

Chapter 15

Project Management

15.0 Introduction

At some point, almost every manager assumes responsibility for the completion of some type of project. The project might be relatively simple—such as planning a company picnic or producing an employee newsletter—or it might be more complex—such as planning the launch of a space shuttle, designing and implementing a large computer information system, or constructing a multistory office building. Successfully completing a project of any size requires a thorough analysis of the physical and logical interdependencies among the tasks involved and accurate estimates of the time and resources required to accomplish these tasks. Keeping track of all these details for even a relatively small project can be overwhelming.

This chapter presents two techniques that were developed to help managers plan, organize, and control projects: the **Critical Path Method** (CPM) and the **Program Evaluation and Review Technique** (PERT). Both techniques were developed concurrently but independently during the 1950s. CPM was developed by representatives of DuPont and Sperry-Rand to assist in the management of industrial projects in which the time required to perform each activity in the project could be estimated with a high degree of accuracy. The focus of CPM is to determine when a project should be completed and to schedule when each activity in the project must begin in order to keep the project on schedule. PERT was developed jointly by representatives of the U.S. Navy, Lockheed, and the consulting firm of Booz, Allen, and Hamilton working on the Polaris submarine. Because many of the tasks required in the development of the Polaris had never been done before, the time required to complete the tasks was uncertain. Thus, PERT was designed for projects in which the time required to perform each activity is essentially a random variable. PERT focuses on estimating the probability of completing a project by a given deadline.

Over the years, the two techniques have blended together, so that most practitioners today refer to them collectively as PERT/CPM or CPM/PERT. The fact that these techniques have stood the test of time and are still widely used is a tribute to their usefulness and simplicity. Indeed, the basic ideas behind PERT/CPM provide the underpinnings for a number of project-management software tools. In this chapter, we explore some of the fundamental concepts of CPM and PERT using a spreadsheet. We will then see how many of these concepts and techniques are implemented in Microsoft Project (see <http://www.microsoft.com/office/project>).

15.1 An Example

One of the key ideas in both CPM and PERT is that any project can be broken down into component activities that require different amounts of time and that must be accomplished in a specific order. The major difference between CPM and PERT involves how

FIGURE 15.1

Activities for the
Lightner
Construction
home-building
project

Activity	Description	Time Required (in days)	Immediate Predecessor Activity
A	Excavate	3	—
B	Lay foundation	4	A
C	Rough plumbing	3	B
D	Frame	10	B
E	Finish exterior	8	D
F	Install HVAC	4	D
G	Rough electric	6	D
H	Sheetrock	8	C, E, F, G
I	Install cabinets	5	H
J	Paint	5	H
K	Final plumbing	4	I
L	Final electric	2	J
M	Install flooring	4	K, L

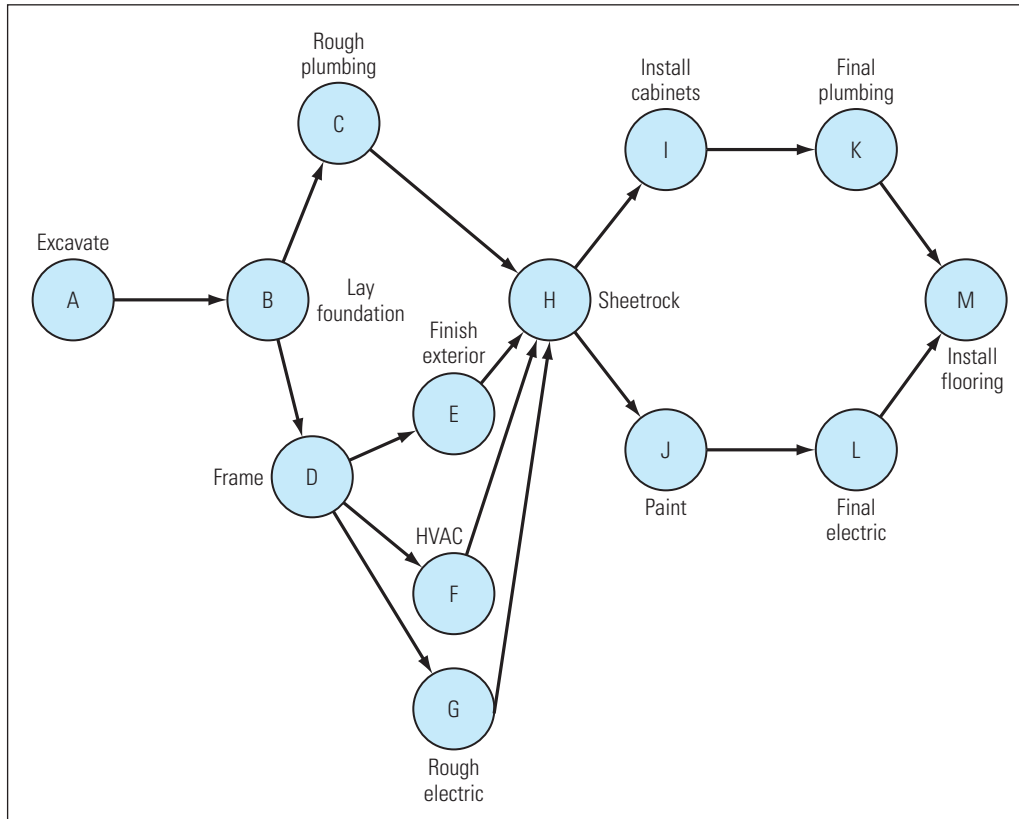
the time element of the activities is determined. However, both CPM and PERT require a detailed network of the project that clearly indicates each of the main activities in the project and their precedence relationships. The following example illustrates how to create such a project network.

