BUSS 230: Managerial Economics
Fall 2011-2012
Assignment 1 Answer Key
Sections 1 to 6

Question 1

a. Ep = $(\%\Delta Q\ /\ \%\Delta P)$ = -8% / 10% = -0.8

b. Inelastic.

c. Less than.

Question 2

a. Decrease, Fall.

b. Increase, Fall.

c. Decrease, not change.

d. Inelastic.

e. Unitary elastic.

f. Inelastic.

Question 3

a. The inverse demand function is given by solving for P in term of Q as:
$-0.5P = Q - 20 \Rightarrow P = (20 - Q)/0.5$, which in turn implies that,
$P = P(Q) = 40 - 2Q$

b. The total revenue function is given by:
TR(Q) = P(Q)×Q
Or,
TR(Q) = (40 – 2Q)Q = 40Q – $2Q^2$

c. The marginal revenue function is the first derivative of the total revenue function and is given by:

$$MR(Q) = \frac{dTR(Q)}{dQ} = 40 - 4Q$$

d. The quantity, Q*, that maximizes total revenue is given by setting MR equal to zero:
MR=0 => 40-4Q*=0 => Q*=10

e. No, total revenue maximization does not qualify as an example of constrained optimization. Since there is no constraint you have to account for (a cost constraint for example), revenue maximization is an unconstrained optimization problem.

Question 4

a. The total revenue function is given by:

$$TR(Q) = P(Q) \times Q = (3000 - 40Q)Q = 3000Q - 40Q^2$$

This implies that the marginal revenue function, being it first derivative, is:

$$MR(Q) = \frac{dTR}{dQ} = 3000 - 80Q$$

b. The demand for the firm's good will be elastic when MR(Q)>0 which, in turn, implies that 3000-80Q>0 or that Q<37.5. By plugging quantity back into the demand equation, this implies that $P \geq \$1500$.

c. Total revenue is maximized when marginal revenue, being its first derivative, equals zero. Setting MR(Q) = 0 yields Q = 37.5 or equivalently $P = \$1500$.

Question 5

a. These goods are extremely close substitutes. A 1 percent increase (decrease) in the price of one good results in a 1.2 percent increase (decrease) in the quantity demanded for the other.

b. 1.2 = % ΔQ / +5%, so that %ΔQ = 6%. This implies that the demand for the good rises by 6% as a result of a 5% increase in the price of the substitute.

Question 6

a. At P = \$10, by plugging back into the demand function we get:
Q= 5000 – 50(10) = 4500

b. At P = \$20, by plugging back into the demand function we get:
Q= 5000 – 50(20) = 4000
At P = \$30, by plugging back into the demand function we get:
Q= 5000 – 50(30) = 3500

c. The arc price elasticity, E_D, between \$10 and \$20 is given by:

$$E_D = \frac{\Delta Q}{\Delta P}\frac{(P_1 + P_2)}{(Q_1 + Q_2)} = \frac{\Delta Q}{(Q_1 + Q_2)} \Bigg/ \frac{\Delta P}{(P_1 + P_2)}$$

Substituting in the numbers, we get:
E_D = [(4000-4500)/(4000+4500)] / [(20-10)/(20+10)] = -0.176

The arc price elasticity, E_D, between \$20 and \$30:
E_D = [(3500-4000)/(3500+4000)] / [(30-20)/(30+20)] = -0.333

d. The point price elasticities are given by:

E_D = Slope of demand × (P/Q), evaluated at the appropriate price and quantity.

E_D = -50(10/4500) = 0.111
E_D = -50(20/4000) = 0.250
E_D = -50(10/3500) = 0.429

Question 7

The arc price elasticity can be computed using: $E_D = \frac{\Delta Q}{\Delta P}\frac{(P_1+P_2)}{(Q_1+Q_2)}$, while marginal revenue is $MR = \frac{\Delta TR}{\Delta Q}$ and total revenue is $TR = P \times Q$. Note that the changes considered here are changes of 10 units, so that you would have to divide changes in TR by 10 to get the MR.

Price	Quantity	Arc Elasticity	Total Revenue	Marginal Revenue
12	30		360	
11	40	-3.29	440	8
10	50	-2.33	500	6
9	60	-1.727	540	4
8	70	-1.308	560	2
7	80	-1	560	0
6	90	-0.765	540	-2
5	100	-0.579	500	-4
4	110	-0.429	440	-6

Question 8

a. Plugging the values P = \$200, M = \$60,000 and P_R = \$100 into the demand function yields:
Q = 250,000 – 500 (200) – 1.50 (60,000) – 240 (100) = 36,000 units

b. The price elasticity of demand is given by:
$E_p = \Delta Q/\Delta P \times P/Q = -500 \times (200/36000) = -2.77$
The above expression uses the fact that $\Delta Q/\Delta P$, being the slope of the demand function with respect to P, is equal to -500.

c. Similarly to the price elasticity of demand, the income elasticity of demand is given by:
$E_M = \Delta Q/\Delta M \times M/Q = -1.5 \times (60000/36000) = -2.5$
Again, this above uses the fact that $\Delta Q/\Delta M$, being the slope of the demand function with respect to M, is equal to -1.5.

