# BUSS 230: Managerial Economics
# Fall 2011-2012
# Regression Assignment
# ANSWER KEY
# Sections 1 to 6

## Question 1

**a.** $\beta_2$ is the output elasticity of labor. It measures the percentage change in output due to a percentage change in labor.

$\beta_3$ is the output elasticity of capital. It measures the percentage change in output due a to a percentage change in capital.

**b.**

Regression Analysis

| | |
|---|---|
| $R^2$ | 0.688 |
| Adjusted $R^2$ | 0.668 |
| R | 0.830 |
| Std. Error | 0.217 |
| 33 | observations |
| 2 | predictor variables |
| LNQ | is the dependent variable |

ANOVA table

| *Source* | *SS* | *df* | *MS* | *F* | *p-value* |
|---|---|---|---|---|---|
| Regression | 3.1123 | 2 | 1.5561 | 33.12 | 2.55E-08 |
| Residual | 1.4094 | 30 | 0.0470 | | |
| Total | 4.5217 | 32 | | | |

Regression output

| | | | | | *confidence interval* | |
|---|---|---|---|---|---|---|
| *variables* | *coefficients* | *std. error* | *t (df=30)* | *p-value* | *95% lower* | *95% upper* |
| intercept | -0.1287 | 0.5461 | -0.236 | .8153 | -1.2440 | 0.9867 |
| LNL | 0.5590 | 0.8164 | 0.685 | .4988 | -1.1084 | 2.2264 |

| | | | | | | |
|---|---|---|---|---|---|---|
| LNK | 0.4877 | 0.7039 | 0.693 | .4937 | -0.9498 | 1.9252 |

**c.** We can use p-values to test the significance of the coefficients $\beta_2$ and $\beta_3$ at the 5% level.
For $\beta_2$: p-value = 0.49>0.05=α. So we do not reject the null that $\beta_2$ is different from zero.
Conclusion: $\beta_2$ is insignificant
For $\beta_3$: p-value = 0.49>0.05=α. So we do not reject the null that $\beta_3$ is different from zero.
Conclusion: $\beta_3$ is insignificant.

**d.** For the joint significance of the variables in this regression, we need to use the p-value of the F-test.
p-value of F-test = $2.55\times10^{-8} < 0.05$. So we reject the null that the variables in this regression are jointly insignificant. Conclusion: the variables in this regression are jointly significant.

**e.** The $R^2$ of this regression is 68% and is reasonably high. This regression exhibits good fir. Note that $R^2$ can be interpreted as 68% of the variation in log quantity being due to variation in log labor and log capital.

**f.** This production function exhibits increasing returns to scale. This is due to the fact that $\hat{\beta}_2 + \hat{\beta}_3$ =0.5590+0.4877>1.

**g.** The variables are individually insignificant but jointly highly significant. This is not very intuitive and might suggest the presence of multicollinearity.

**h.** We can compute the correlation between the two independent variables ln(K) and ln(L). The correlation coefficient is 0.98 and this indicates the presence of multicollinearity.

## Question 2

**a.** Due to the law of demand, $\beta_{12}$, $\beta_{22}$ and $\beta_{32}$ are expected to be negative. The signs of $\beta_{13}$, $\beta_{23}$ and $\beta_{33}$ depend on whether we expect the good to be an inferior or normal good.
Meat is expected to be a normal good so $\beta_{13}$ is expected to be positive.
Fruits and vegetables are also expected to be normal goods so $\beta_{23}$ is expected to be positive.
It can be argued that cereals and bakery products are either a normal or inferior good. Therefore, $\beta_{33}$ can be either positive or negative.

**b.** $\beta_{12}$, $\beta_{22}$ and $\beta_{32}$ are, respectively, the price elasticities of demand for meat, fruits and vegetables and cereals and bakery products.

$\beta_{13}$, $\beta_{23}$ and $\beta_{33}$ are, respectively, the income elasticity of demand for meat, fruits and vegetables and cereals and bakery products.

