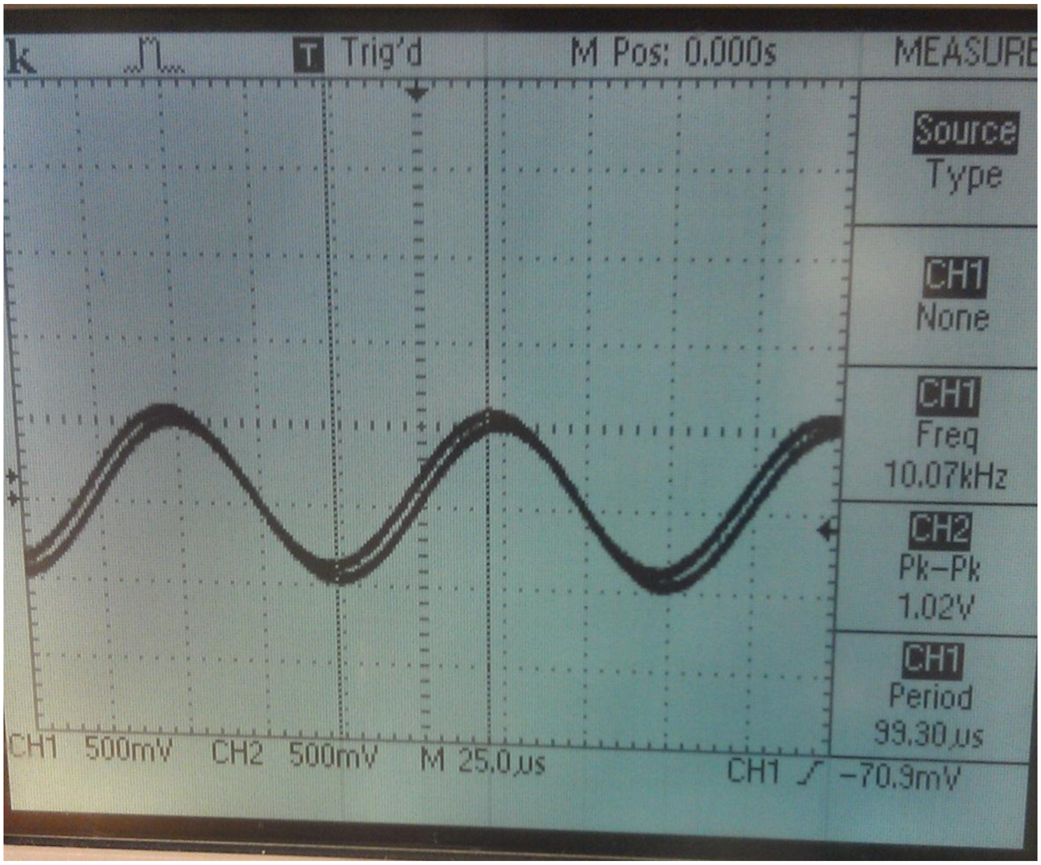
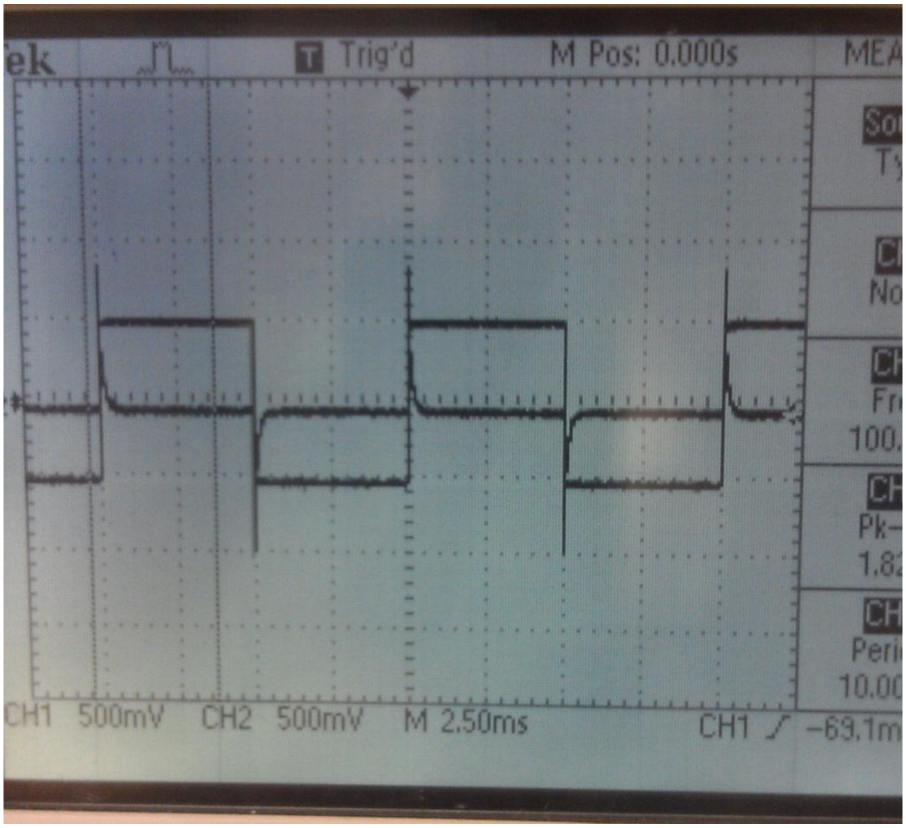
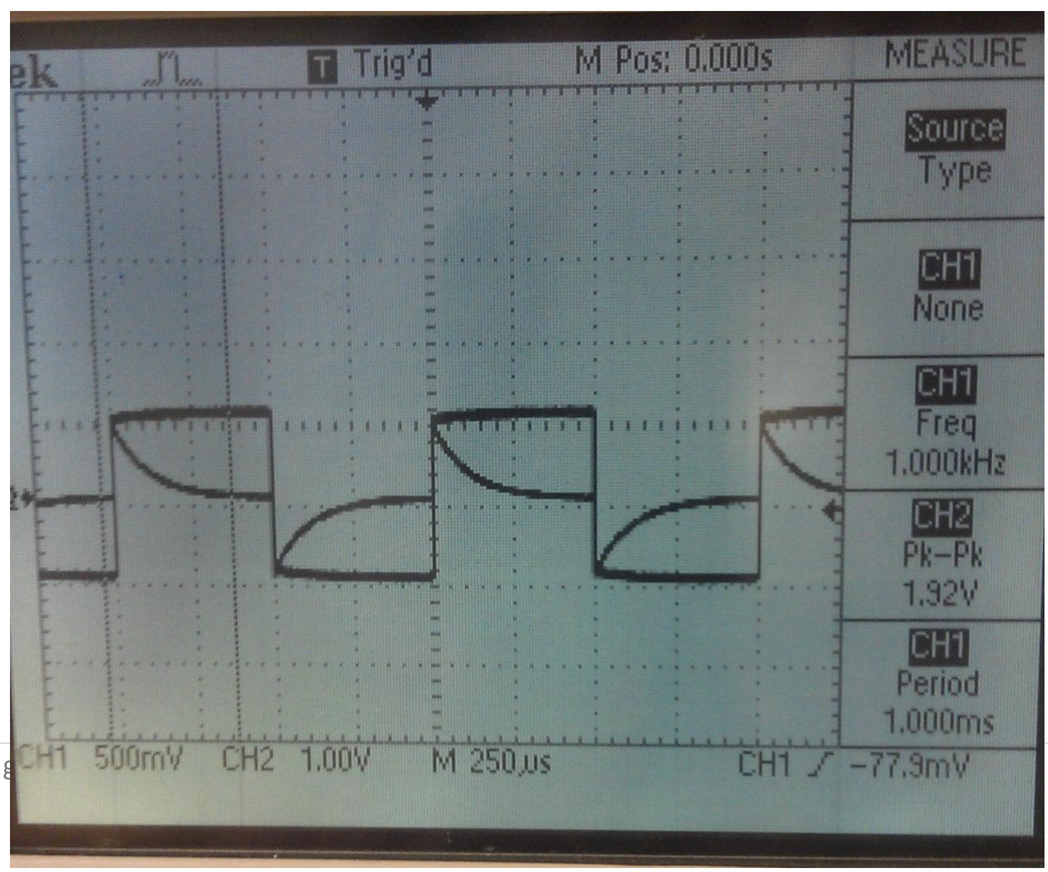
CH1
Freq
998.0Hz
CH2
Pk-Pk
568mV
CH1
Period
1.002ms
500mV CH2 200mV M 250µs
CH1 -77.9mV