Tom Lightner is the owner of Lightner Construction, a general contracting company that specializes in the construction of single-family residences and small office buildings. Tom frequently has numerous construction projects going on at the same time and needs a formal procedure for planning, monitoring, and controlling each project. He is aware of various project-scheduling techniques but has never used them. He wants to see how he might apply such techniques to one of the home-building projects he will be undertaking in the near future. Tom has identified each of the major activities required for this project and has estimated how much time each activity will require. Figure 15.1 summarizes this information along with the precedence relationships among the activities.

15.2 Creating the Project Network

The activities in a project can be represented as a network in one of two ways. As noted in Chapter 5, a network is a set of nodes that are connected in various ways by directed arcs. Thus, if we let the nodes in a network represent the activities in a project, we are employing the **Activity-On-Node** (AON) network design. Figure 15.2 shows this type of network for our example problem.

The nodes in the network shown in Figure 15.2 correspond to each project activity in Figure 15.1. The arcs in this network indicate the precedence relationships between the activities (or nodes). For example, Figure 15.2 shows that we cannot start laying the foundation (activity B) until the excavation (activity A) is complete. Similarly, the arcs leading into node H indicate that the sheetrock cannot be installed until the activities represented by nodes C, E, F, and G are *all* complete.

**FIGURE 15.2**

Activity-On-Arrow (AOA) representation of the home-building project

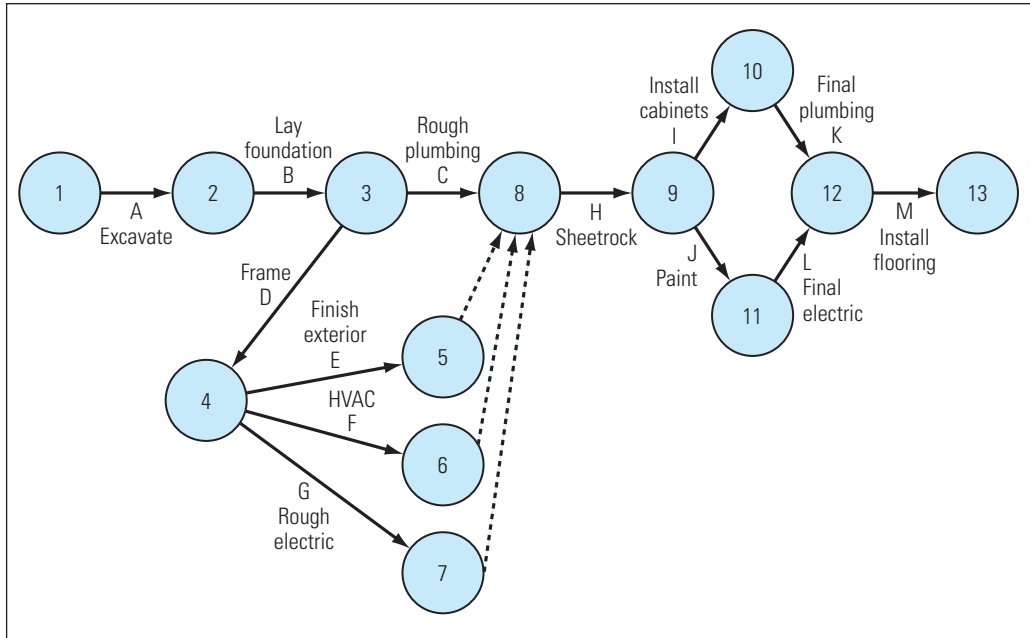
In this type of network, activity A is called the **immediate predecessor** of activity B, and activities C and D are called the **immediate successors** of activity B. Throughout this chapter, we use the shortened terms **predecessor** and **successor** to mean immediate predecessor and immediate successor.