**c.**

## Regression Analysis

| | |
|---|---|
| $R^2$ | 0.623 |
| Adjusted $R^2$ | 0.595 |
| R | 0.789 |
| Std. Error | 0.394 |
| 30 | observations |
| 2 | predictor variables |
| LNq1 | is the dependent variable |

ANOVA table

| *Source* | *SS* | *df* | *MS* | *F* | *p-value* |
|---|---|---|---|---|---|
| Regression | 6.9437 | 2 | 3.4719 | 22.33 | 1.89E-06 |
| Residual | 4.1977 | 27 | 0.1555 | | |
| Total | 11.1415 | 29 | | | |

| Regression output | | | | | *confidence interval* | |
|---|---|---|---|---|---|---|
| *variables* | *coefficients* | *std. error* | *t (df=27)* | *p-value* | *95% lower* | *95% upper* |
| intercept | 1.0174 | 1.3541 | 0.751 | .4590 | -1.7611 | 3.7958 |
| LNy | 1.4339 | 0.2288 | 6.267 | 1.05E-06 | 0.9644 | 1.9033 |
| LNp1 | -0.5670 | 0.2149 | -2.639 | .0136 | -1.0079 | -0.1261 |

## Regression Analysis

| | |
|---|---|
| $R^2$ | 0.541 |
| Adjusted $R^2$ | 0.507 |
| R | 0.736 |
| Std. Error | 0.451 |
| 30 | observations |
| 2 | predictor variables |
| LNq2 | is the dependent variable |

ANOVA table

| *Source* | *SS* | *df* | *MS* | *F* | *p-value* |
|---|---|---|---|---|---|
| Regression | 6.4654 | 2 | 3.2327 | 15.92 | 2.71E-05 |
| Residual | 5.4830 | 27 | 0.2031 | | |

| | | | | | | |
|---|---|---|---|---|---|---|
| Total | 11.9484 | 29 | | | | |

Regression output

| variables | coefficients | std. error | t (df=27) | p-value | confidence interval 95% lower | 95% upper |
|---|---|---|---|---|---|---|
| intercept | 2.4628 | 1.4529 | 1.695 | .1016 | -0.5183 | 5.4439 |
| LNp2 | -0.6482 | 0.1875 | -3.456 | .0018 | -1.0330 | -0.2634 |
| LNy | 1.1435 | 0.2612 | 4.378 | .0002 | 0.6075 | 1.6794 |

## Regression Analysis

| | |
|---|---|
| $R^2$ | 0.915 |
| Adjusted $R^2$ | 0.909 |
| R | 0.956 |
| Std. Error | 0.187 |
| 30 | observations |
| 2 | predictor variables |
| LNq3 | is the dependent variable |

ANOVA table

| Source | SS | df | MS | F | p-value |
|---|---|---|---|---|---|
| Regression | 10.1149 | 2 | 5.0575 | 145.07 | 3.60E-15 |
| Residual | 0.9413 | 27 | 0.0349 | | |
| Total | 11.0562 | 29 | | | |

Regression output

| variables | coefficients | std. error | t (df=27) | p-value | confidence interval 95% lower | 95% upper |
|---|---|---|---|---|---|---|
| intercept | 4.8696 | 0.5467 | 8.908 | 1.60E-09 | 3.7479 | 5.9913 |
| LNp3 | -0.9639 | 0.0653 | -14.769 | 1.87E-14 | -1.0978 | -0.8300 |
| LNy | 0.8713 | 0.1082 | 8.050 | 1.19E-08 | 0.6492 | 1.0934 |

**d.** For $\beta_{12}$, p-value = 0.0136<0.05=α, therefore we reject the null that $\beta_{12}$ is insignificant (i.e. it is significant)

For $\beta_{13}$, p-value=$1.05\times10^{-6}$<0.05=α, therefore we reject the null that $\beta_{13}$ is insignificant (i.e. it is significant)

For $\beta_{22}$, p-value=0.0018<0.05=α, therefore we reject the null that $\beta_{22}$ is insignificant (i.e. it is significant)