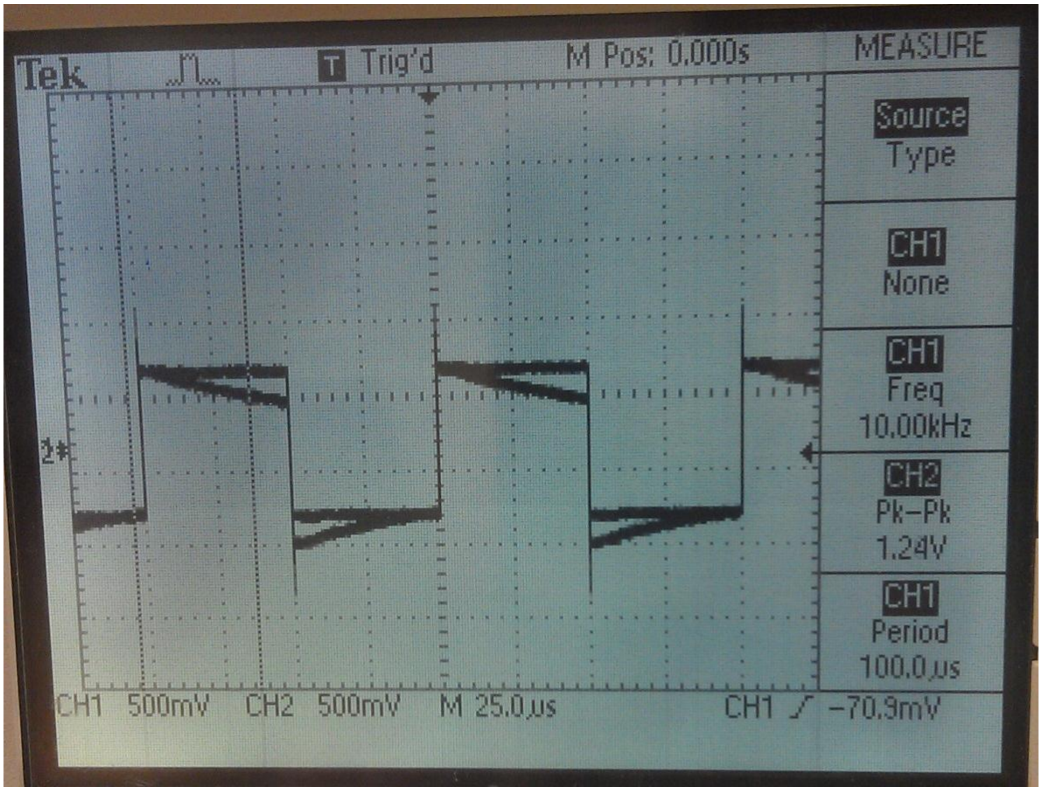


For square signals (Lead) :

Lead Network (Measured)		
Frequency	Input Voltage	Output Voltage V_{Pk-Pk}
100 Hz	1 V_{Pk-Pk}	Vpp= 1820 V
1 KHz	1 V_{Pk-Pk}	Vpp= 1920 V
10 KHz	1 V_{Pk-Pk}	Vpp= 1240 V


Trig'd
M Pos: 0.000s
CH1 500mV
CH2 500mV
M 2.50ms
CH1


Trig'd
M Pos: 0.000s
MEASURE
Source
Type
CH1
None
CH1
Freq
1.000kHz
CH2
Pk-Pk
1.92V
CH1
Period
1.000ms
CH1 500mV
CH2 1.00V
M 250µs
CH1 -77.9mV



Network	Lag			Lead		
Frequency (Hz)	100	1000	10000	100	1000	10000
Error (%)	6.21	4.02	1.91	24.4	6.76	3.29

B4. Discussions

- For both lead and lag networks, the peak to peak value of the square wave output was greater than the input! This is mainly due the capacitor changing its state from charging to discharging while keeping the same voltage in the transition state (so we may reach a voltage double that of the input). From mathematical point of view, lead networks at low frequencies act as differentiators, and the derivative of a step of magnitude A is a step of magnitude A. Similarly, for lag networks at high frequencies the signal would be integrated yielding a signal with larger magnitude.