The second way to represent a project as a network is the **Activity-On-Arrow (AOA)** design, in which the arcs represent the project's activities. Figure 15.3 shows an AOA network for our example problem.

The nodes in an AOA network represent the start and finish points (or milestones) for each activity. For example, in Figure 15.3, activity A starts at node 1 and ends at node 2. Activity B starts at node 2 and ends at node 3, and so on. Unfortunately, an AOA representation often produces a more complicated network when multiple activities start and end at common nodes. For example, because activity H (which begins at node 8) cannot begin until activities C, E, F, and G are complete, we might be inclined to draw the arcs for activities E, F, and G as starting at node 4 and finishing at node 8 (eliminating nodes 5, 6, and 7). However, an AOA network does not allow multiple arcs with common start and finish nodes. To get around this problem, we must include the "phantom" activities indicated by the dashed lines connecting nodes 5, 6, and 7 to node 8. AOA networks are mentioned here so that you will be aware of their existence (and shortcomings). In this chapter, we use only the AON network representation, which many believe to be superior.

FIGURE 15.3

Activity-On-Arc (AOA) representation of the home-building project

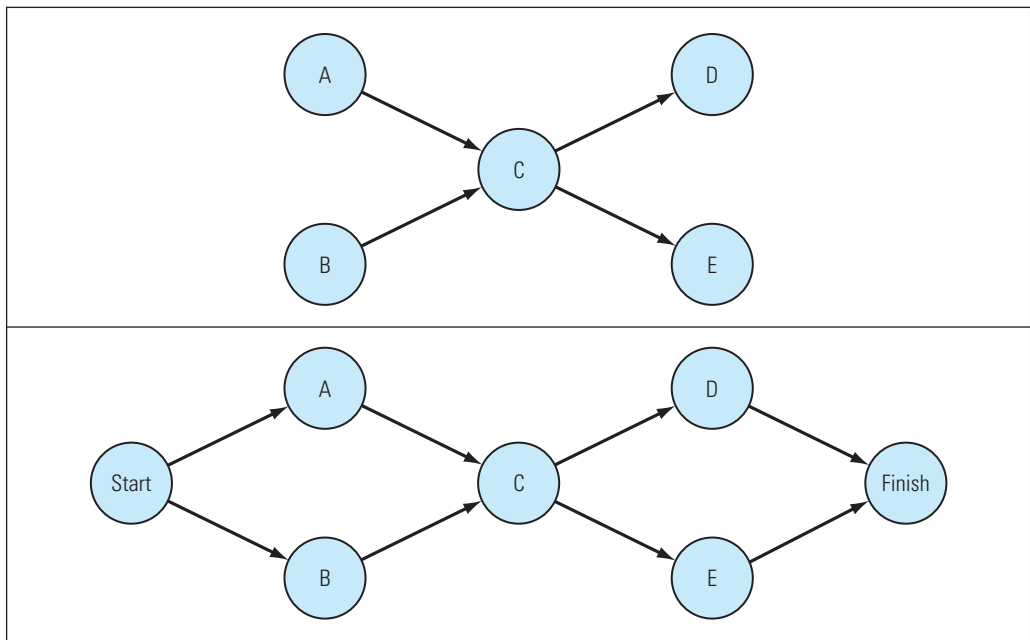


15.2.1 START AND FINISH POINTS

The techniques discussed in this chapter require that all projects have one unique start activity and one unique finish activity. This requirement is met by our example project shown in Figure 15.2 in which activity A is the unique start point, and activity M is the unique finish point. However, some projects involve multiple start and/or finish activities, as illustrated in Figure 15.4.

FIGURE 15.4

Using artificial start and finish activities to produce a network with unique start and finish points



The first network shown in Figure 15.4 represents a project with five activities: A, B, C, D, and E. However, this project does not have a unique start or finish activity. In such cases, it is necessary to create artificial start and/or finish activities, as shown by the second network in Figure 15.4. Here, we created an artificial start activity that precedes activities A and B. Similarly, we inserted an artificial finish activity that follows the completion of activities D and E. These activities are artificial because they require no time to complete and merely serve to give a project a unique start and finish point. Thus, by using artificial start and finish activities, we can ensure that the network for any project has a unique start and finish point.

15.3 CPM: An Overview

After creating a network representation of a project, the next step in the CPM technique is to determine the earliest time that each activity in the network can start and finish. We determine these times by making what is called a **forward pass** through the network. Ultimately, this analysis determines the earliest time that the project itself can be completed. Next, we make a **backward pass** through the network to determine the latest time that each activity can start and finish without delaying the completion of the project.

