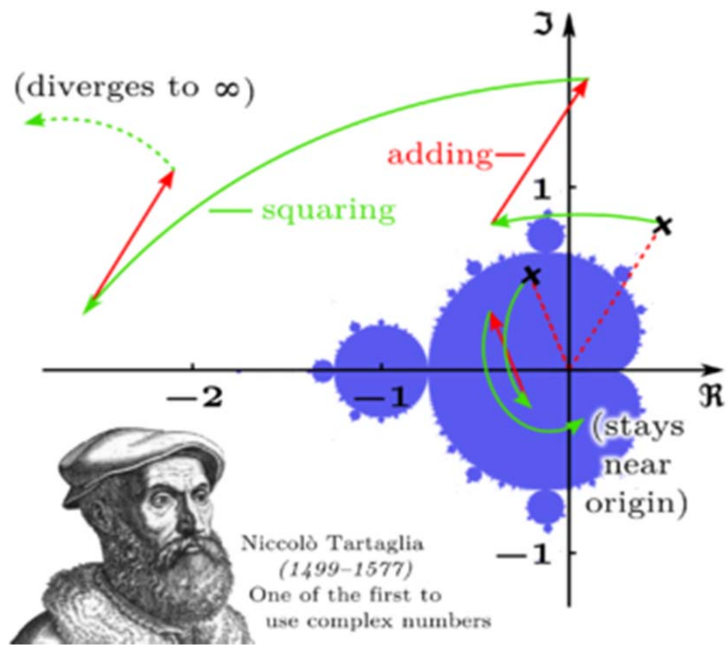
Lecture 8
Introduction to Complex Numbers

October 21, 2010



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Definitions

- The set of all complex numbers is usually denoted by **C**.
- Complex numbers are usually written in the form: **z=a+jb**
- where *a* and *b* are real numbers, and *j* is the imaginary unit, which has the property $j^2 = -1$. The real number *a* is called the *real part* of the complex number, and the real number *b* is the *imaginary part*.
- Example: 3 + j2 is a *complex number*, with real part 3 and **imaginary** part 2. If $z = a + bi$, the real part *a* is denoted Re(*z*) or $\Re(z)$, and the imaginary part *b* is denoted Im(*z*) or $\Im(z)$.

Definitions: a+jb

- If b=0, the complex number wil thenbe a real number.
- If a=0, the Complex numbers are then called i*maginary numbers.*

Notation

- In some disciplines (in particular, electrical engineering, where *i* is a symbol for current), the imaginary unit *i* is instead written as *j*, so complex numbers are sometimes written as *a* + *bj* or *a* + *jb*.

Equality of Complex Numbers

- Two complex numbers are said to be equal if and only if their real parts are equal *and* their imaginary parts are equal.

$x+jy = 2 + j5$. This implies, $x=2$ and $y=5$

Operations on Complex Numbers: Addition

- **Addition**

$$(a + jb) + (c + jd) := (a + c) + j(b + d)$$

- **Multiplication**

$$(a + jb)(c + jd) := (ac + jad + jbc + j^2bd)$$
$$= (ac - bd) + j(bc + ad)$$

- **Division**

$$\frac{a+bi}{c+di} = \frac{(a+bi)(c-di)}{(c+di)(c-di)} = \frac{ac+bci-adi+bd}{(c+di)(c-di)}$$
$$= \left(\frac{ac+bd}{c^2+d^2}\right) + \left(\frac{bc-ad}{c^2+d^2}\right)i,$$

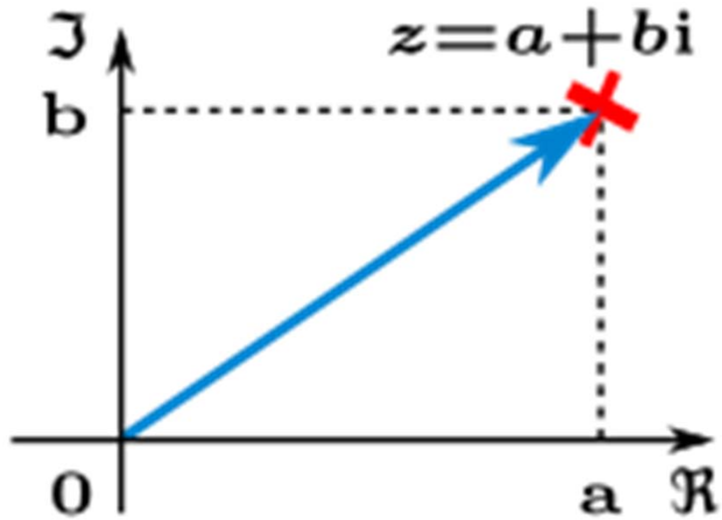
Conjugate

The *conjugate* of $z = x + jy$ is $x - jy$ *and* written as or z^* .