For $\beta_{23}$, p-value = 0.0002 <0.05=α, therefore we reject the null that $\beta_{23}$ is insignificant (i.e. it is significant)

For $\beta_{32}$, p-value = $1.87\times10^{-14}<0.05=\alpha$, therefore we reject the null that $\beta_{32}$ is insignificant (i.e. it is significant)

For $\beta_{33}$, p-value = $1.19\times10^{-8}<0.05=\alpha$, therefore we reject the null that $\beta_{33}$ is insignificant (i.e. it is significant)

**e.** All three regressions have a high $R^2$ ranging from 54% to 91%. The best "fit" corresponds to the third demand equation (that of cereals and bakery with an $R^2$ of around 91%)

## Question 3

**a.** The portfolio manager is postulating that S&P500 prices follow a linear (or secular) trend model with seasonal variation.

**b.** See excel output.

Regression Analysis

| | |
|---|---|
| R² | 0.820 |
| Adjusted R² | 0.814 |
| R | 0.906 |
| Std. Error | 198.928 |
| 123 | observations |
| 4 | predictor variables |
| P | is the dependent variable |

ANOVA table

| *Source* | *SS* | *df* | *MS* | *F* | *p-value* |
|---|---|---|---|---|---|
| Regression | 21,330,410.8066 | 4 | 5,332,602.7017 | 134.76 | 4.98E-43 |
| Residual | 4,669,552.1591 | 118 | 39,572.4759 | | |
| Total | 25,999,962.9657 | 122 | | | |

| Regression output | | | | | *confidence interval* | |
|---|---|---|---|---|---|---|
| *variables* | *coefficients* | *std. error* | *t (df=118)* | *p-value* | *95% lower* | *95% upper* |
| intercept | -26.4871 | 47.9633 | -0.552 | .5818 | -121.4675 | 68.4933 |
| t | 11.7296 | 0.5053 | 23.214 | 8.24E-46 | 10.7290 | 12.7302 |
| D1 | -5.0832 | 50.9497 | -0.100 | .9207 | -105.9774 | 95.8110 |
| D2 | -2.9312 | 50.9472 | -0.058 | .9542 | -103.8205 | 97.9581 |
| D3 | -20.1559 | 50.9497 | -0.396 | .6931 | -121.0502 | 80.7383 |

**c.** The parameter estimate for b is positive. We need to test for the significance of b at the 5% to establish whether a time trend exists. p-value=$8.24\times10^{-48}$ <0.05=α, therefore we reject the null that b is insignificant (i.e. it is significant). Therefore, b is positive and significant and there is evidence of a time trend in S&P500 prices.

**d.** p-value of c1 = 0.9207 > 0.05, we do not reject the null that c1 is insignificant.
p-value of c2 = 0.9542>0.05, we do not reject the null that c2 is insignificant.
p-value of c3 = 0.6937>0.05, we do not reject the null that c2 is insignificant.
Given that all three seasonal dummy variables are insignificant, we conclude that there is no evidence of seasonality in S&P500 prices.