One of the primary goals in CPM is to determine the critical path through the network. Figure 15.2 shows several paths that could be followed from the initial activity (node A) to the final activity (node M). For example, one path is given by the following set of activities: $A \rightarrow B \rightarrow C \rightarrow H \rightarrow I \rightarrow K \rightarrow M$. Another path through the network is given by: $A \rightarrow B \rightarrow D \rightarrow F \rightarrow H \rightarrow J \rightarrow L \rightarrow M$. Figure 15.2 shows a total of eight paths through the network, each representing a set of activities that must be completed before the project is complete. Therefore, the earliest time that the project can be completed is the time required to complete the set of activities along the *longest* path through the network. The **critical path** is the longest path through a project network.

Any delays in the start or finish times of the activities on the critical path (also known as **critical activities**) delay the completion of the project. A project manager should always identify the critical activities in a project to focus attention on these activities and to avoid delays on these activities when possible.

In conducting the forward pass through the network, we will use the activity times to determine the earliest possible start time and the earliest possible finish time for each activity. The following notations represent these quantities:

t_i = amount of time required to perform activity i

EST_i = earliest start time for activity i

EFT_i = earliest finish time for activity i

During the backward pass, we will determine the latest time that each activity can be started and finished without delaying the project. These quantities are represented as:

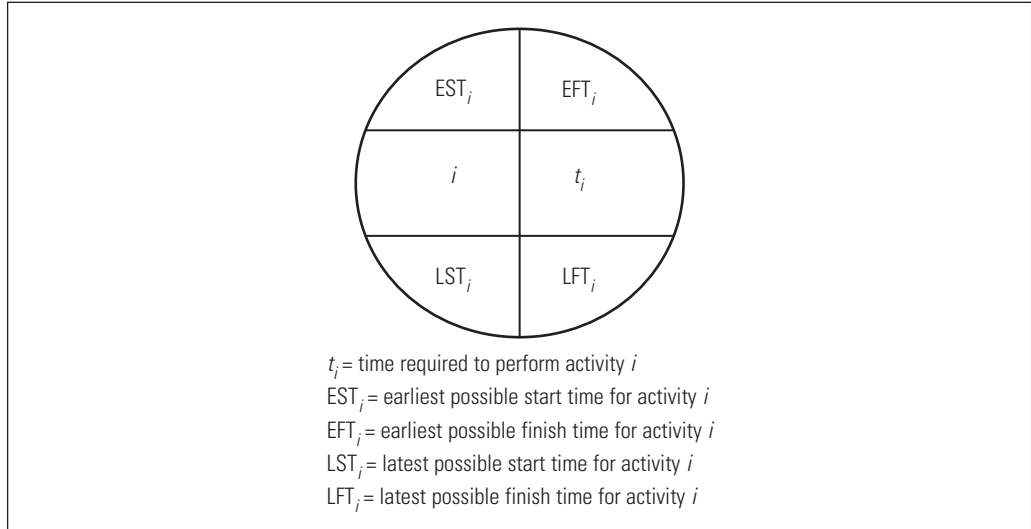
LST_i = latest start time for activity i

LFT_i = latest finish time for activity i

We will record all the information about each project activity on the corresponding network node in the format shown in Figure 15.5.

FIGURE 15.5

Information recorded on the node for each project activity



15.4 The Forward Pass

The first step in the CPM technique (after constructing the project network) is to determine the earliest time at which each activity in the network can start and finish. The term **time zero** identifies the time period at which the first project activity begins—just as the time on a stopwatch starts from zero. Thus, in our example problem, the earliest time at which activity A (excavation) can begin is time zero, or $EST_A = 0$.

In general, the earliest time at which an activity can finish is the earliest time at which it can start plus the expected time required to perform the activity. That is, for any activity i :

$$EFT_i = EST_i + t_i$$

For activity A, $EFT_A = EST_A + t_A = 0 + 3 = 3$. The project network in Figure 15.6 shows the earliest start and finish times for activity A.

Now consider activity B. Because activity A must be completed before activity B can begin, the earliest start time for activity B equals the earliest finish time for activity A. That is, $EST_B = EFT_A = 3$. Activity B is expected to take four units of time; therefore, its earliest finish time is $EFT_B = EST_B + t_B = 3 + 4 = 7$.

Now consider activity C. Because activity B must be completed before activity C can begin, the earliest start time for activity C equals the earliest finish time for activity B. That is, $EST_C = EFT_B = 7$. Activity C is expected to take three units of time; therefore, its earliest finish time is $EFT_C = EST_C + t_C = 7 + 3 = 10$.

As you can see from these examples, calculating the earliest start and finish times for activities with a single predecessor activity is a simple process. We could apply this same logic to determine the earliest start and finish times for activities D, E, F, and G shown in Figure 15.6. You should verify the calculations for these activities on your own to make sure you understand the process. Figure 15.7 provides details of all the forward pass calculations for this example.