- For the lag network not to distort a square wave, the frequency should be very low (f<<1/RC). This is because according to Fourier series expansion a square wave can be represented as:

  $$f_{sq}(t) = \frac{4A_m}{\pi}\left[\cos\omega_0 t - \frac{1}{3}\cos 3\omega_0 t + \frac{1}{5}\cos 5\omega_0 t - \frac{1}{7}\cos 7\omega_0 t + \ldots\right]$$

  Thus, the higher is the frequency, the more the harmonics would be attenuated by the low-pass filter (i.e. lag network), and consequently more distortion of the signal will take place. For lowest attenuation the fundamental frequency w_0 must be way less than the cutoff frequency.

  The lag network acts as integrator at high frequencies since $H(s) = \frac{1}{1+sRC} \approx \frac{1}{sRC}$ and division by s in Laplace domain is an equivalent integration in time domain (i.e. $Vo(s) = \approx \frac{1}{sRC} Vi(s)$).

  For the lead network not to distort the square wave the frequency should be relatively high (f>>1/RC) for the same reasoning as for the lag network. The higher is the frequency, the more harmonics will be kept and thus the signal would be preserved.

  The lead network acts a differentiator for low frequencies since $H(s) = \frac{sRC}{1+sRC} \approx \frac{sRC}{1}$ and multiplication by s in Laplace domain is an equivalent differentiation in time domain.

- For the lag network not to distort a sinusoidal wave, the frequency should be very low (f<<1/RC). This is for the same reasoning of the square wave; a low pass filter remarkably attenuates signals after its cutoff frequency, and before that the magnitude is approximately 1.

  For the lead network not to distort the sinusoidal wave the frequency should be relatively high (f>>1/RC). Clearly, a high pass filter wouldn't

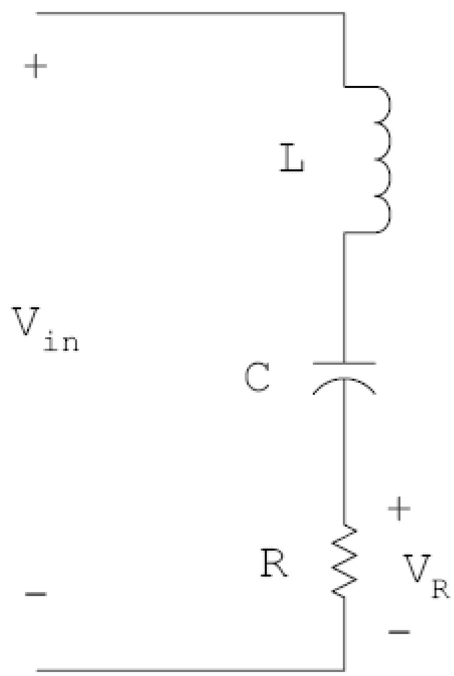
attenuate frequencies after its cutoff frequencies and the magnitude would be approximately one.
We can conclude that what applies for sinusoids also applies for square waves since as mentioned in the point before a square wave is constituted of an infinite sum of sinusoids.

- A low pass filter is a filter that passes all frequencies below its cutoff frequency and attenuates others after the cutoff. By setting the magnitude transfer function $|H(jw)| = \frac{1}{\sqrt{1+\omega^2 R^2 C^2}}$ to $\frac{1}{\sqrt{2}}$ we get the cutoff frequency $\omega_c = \frac{1}{RC}$ radians.
  A high pass filter is a filter that passes all frequencies beyond its cutoff frequency and remarkably attenuates others. Also setting the magnitude transfer function to $\frac{1}{\sqrt{2}}$ would give the same result for the cutoff frequency.

- Since the signal has an average DC value, then the capacitor would be completely charged after approximately 5RC s. Thus $V_c = V_{in}$ and $V_r=0$.
  Note: Sinusoidal signals we dealt with in this lab did not have DC value (or they have a DC value of zero) and that's why the capacitor kept charging and discharging.