**e.** Forecast of S&P500 for 2010Q4:

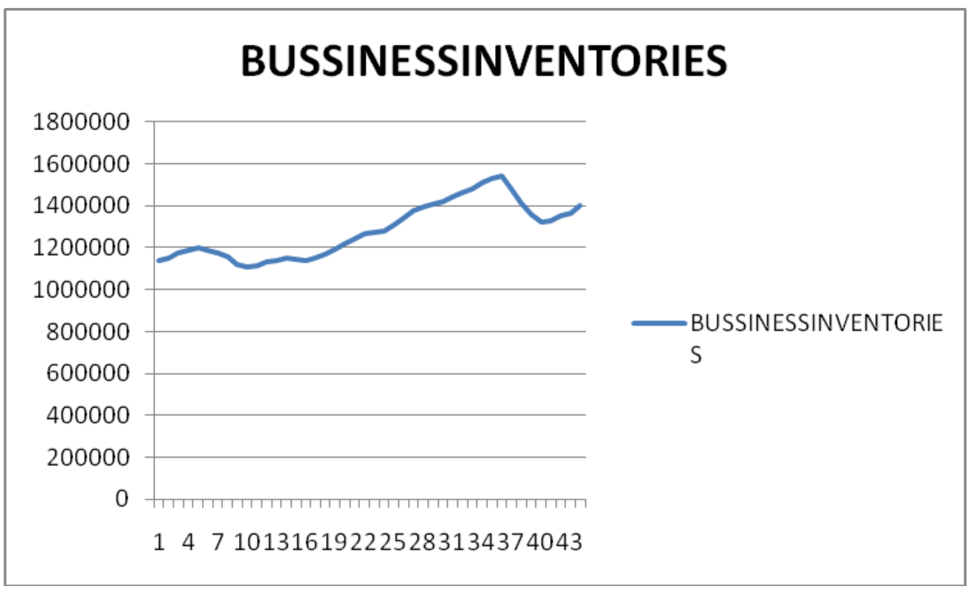
P(2010Q4) = -26.4871+11.72×124=1428.80

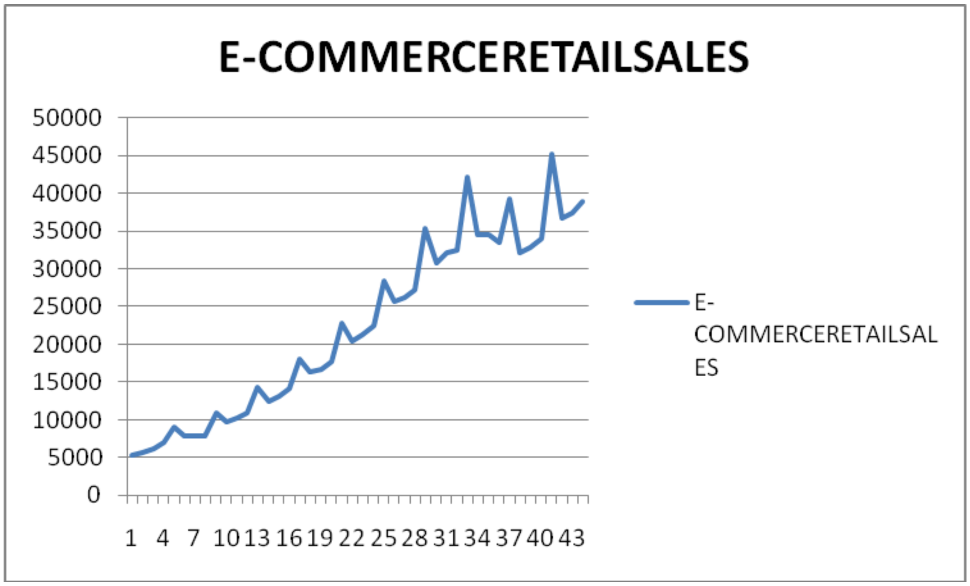
**f.** 3 quarter MA forecast is: 1113.78, while 5 quarter moving average forecast is: 1102.704

**g.** The adjusted closing price on 31 December 2010 is 1,257.64. The best forecasting method is the 3 quarter moving average as it is the closest to the actual value that materializes in December 2010.

**Question 4**

**a.**






**b.** The correlation between E-commerce retail sales and business inventories is 0.879. This is a high correlation coefficient indicating a strong positive linear relationship between the variables.

**c.** Yes, a seasonal pattern is expected in both variables. Sales are expected to increase during the holiday season (In Christmas, the $4^{th}$ quarter of the year) while business are expected to hold higher inventories in the quarter preceding Christmas ($3^{rd}$ quarter) in anticipation for the increase in sales.

