## RC and RLC Circuits

## Objectives:

In this experiment you will learn how to:

- Investigate the frequency response and time response of RC circuits.
- Investigate the frequency response of series RLC circuits.
- Use the oscilloscope to do frequency, time, and phase measurements.


## Circuit Diagrams



Fig. 1

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Fig. 3


Fig. 2


Fig. 4


Fig. 5

## A. Phase Shift Measurements

## Procedure

Using the function generator, apply a sinusoidal voltage $\left(\mathrm{V}_{\mathrm{af}}=6 \mathrm{~V}\right.$ peak-topeak) of frequency 5 KHz to the input of the circuit shown in Fig. 1. Apply $\mathrm{V}_{\mathrm{BA}}$ to CH 1 of the oscilloscope and $\mathrm{V}_{\mathrm{DA}}$ to CH 2.
A.1. Superpose the two traces of $\mathrm{V}_{\mathrm{BA}}$ and $\mathrm{V}_{\mathrm{DA}}$ to have the same horizontal axis and adjust the VOLT/DIV and SEC/DIV settings to get stable traces.

- Measure the phase difference $\phi$ on the oscilloscope.

Note The phase difference can be measured from the time instants at which the waveforms cross the time axis. Consider $V_{\text {af }}$ to be of the form $3 \sin (\omega \mathrm{t}) V$ and $V_{B A}$ to be of the form $\mathrm{V}_{\mathrm{m}} \sin (\omega \mathrm{t}+\phi) V$.
A.2. The phase angle $\phi$ can also be calculated using the formula:

$$
\tan \phi=\frac{\mathrm{X}_{\mathrm{C}}}{\mathrm{R}} \text { where } \mathrm{X}_{\mathrm{C}}=\frac{1}{\omega \mathrm{C}}=\frac{1}{2 \pi \mathrm{fC}},
$$

with $\mathrm{f}=5 \mathrm{KHz}, \mathrm{C}=1 \mathrm{nF}$ and $\mathrm{R}=20 \mathrm{~K} \Omega$.

- Compare the measured value of Part A.1. with the calculated value.
A.3. Leaving the connections the same as in Part A.1., set the sweep rate to X-Y mode. $\mathrm{V}_{\mathrm{BA}}$ and $\mathrm{V}_{\mathrm{DA}}$ will be connected to the X and Y channels of the oscilloscope. An ellipse (called the Lissajous figure) will be observed on the oscilloscope screen resulting from the superposition of two perpendicular sinusoidal signals $\mathrm{V}_{\mathrm{BA}}$ and $\mathrm{V}_{\mathrm{DA}}$. Adjust the VOLTS/DIV controls of X and Y and use the vertical and horizontal POSITION knobs to center the ellipse symmetrically as shown in Fig. 2.

It can be shown that $\sin \phi=\frac{\mathrm{B}}{\mathrm{A}}$.

- Measure 2B and 2A and calculate $\phi$. Compare to the value calculated using the formula of Part A.2.

Change the frequency of the input and observe how the shape of the ellipse changes with frequency.

- For what range of frequencies does the ellipse look like a full circle?
- For what range of frequencies does the ellipse look like a straight line?


## B. Lead and Lag Networks

## Procedure

B.1. Starting with a frequency of 100 Hz on the function generator, apply a square wave input of amplitude 1 V to the lag network shown in Fig. 3. Observe the input and output waveforms on the oscilloscope and record the results. Repeat for square waves with frequencies of 1 KHz and 10 KHz .
B.2. Repeat the above procedure with the lead network shown in Fig. 4, but starting with a square wave with frequencies of 10 KHz , then 1 KHz and 100 Hz .
B.3. Apply a sinusoidal waveform of 100 Hz frequency and 1 V peak-to-peak amplitude to the lag network and measure the amplitude of the output voltage on the oscilloscope and compare this value with the theoretical value. Repeat for frequencies of 1 KHz and 10 KHz .
B.4. Repeat Part B.3. for the lead network.

## Discussion

B.D1. Explain the shape of the output waveforms of the lag and lead networks to square wave inputs of various frequencies, with particular reference to the fundamental property of a capacitor not changing its voltage instantaneously.
B.D2. What should be the relationship between the RC time constant and the frequency of the square wave so that:

- The lag network does not appreciably distort the square wave.
- The lag network acts as an integrator.
- The lead network does not appreciably distort the square wave.
- The lead-network acts as a differentiator.
B.D3. What should be the relationship between the RC time constant and the frequency of the sinusoidal input so that:
- The lag network does not introduce appreciable attenuation.
- The lead network does not introduce appreciable attenuation.
- How do these relationships compare with those for the square wave? What is the relationship between a periodic waveform (such as the square wave) and sinusoids? (Refer to Fourier's Theorem).
B.D4. The lag and lead networks are also referred to as low-pass and high-pass filters, respectively. Explain what these terms mean and indicate the cutoff frequency in each case.