C. Series RLC Circuits

C1. Circuit Diagrams:



C2. Detailed Experimental Procedure

We setup the circuit shown above and we connect V_R and V_{in} to both channel of the oscilloscope. Then we sweep the frequency and measure the frequency with highest peak; this frequency is the resonant frequency, were the circuit is purely resistive. Also, we measure the time shift between both signals.

C3. Measurements and Results

R = 100 ohms, L= 220 uH, and C= 1 uF

Freq (Hz)	Vout	Vin	Vout/Vin	delta T	Theoretical
1000	0.35	0.8	0.4375	0.00014	0.535347504
1400	0.432	0.72	0.6	0.000092	0.666853924
2000	0.456	0.656	0.695122	0.000048	0.793046174
2800	0.504	0.664	0.759036	0.000028	0.883680343
4000	0.424	0.52	0.815385	0.000013	0.946021908
5400	0.488	0.504	0.968254	0.000005	0.976626606
7500	0.472	0.504	0.936508	0.000002	0.994161711
10000	0.52	0.584	0.890411	0	0.999781147
14000	0.552	0.608	0.907895	0	0.996827944
20000	0.52	0.576	0.902778	1.1E-06	0.981164458
28000	0.488	0.584	0.835616	1.5E-06	0.949571222
40000	0.48	0.592	0.810811	1.7E-06	0.889705065
54000	0.48	0.616	0.779221	1.5E-06	0.812700667
75000	0.48	0.696	0.689655	1.68E-06	0.701646375
100000	0.432	0.752	0.574468	1.48E-06	0.59058919

R = 100 ohms, L= 470 uH, and C= 1 uF

Freq(Hz)	Vout	Vin	Vout/Vin	delta T	Theoretical
1000	0.368	0.82	0.44878	0.000172	0.539171572
1400	0.416	0.72	0.577778	0.00009	0.674188936
2000	0.44	0.656	0.670732	0.00004	0.805105561
2800	0.456	0.64	0.7125	0.000024	0.899504798
4000	0.472	0.6	0.786667	0.000012	0.963023138
5400	0.488	0.6	0.813333	0.000005	0.990975457
7500	0.496	0.584	0.849315	0.000003	0.999956984
10000	0.472	0.584	0.808219	0	0.990857854
14000	0.48	0.6	0.8	0.000002	0.957891777
20000	0.488	0.644	0.757764	2.4E-06	0.890459639
28000	0.472	0.664	0.710843	3.2E-06	0.79231934
40000	0.472	0.736	0.641304	2.8E-06	0.658963977
54000	0.416	0.776	0.536082	2.9E-06	0.538393698
75000	0.368	0.856	0.429907	2.4E-06	0.414802429
100000	0.312	0.904	0.345133	2.1E-06	0.322295104