Good X is inferior since an increase in income, M, leads to a decrease in demand Q.

The effect of a 4% increase in income can be calculated using:
$E_M = \%\Delta Q/\%\Delta M = -2.5$ which implies that $\%\Delta Q/4\% = -2.5$, which in turn implies that $\%\Delta Q = -10\%$. This implies that a 4% increase in come leads to a 10% decrease in demand.

d. The cross-price elasticity is given by:
$E_{XR} = \Delta Q/\Delta P_R \times P_R/Q = -240 \times (100/36000) = -0.6667$
The above computation uses the fact that $\Delta Q/\Delta P_R$, being the slope of the demand function with respect to R, is equal to -240.

The negative sign of the cross-price elasticity between X and R indicates that they are complementary goods**.** An increase in the price of R leads to a decrease in the demand for X.

To assess the effect of a 5% decrease in the price of R, holding all other factos fixed, on the demand for X we use:
$E_{XR} = \%\Delta Q/\%\Delta\ P_R = -0.6667$ so that $\%\Delta Q/-5\% = -0.6667$ implying $\%\Delta Q = 3.335\%$. This in turn implies that a 5% decrease in the price of R leads to 3.335% increase in the demand for X.

Question 9

a. X is an inferior good. The negative sign of $\Delta Q/\ \Delta M = -0.6$ implies that an increase in income, M, leads to decrease in demand Q.
b. X and Z are substitutes. The positive sign of $\Delta Q/\Delta P_z = 4$ indicates that the goods are substitutes. An increase in P_Z leads to an increase in the demand for X.
c. At P = 10, M = 30, and $P_z = 6$, by plugging back into the demand function we get:

$Q = 70 – 3.5(10) – 0.6 (30) + 4(6) = 41$

So that,

$E_p = \Delta Q/\Delta P \times P/Q = -3.5 \times (10/41) = 0.85$
$E_{XZ} = \Delta Q/\Delta P_Z \times P_Z/Q = 4 \times (6/41) = 0.5854$
$E_M = \Delta Q/\Delta M \times M/Q = -0.6 \times (30/41) = -0.439$

Question 10

a. Law of demand stipulates that *b* should be negative (there's a negative relationship between the price of the good and quantity demanded). $\hat{b}$ is indeed negative as is predicted theoretically and equals -6.50.

b. $\hat{c} = \Delta Q/\Delta M = 0.13926 > 0$ which implies that the good is normal since demand increases as income increases.

c. The goods are complementary since $\hat{d} = -10.77 = E_{XR} = \Delta Q/\Delta P_R < 0$. This implies that an increase in the price of R leads to a decrease in the demand for X.

d. *First, using P-values:*
<u>(On an exam, you should state the null and alternative hypotheses and provide details similar to the ones provided here)</u>
Decision rule: Reject H_0 if p-value < α

$\alpha = 5\% = 0.05$

First, starting with the intercept, the null and alternative hypotheses are:
H_0: a = 0
H_1: a ≠ 0
p-value for $\hat{a}$ is 0.0001 < 0.05
This implies that we reject H_0, â is significant.

To test the significance of own price of the good, we state the null and alternative hypotheses:
H_0: b = 0
H_1: b ≠ 0
P-value is 0.0492 < 0.05. This implies we reject H_0, so that the price of the good significantly affects quantity demanded.

To test the significance of income, we state the null and alternative hypotheses:
H_0: c = 0

H_1: c $\neq$ 0
P-value is 0.0001 < 0.05. This implies we reject H_0, so that income significantly affects demand.

Regarding the price of the related good R, the null and alternative hypotheses are:
H_0: d = 0
H_1: d $\neq$ 0
P-value is 0.0002 < 0.05. This implies we reject H_0, so that the price of related good R significantly affects demand.

Now, we test the same hypotheses stated above, *using t-statistics*:

Decision rule: Reject H_0 if the calculated t-statistics, in absolute value, exceed the critical value.

The calculated t-statistics are:

$$t_{\hat{a}} = \frac{\hat{a}}{se(\hat{a})} = (68.38/12.65) = 5.41$$

$$t_{\hat{b}} = \frac{\hat{b}}{se(\hat{b})} = (-6.50/3.15) = -2.06$$

$$t_{\hat{c}} = \frac{\hat{c}}{se(\hat{c})} = (0.1392/0.0131) = 10.63$$

$$t_{\hat{d}} = \frac{\hat{d}}{se(\hat{d})} = (-10.77/2.45) = -4.40$$

Critical value at α=5% with degrees of freedom df = *(n – k)* = 24 – 4 = 20 is obtained from the t-table as: $c_{\alpha/2, n-k}$ = 2.086 (Note *n=24* as shown next to observations on the computer output whereas *k = 4*).

$|t_{\hat{a}}|$ = 5.41 > 2.08 so that we reject H_0, meaning that the intercept is significant.

$|t_{\hat{b}}|$ =2.06 < 2.08 meaning we do not reject H_0. This implies that the price of the good affects quantity demanded significantly.