Notice that the calculation of the earliest start time for activity H differs from the previous activities because it has *multiple* predecessors rather than a single predecessor. Activity H cannot begin until activities C, E, F, and G are *all* completed. Thus, if we consider the earliest finish times of activities C, E, F, and G, the latest of these times determines the earliest time at which activity H can start:

$$EST_H = \text{MAX}(EFT_C, EFT_E, EFT_F, EFT_G) = \text{MAX}(10, 25, 21, 23) = 25$$

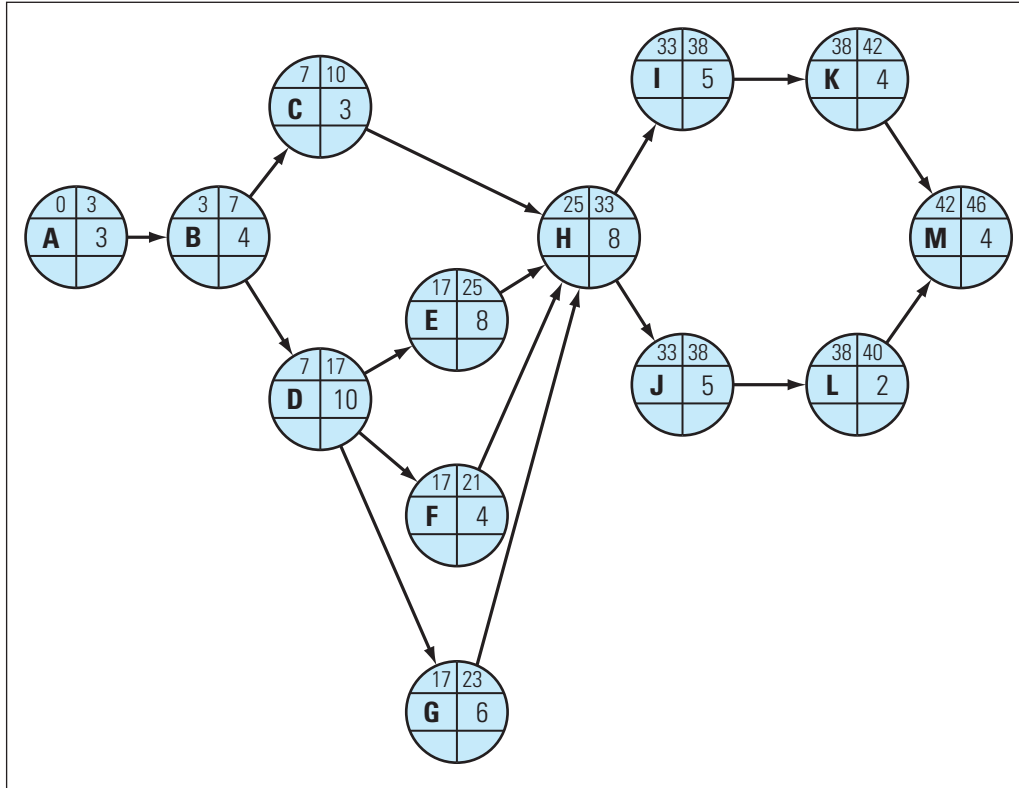


FIGURE 15.6

Results of forward pass calculation of earliest start and finish times

Earliest Start Times	Earliest Finish Times
$EST_A = 0$	$EFT_A = EST_A + t_A = 0 + 3 = 3$
$EST_B = EFT_A = 3$	$EFT_B = EST_B + t_B = 3 + 4 = 7$
$EST_C = EFT_B = 7$	$EFT_C = EST_C + t_C = 7 + 3 = 10$
$EST_D = EFT_B = 7$	$EFT_D = EST_D + t_D = 7 + 10 = 17$
$EST_E = EFT_D = 17$	$EFT_E = EST_E + t_E = 17 + 8 = 25$
$EST_F = EFT_D = 17$	$EFT_F = EST_F + t_F = 17 + 4 = 21$
$EST_G = EFT_D = 17$	$EFT_G = EST_G + t_G = 17 + 6 = 23$
$EST_H = \text{MAX}(EFT_C, EFT_E, EFT_F, EFT_G)$ $= \text{MAX}(10, 25, 21, 23) = 25$	$EFT_H = EST_H + t_H = 25 + 8 = 33$
$EST_I = EFT_H = 33$	$EFT_I = EST_I + t_I = 33 + 5 = 38$
$EST_J = EFT_H = 33$	$EFT_J = EST_J + t_J = 33 + 5 = 38$
$EST_K = EFT_I = 38$	$EFT_K = EST_K + t_K = 38 + 4 = 42$
$EST_L = EFT_J = 38$	$EFT_L = EST_L + t_L = 38 + 2 = 40$
$EST_M = \text{MAX}(EFT_K, EFT_L)$ $= \text{MAX}(42, 40) = 42$	$EFT_M = EST_M + t_M = 42 + 4 = 46$

FIGURE 15.7

Summary of forward pass calculations

The earliest start time of an activity is equal to the latest (or maximum) early finish time of the activities directly preceding it. This is true regardless of the number of predecessors an activity has.