R = 56 ohms, L= 220 uH, and C= 1 uF

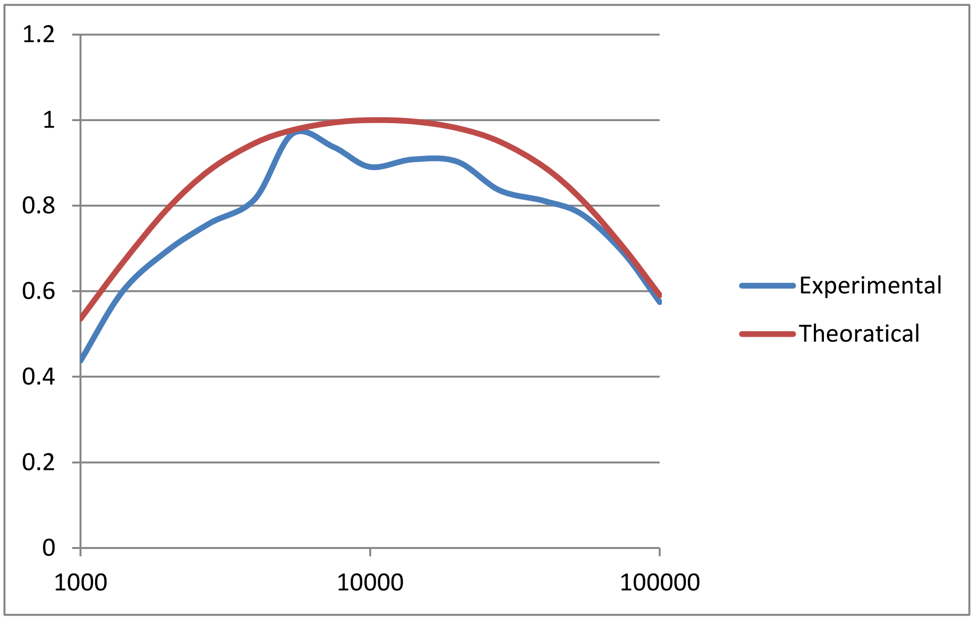
Freq(Hz)	Vout	Vin	Vout/Vin	delta T	Theoretical
1000	0.35	0.8	0.4375	0.00014	0.334495571
1400	0.432	0.72	0.6	0.000092	0.448023435
2000	0.456	0.656	0.695122	0.000048	0.589107703
2800	0.504	0.664	0.759036	0.000028	0.726483727
4000	0.424	0.52	0.815385	0.000013	0.853028855
5400	0.488	0.504	0.968254	0.000005	0.9307022
7500	0.472	0.504	0.936508	0.000002	0.981731763
10000	0.52	0.584	0.890411	0	0.999302627
14000	0.552	0.608	0.907895	0	0.989989053
20000	0.52	0.576	0.902778	1.1E-06	0.943393776
28000	0.488	0.584	0.835616	1.5E-06	0.861401739
40000	0.48	0.592	0.810811	1.7E-06	0.737286443
54000	0.48	0.616	0.779221	1.5E-06	0.615553768
75000	0.48	0.696	0.689655	1.68E-06	0.482893173
100000	0.432	0.752	0.574468	1.48E-06	0.379226917

C4. Discussions

Magnitude transfer function: $|H(jw)| = \dfrac{R}{\sqrt{R^2 + (\omega L - \dfrac{1}{\omega C})^2}}$

The calculated theoretical values are found in the tables above.

Below are the graphs of each set plotted respectively using Microsoft Excel: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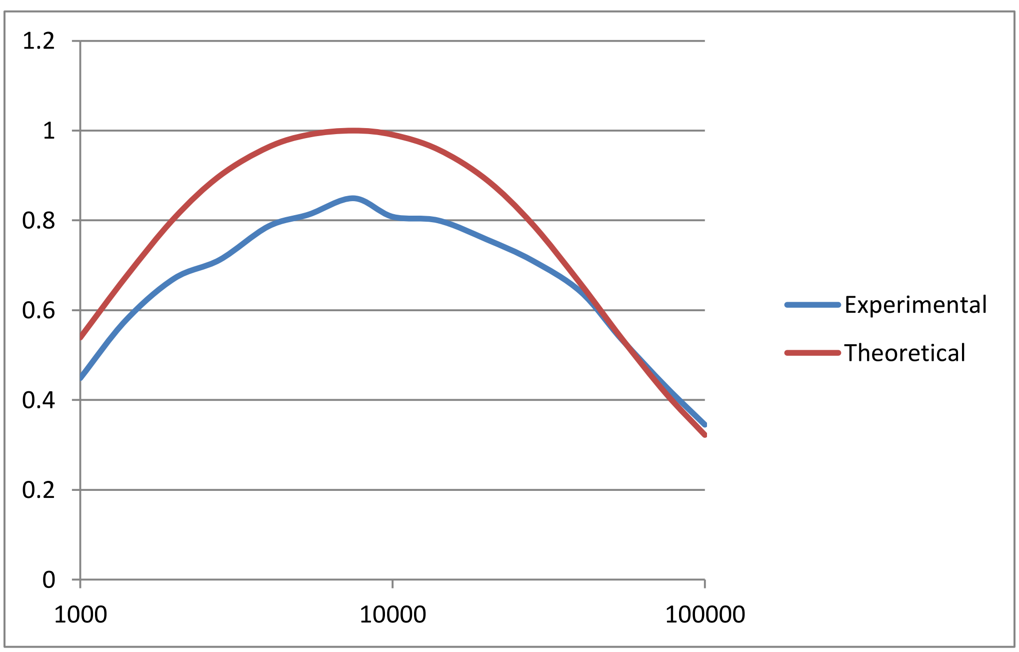
0.8
0.6
0.4
0.2
0
1000
10000
100000
Experimental
Theoratical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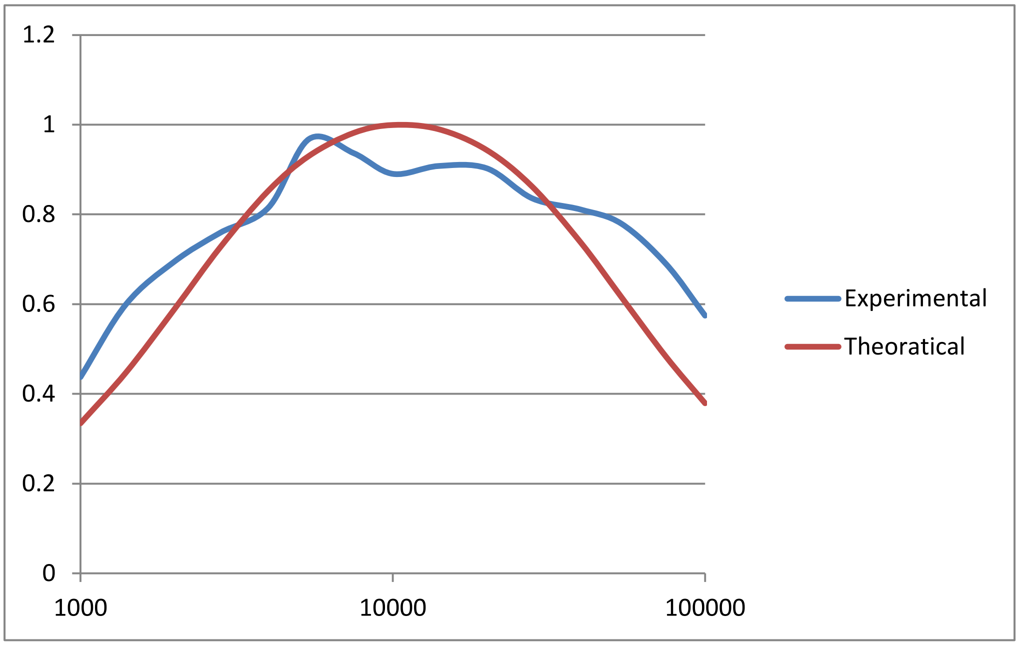