Note The cutoff frequency is defined as the frequency at which the output amplitude is $\frac{1}{\sqrt{2}}$ times its maximum value.
B.D5. Considering one of the RC elements to be a source impedance, and the other to be a load impedance, explain the integrating and differentiating action of these networks on the basis of the relationship between source and load impedances in the $s$ domain.
B.D6. If the input voltage to either networks has an average value of $\mathrm{V}_{\mathrm{DC}}$, what will be the average value of the voltage across the resistor and the capacitor? What will be the relationship between these three voltage values?

## C. Series RLC circuits

## Procedure

C.1. For the circuit of Fig. 5., and with $R=10 \Omega, L=220 \mu H$, and $C=1 \mu \mathrm{~F}$, measure the magnitude and phase angle of the transfer function $\mathrm{V}_{\mathrm{R}} / \mathrm{V}_{\mathrm{in}}$, and plot them versus frequency on semi-log paper for the range of values of frequency shown in Table C.1. Use a 1 V peak-to-peak sinusoidal voltage for $\mathrm{V}_{\mathrm{in}}$.
C.2. Repeat Part C.1. with $\mathrm{R}=100 \Omega$. ( L and C are unchanged).
C.3. Repeat Part C. 1 with $\mathrm{R}=10 \Omega, \mathrm{~L}=470 \mu \mathrm{H}$, and $\mathrm{C}=1 \mu \mathrm{~F}$.
C.4. Repeat Part C. 1 with $\mathrm{R}=10 \Omega, \mathrm{~L}=470 \mu \mathrm{H}$, and $\mathrm{C}=0.1 \mu \mathrm{~F}$.

TABLE C.1.

|  | $\begin{aligned} & \mathrm{R}=10 \Omega, \mathrm{~L}= \\ & 220 \mu \mathrm{H}, \mathrm{C}=1 \mu \mathrm{~F} \end{aligned}$ |  | $\begin{aligned} & \mathrm{R}=100 \Omega, \mathrm{~L}= \\ & 220 \mu \mathrm{H}, \mathrm{C}=1 \mu \mathrm{~F} \end{aligned}$ |  | $\begin{aligned} & R=10 \Omega, L= \\ & 470 \mu H, C=1 \mu F \end{aligned}$ |  | $\begin{aligned} & R=10 \Omega, L= \\ & 470 \mu \mathrm{H}, \mathrm{C}=0.1 \mu \mathrm{~F} \end{aligned}$ |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\underset{\mathbf{K H z}}{\mathbf{f}}$ | $\begin{gathered} \mathrm{V}_{\mathrm{R}} / V_{\text {in }} \\ \mathrm{mag} \end{gathered}$ | $V_{R} / v_{\text {in }}$ <br> angle | $\begin{gathered} \mathrm{V}_{\mathrm{R}} / \mathrm{V}_{\text {in }} \\ \mathrm{mag} \end{gathered}$ | $V_{R} / v_{\text {in }}$ angle | $\begin{gathered} V_{\mathrm{R}} / V_{\text {in }} \\ \mathrm{mag} \end{gathered}$ | $\begin{aligned} & V_{R} / V_{\text {in }} \\ & \text { angle } \end{aligned}$ | $\begin{gathered} V_{\mathrm{R}} / V_{\text {in }} \\ \mathrm{mag} \end{gathered}$ | $\begin{aligned} & V_{\mathrm{R}} / V_{\text {in }} \\ & \text { angle } \end{aligned}$ |
| 1 |  |  |  |  |  |  |  |  |
| 1.4 |  |  |  |  |  |  |  |  |
| 2 |  |  |  |  |  |  |  |  |
| 2.8 |  |  |  |  |  |  |  |  |
| 4 |  |  |  |  |  |  |  |  |
| 5.4 |  |  |  |  |  |  |  |  |
| 7.5 |  |  |  |  |  |  |  |  |
| 10 |  |  |  |  |  |  |  |  |
| 14 |  |  |  |  |  |  |  |  |
| 20 |  |  |  |  |  |  |  |  |
| 28 |  |  |  |  |  |  |  |  |
| 40 |  |  |  |  |  |  |  |  |
| 54 |  |  |  |  |  |  |  |  |
| 75 |  |  |  |  |  |  |  |  |
| 100 |  |  |  |  |  |  |  |  |

## Discussion

C.D1. Calculate the resonance frequencies for the four cases in Table C. 1 and compare them with the measured values.
C.D2. Measure the bandwidth from your plots for $\mathrm{R}=10 \Omega$ and $\mathrm{R}=100 \Omega$.

- How does bandwidth vary with R?
- Deduce the value of the effective series resistance for the two R values. Why does this value differ from R ? What is the DC resistance of the coil?

Note $\quad$ Bandwidth is defined as $f_{2}-f_{1}$, where $f_{2}$ and $f_{1}\left(f_{2}>f_{1}\right)$ are the frequencies where the magnitude of the transfer function is $\frac{1}{\sqrt{2}}$ times its maximum value.
C.D3. Compare your plots with the theoretical values of resonance frequency and bandwidth and explain any discrepancies. How does the bandwidth vary with varying $L$ and with varying $C$ ? Explain.