The earliest start and finish times for the remaining activities are calculated using this same logic. Notice that the earliest start times for nodes I, J, K, and L are the earliest

finish times of their respective predecessor activities. Because activity M has multiple predecessor activities, its earliest start time is the maximum of the earliest finish times of activities K and L.

The Forward Pass

- The earliest start time for the initial activity in a project is time zero.
- The earliest start time of an activity is equal to the latest (or maximum) early finish time of the activities directly preceding it.
- The earliest finish time of an activity is equal to its earliest start time plus the time required to perform the activity.

At the end of our forward pass, we have determined the earliest possible start and finish times of each activity in the network. Note that the last activity in the project, activity M, can be completed no earlier than 46 time units (in this case, days) from the beginning of the project. Thus, the earliest the project can be completed is 46 days.

15.5 The Backward Pass

After completing the forward pass, the next step in the CPM technique is to make a backward pass to determine the *latest* times at which the project activities can start and finish without delaying the project. This process identifies those activities that *must* start and finish at their earliest possible times in order to complete the project in the minimum possible time identified by the forward pass. These activities constitute the critical path through the network.

The logic behind the backward pass is similar to that of the forward pass. Here, however, we start at the final activity in the network and work backwards through the network to determine the latest time at which each activity could be finished without delaying the project. Consider activity M. If the project can be completed within 46 days, the latest finish time for activity M is 46, or $LFT_M = 46$.

In general, the latest time at which an activity can start without delaying a project is the latest time by which it must be finished minus the time required to perform the activity. That is, for any activity i :

$$LST_i = LFT_i - t_i$$

For activity M, $LST_M = LFT_M - t_M = 46 - 4 = 42$. Figure 15.8 shows the latest start and finish times for activity M.

Now, let's consider activity L. Because activity L must be completed before activity M can begin, and activity M must begin no later than time period 42, activity L must be finished no later than time period 42. That is, $LFT_L = LST_M = 42$. Activity L is scheduled to take two units of time; therefore, the latest time at which this activity could start without delaying the project is $LST_L = LFT_L - t_L = 42 - 2 = 40$.

As another example, consider activity J. Because activity J must be completed before activity L can begin, and activity L must begin no later than time period 40, activity J must be finished no later than time period 40. That is, $LFT_J = LST_L = 40$. Activity J is scheduled to take five units of time; therefore, the latest time at which this activity could start without delaying the project is $LST_J = LFT_J - t_J = 40 - 5 = 35$.