- For each graph we draw the line $|H(jw)| = \frac{1}{\sqrt{2}}$. This line will intersect the curve into points f1 and f2 where f2-f1 is the bandwidth. The resonant frequency is the frequency where the curve peaks.

- For set 1: BW=50 KHz and F=5400 Hz

  For set 2: BW= 25.2 KHz and F= 7500 Hz

  For set 3: BW=70 KHz and F=5400 Hz

- Theoretically (Calculated in the Inlab):

  For set 1: BW=72.34 KHz and F= 10.73 KHz

  For set 2: BW= 33.86 KHz and F= 23.215 KHz

  For set 3: BW= 40.5 KHz and F= 10.73 KHz

- There is a significant difference between the theoretical and experimental values and that might be due to reading errors and presence of noise signals in the channels of the oscilloscope.
- Since $BW = \frac{R}{2\Pi L}$ then any increase in R would imply an increase in the bandwidth and vice versa. The latter is verified by comparing Sets 1 and 3 (Set 1 has a higher value of R and a higher BW).
- Similarly, BW is inversely proportional to L, so as L increases BW would decrease. This is verified by comparing sets 1 and 2. In addition, BW is independent of the value of C.
- Sets 1 and 3 have the same resonant frequency since they have the same values of L and C, and set 1 has a larger bandwidth since it has a larger R. Set 1 has a larger bandwidth and resonant frequency than set 2 since it has a smaller value of L. It is irrelevant to compare the bandwidth of sets 2 and 3, however, set 3 has a higher resonant frequency since it has a lower value of L.

VI. **References:**

Sedra Smith – Microelectronic Circuits 6th Edition

VII. **Mistakes faced in lab:**

We tried to connect the oscilloscope's probes differentially to the components, but the instructor reassured that this method does not work with this oscilloscope model.

"I HAVE NEITHER GIVEN NOR RECEIVED AID ON THIS REPORT NOR HAVE I CONCEALED ANY VIOLATION OF THE AUB STUDENT CODE OF CONDUCT."