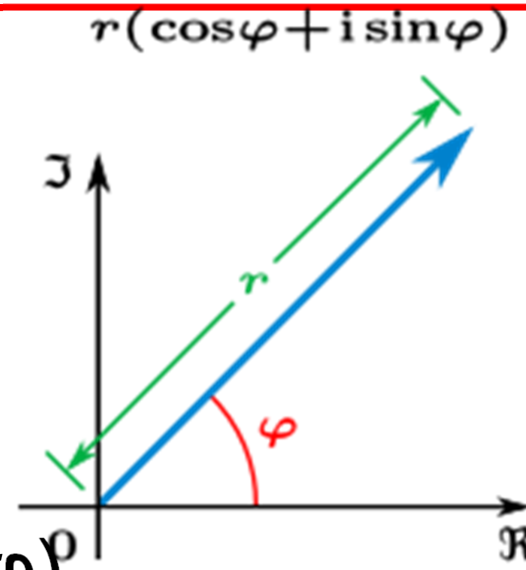
Example: The conjugate of 3-j4 is 3+j4

The complex plane

A complex number can be viewed as a point or position vector in a two-dimensional Cartesian coordinate system called the complex plane or Argand diagram, named after Jean-Robert Argand. The numbers are conventionally plotted using the real part as the horizontal component, and imaginary part as vertical .



Polar form



From above:

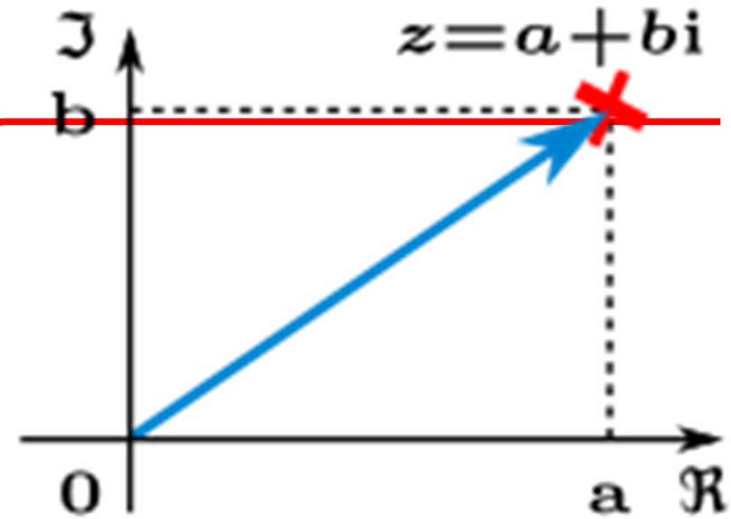
The real part: $a = r\cos(\varphi)$

The imaginary part: $b = r\sin(\varphi)$

$z = r(\cos\varphi + j\sin\varphi) = re^{j\varphi}$

r is called the magnitude (Envelope) and φ is called the *argument* or *phase* of a complex number *z*.

Computation of r and φ



The magnitude and Argument of z are given by:

$$r = |z| = \sqrt{a^2 + b^2}$$

$$\varphi = \tan^{-1}\left(\frac{b}{a}\right)$$

Operations in polar form

- Multiplication and division have simple formulas in polar form:

$$(r_1 e^{i\varphi_1}) \cdot (r_2 e^{i\varphi_2}) = (r_1 r_2) e^{i(\varphi_1 + \varphi_2)}$$

$$\frac{(r_1\, e^{i\varphi_1})}{(r_2\, e^{i\varphi_2})} = \left(\frac{r_1}{r_2}\right) e^{i(\varphi_1 - \varphi_2)}.$$

Example

$z_1=3+j4$ and $z_2=3+j3$

In Polar form:

$z_1=5<53.13^0$ and $z_2=4.24<45^0$

Computation in algebraic form:

$z_1z_2=(3+j4)(3+j3)=(9-12)+j(9+12)=-3+j21$

$z_1z_2=(5<53.13^0)(4.24<45^0)=21.213<98.13^0$

$= 21.213(\cos 98.13^0+j\sin 98.13^0)$

$=21.213(-0.14142+j0.9899)=-3+j21$

References

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