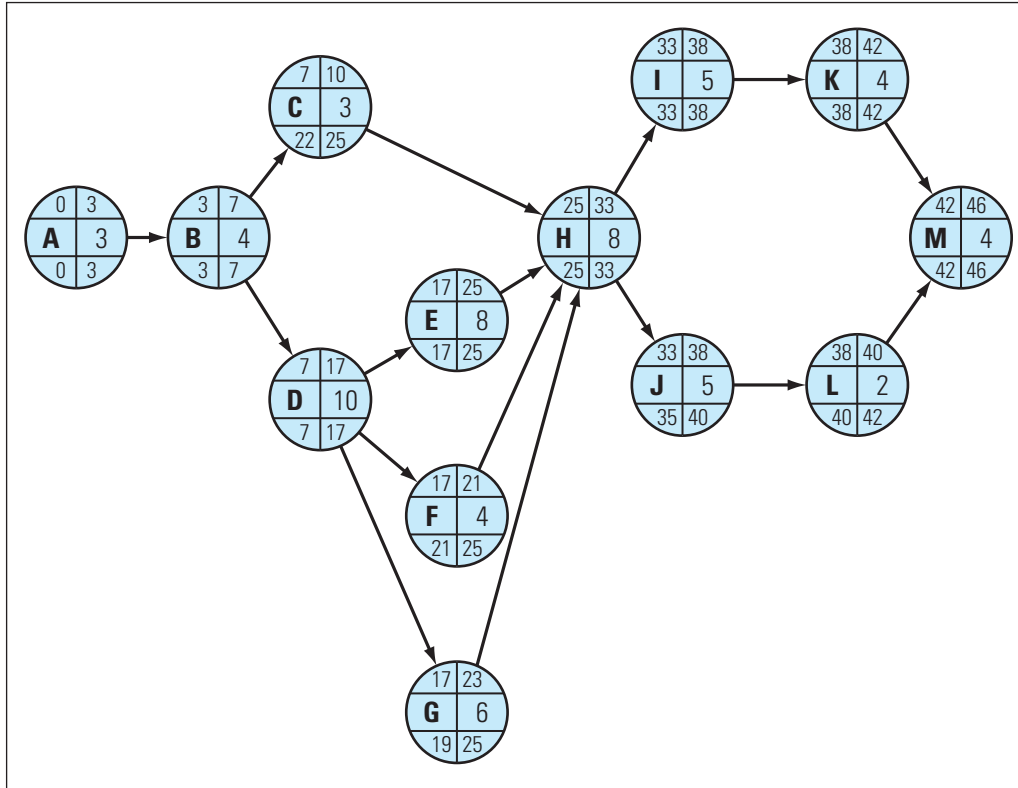


FIGURE 15.8

Results of backward pass calculation of latest start and finish times

Calculating the latest start and finish times for activities with a single successor activity is a simple process. We could apply this same logic to determine the earliest start and finish times for activities I and K, as shown in Figure 15.8. Again, you should verify the calculations for these activities on your own to make sure you understand the process. Figure 15.9 provides the details of the backward pass calculations for this example.

Notice that the calculation of the latest finish time for activity H differs from the previous activities because it has *multiple* successors rather than a single successor. Activity H must be finished in time for activities I and J to begin by their latest start times. Thus, if we consider the latest start times for activities I and J, the earliest of these times determines the latest time by which activity H must be finished:

$$LFT_H = \text{MIN}(LST_I, LST_J) = \text{MIN}(33, 35) = 33$$

The latest finish time of an activity is equal to the earliest (or minimum) late start time of the activities directly following (or succeeding) it. This is true regardless of the number of successors an activity has.

The latest start and finish times for the remaining activities are calculated using this same logic. Notice that the latest finish times for nodes C, E, F, and G are the latest start times of their successor activities. Because activity D has multiple successor activities, its latest finish time is the minimum of the latest start times of activities E, F, and G. Similarly, because activity B has multiple successor activities, its latest finish time is the minimum of the latest start times of activities C and D. Finally, the latest finish time for activity A is the latest start time for activity B ($LFT_A = LST_B = 3$), and the latest start time for activity A is then determined by $LST_A = LFT_A - t_A = 0$.

FIGURE 15.9

Summary of
backward pass
calculations

Latest Finish Times	Latest Start Times
$LFT_M = EFT_M = 46$	$LST_M = LFT_M - t_M = 46 - 4 = 42$
$LFT_L = LST_M = 42$	$LST_L = LFT_L - t_L = 42 - 2 = 40$
$LFT_K = LST_M = 42$	$LST_K = LFT_K - t_K = 42 - 4 = 38$
$LFT_J = LST_L = 40$	$LST_J = LFT_J - t_J = 40 - 5 = 35$
$LFT_I = LST_K = 38$	$LST_I = LFT_I - t_I = 38 - 5 = 33$
$LFT_H = \min(LST_L, LST_J)$ $= \min(33, 35) = 33$	$LST_H = LFT_H - t_H = 33 - 8 = 25$
$LFT_G = LST_H = 25$	$LST_G = LFT_G - t_G = 25 - 6 = 19$
$LFT_F = LST_H = 25$	$LST_F = LFT_F - t_F = 25 - 4 = 21$
$LFT_E = LST_H = 25$	$LST_E = LFT_E - t_E = 25 - 8 = 17$
$LFT_D = \min(LST_E, LST_F, LST_G)$ $= \min(17, 21, 19) = 17$	$LST_D = LFT_D - t_D = 17 - 10 = 7$
$LFT_C = LST_H = 25$	$LST_C = LFT_C - t_C = 25 - 3 = 22$
$LFT_B = \min(LST_C, LST_D)$ $= \min(22, 7) = 7$	$LST_B = LFT_B - t_B = 7 - 4 = 3$
$LFT_A = LST_B = 3$	$LST_A = LFT_A - t_A = 3 - 3 = 0$

The Backward Pass

- The latest finish time for the final activity in a project is equal to its earliest finish time as determined by the forward pass.
- The latest finish time for any other activity is equal to the earliest (or minimum) late start time of the activities directly following (or succeeding) it.
- The latest start time of an activity is equal to its latest finish time minus the time required to perform the activity.