$|t_{\hat{c}}|$ =10.63 > 2.08 → Reject H_0, meaning that income affect demand significantly.

$\left| t_{\hat{d}} \right|$ =4.40 > 2.08 → Reject $H_{0,}$ meaning that the price of good R affects demand significantly.

Note that using p-values and t-tests leads to different conclusions regarding the significance of the price of the good. This is an inconsistent result and should not typically occur.

e. By plugging back P = 225, M = 24,000 and P_R= 60 into the demand function we get the quantity:
Q = 68.38 – (6.50 ×225) + (0.13926 × 24000) – (10.77 × 60) = 1,301.92

To compute the price elasticity of demand, we get:

E_p = ΔQ/ΔP × P/Q = $\hat{b}$ × (P/Q) = -6.50 × (225/1301.92)=-1.12

Whereas the income elasticity of demand and the cross-price elasticity of demand can be computed similarly as:

E_M = ΔQ/ΔM× M/Q= $\hat{c}$ ×(24,000/1301.92)= 2.57
E_{XR} = $\Delta Q/\Delta P_R \times P_R/Q$ = $\hat{d} \times (24{,}000/1301.92)$ = -0.49

Question 11

a. *First, using P-values:*

Decision rule: Reject H_0 if p-value < α

α= 5% = 0.05 (since it has not been specified)

First, starting with the intercept, the null and alternative hypotheses are:
H_0: a = 0
H_1: a ≠ 0
p-value for $\hat{a}$ is 0.0716 ? 0.05
This implies that we do not reject $H_{0,}$ or that the intercept is not significant.

To test the significance of own price of the good, we state the null and alternative hypotheses:
H_0: b = 0
H_1: b ≠ 0

P-value is 0.0093 < 0.05. This implies we reject $H_{0,}$ so that the price of the good significantly affects quantity demanded.

To test the significance of income, we state the null and alternative hypotheses:
H_0: c = 0
H_1: c $\neq$ 0

P-value is 0.0009 < 0.05. This implies we reject H_0, so that income significantly affects demand.

Regarding the price of tennis rackets, the null and alternative hypotheses are:
H_0: d = 0
H_1: d $\neq$ 0
P-value is 0.006 < 0.05. This implies we reject H_0, so that the price of tennis rackets significantly affects demand
Note that the same conclusions would hold if we use a level of significance of 1%. At the 10% level, all the variables including the intercept would be significant.
The signs of the parameter estimates are consistent with demand theory: $\hat{b}$ is negative as the law of demand predicts, ĉ is positive indicating tennis balls are a normal good and we expect for most goods, and $\hat{d}$ is negative as it should be for complements (tennis balls and tennis rackets).

Now, we test the same hypotheses stated above, *using t-statistics*:

Decision rule: Reject H_0 if the calculated t-statistics, in absolute value, exceed the critical value.

The calculated t-statistics are:

$$t_{\hat{a}} = \frac{\hat{a}}{se(\hat{a})} = (425120.0/220300.0) = 1.92$$

$$t_{\hat{b}} = \frac{\hat{b}}{se(\hat{b})} = (-37260.6/12587) = -2.96$$

$$t_{\hat{c}} = \frac{\hat{c}}{se(\hat{c})} = (1.49/0.3651) = 4.0811$$

$$t_{\hat{d}} = \frac{\hat{d}}{se(\hat{d})} = (-1456.0/460.75) = -3.16$$

Critical value at α=5% with degrees of freedom df = $(n - k)$ = 20 – 4 = 16 is obtained from the t-table as: $c_{\alpha/2, n-k}$ = 2.12 (Note $n=20$ as shown next to observations on the computer output whereas $k = 4$).

$|t_{\hat{a}}|$ = 1.92 < 2.12 so that we do not reject H_0, meaning that the intercept is notsignificant.

$|t_{\hat{b}}|$ = 2.96 > 2.12 meaning we reject H_0. This implies that the price of the good affects quantity demanded significantly.

$|t_{\hat{c}}|$ =4.0811 > 2.12 → Reject H_0, meaning that income affect demand significantly.

$|t_{\hat{d}}|$ =3.16 > 2.12 → Reject H_0, meaning that the price of tennis rackets affects demand significantly.

The decisions regarding the significance of the variables that we get from p-values and t-statistics are consistent.

- **a.** By plugging back into the demand equation we get:
  240,134 cans per quarter = 425,120 + (-37260.6 x 1.65) + (1.49 x 24,600) + (-1456 × 110)
- **b.** Similarly to question 10, the elasticities can be computed as:
  E_p = -37260.6(1.65/240,143) = -0.256
  E_M = 1.49(24,600/240,143) = 0.153
  E_{XR} = -1456(110/240,143) = -0.667
- **c.** % ΔQ / -15% = -0.256; implying that sales rise by 3.84% due to a decrease in the price of tennis balls by 15%.
- **d.** % ΔQ / 20% = -0.153; implying that sales rise by 2.98% due to an increase in income by 10%.
- **e.** % ΔQ / 25% = -0.667; implying that sales rise by 16.67% due to an increase in the price of rackets by 25%.