**d.** and **e.** Denote business inventories by and E-commerce retail sales by $X_t$ and inventories by $Y_t$. The following 2 equations can be estimated to check for a time trend and seasonal pattern in sales:

$$X_t = a + bt + c_1D_1 + c_2D_2 + c_3D_3 + e_t$$

$$Y_t = a + bt + c_1D_1 + c_2D_2 + c_3D_3 + e_t$$

Where:

$t = 1, 2, \ldots, 44.$

$D_1 = 1$ if $t$ is quarter 1

$D_2 = 1$ if $t$ is quarter 2

$D_3 = 1$ if $t$ is quarter 3

Estimating the 2 equations yields:

## Regression Analysis

| | |
|---|---|
| $R^2$ | 0.691 |
| Adjusted $R^2$ | 0.659 |
| R | 0.831 |
| Std. Error | 77666.022 |
| 44 | observations |
| 4 | predictor variables |
| BUSSINESSINVENTORIES | is the dependent variable |

ANOVA table

| *Source* | *SS* | *df* | *MS* | *F* | *p-value* |
|---|---|---|---|---|---|
| Regression | 525,777,004,336.9070 | 4 | 131,444,251,084.2270 | 21.79 | 1.65E-09 |
| Residual | 235,248,427,936.9800 | 39 | 6,032,010,972.7431 | | |
| Total | 761,025,432,273.8860 | 43 | | | |

Regression output

| | | | | | *confidence interval* | |
|---|---|---|---|---|---|---|
| *variables* | *coefficients* | *std. error* | *t (df=39)* | *p-value* | *95% lower* | *95%* |
| intercept | 1,090,454.5017 | 30,433.8979 | 35.830 | 1.85E-31 | 1,028,896.1334 | 1,152,01 |
| time | 8,617.3960 | 925.6456 | 9.310 | 1.86E-11 | 6,745.1011 | 10,48 |
| Q1 | -2,206.5778 | 33,129.8368 | -0.067 | .9472 | -69,217.9973 | 64,80 |
| Q2 | -3,599.6102 | 33,168.6079 | -0.109 | .9141 | -70,689.4515 | 63,49 |
| Q3 | -3,732.0972 | 33,233.1257 | -0.112 | .9112 | -70,952.4382 | 63,48 |

## Regression Analysis

| | |
|---|---|
| $R^2$ | 0.965 |
| Adjusted $R^2$ | 0.962 |
| R | 0.982 |
| Std. Error | 2324.924 |
| 44 | observations |
| 4 | predictor variables |
| E-COMMERCERETAILSALES | is the dependent variable |

ANOVA table

| *Source* | *SS* | *df* | *MS* | *F* | *p-value* |
|---|---|---|---|---|---|
| Regression | 5,848,914,539.6000 | 4 | 1,462,228,634.9000 | 270.52 | 7.16E-28 |
| Residual | 210,805,548.9455 | 39 | 5,405,270.4858 | | |

| | | |
|---|---|---|
| Total | 6,059,720,088.5455 | 43 |

| Regression output | | | | | confidence interval | |
|---|---|---|---|---|---|---|
| *variables* | *coefficients* | *std. error* | *t (df=39)* | *p-value* | *95% lower* | *95% upper* |
| intercept | 5,613.6409 | 911.0354 | 6.162 | 3.09E-07 | 3,770.8980 | 7,456.3838 |
| time | 905.3591 | 27.7091 | 32.674 | 6.01E-30 | 849.3122 | 961.4060 |
| Q1 | -4,404.1773 | 991.7380 | -4.441 | .0001 | -6,410.1567 | -2,398.1979 |
| Q2 | -4,781.9909 | 992.8986 | -4.816 | 2.23E-05 | -6,790.3179 | -2,773.6639 |
| Q3 | -4,995.4409 | 994.8299 | -5.021 | 1.17E-05 | -7,007.6744 | -2,983.2075 |

Which shows that business inventories have a positive time trend and no clear seasonal pattern (time trend is significant while seasonal dummy variables are not) whereas sales exhibit both a positive and significant time trend and a significant seasonal pattern (both the time trend and seasonal dummies are significant as evidenced by the very small p-vales on the time trend and seasonal dummy variables).