15.6 Determining the Critical Path

As mentioned earlier, one of the key objectives of CPM is to determine the critical path through a project network. The critical path consists of the set of activities that, if delayed in any way, would cause a delay in the completion of the entire project. The activities on the critical path can be identified easily from the results of the forward pass and backward pass. Specifically, the activities whose latest start times equal their earliest start times make up the critical path (or, equivalently, whose latest finish times equal their earliest finish times). As shown by the heavier arcs in Figure 15.10, the critical path of activities in our example problem is:

$$A \rightarrow B \rightarrow D \rightarrow E \rightarrow H \rightarrow I \rightarrow K \rightarrow M$$

If any of the activities in this path do not start by their earliest start time, the overall time required to complete the project will increase—unless management intervenes in some way. (Note that it is possible for an activity network to have more than one critical path, although this is not illustrated in our example.)

The noncritical activities in a project are distinguished by the presence of slack. **Slack** is the amount of time by which the start of an activity can be delayed without delaying

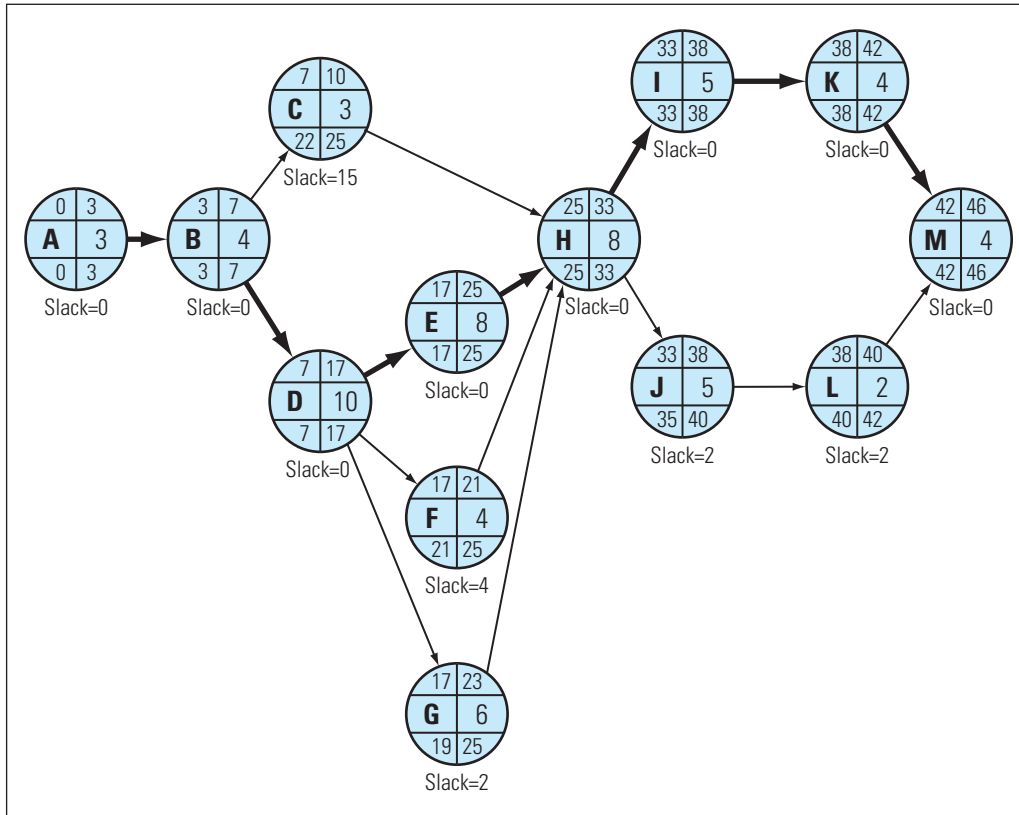


FIGURE 15.10

The critical path and slack conditions

the project. Critical activities have zero slack, and noncritical activities have slack values that are strictly positive. The amount of slack for any activity i is defined as:

$$\text{Slack for activity } i = \text{LST}_i - \text{EST}_i$$

or, equivalently, as:

$$\text{Slack for activity } i = \text{LFT}_i - \text{EFT}_i$$

In our example, activities C, F, G, J, and L are noncritical. Thus, we compute the slack for activity C as:

$$\text{Slack for activity C} = \text{LST}_C - \text{EST}_C = 22 - 7 = 15$$

This indicates that activity C could be delayed up to 15 days beyond its earliest start time without delaying the project. The slack for activities F, G, J, and L are:

$$\text{Slack for activity F} = \text{LST}_F - \text{EST}_F = 21 - 17 = 4$$

$$\text{Slack for activity G} = \text{LST}_G - \text{EST}_G = 19 - 17 = 2$$

$$\text{Slack for activity J} = \text{LST}_J - \text{EST}_J = 35 - 33 = 2$$

$$\text{Slack for activity L} = \text{LST}_L - \text{EST}_L = 40 - 38 = 2$$

15.6.1 A NOTE ON SLACK

In Figure 15.10, note that activities J and L both appear to have 2 units of slack. Now suppose that the start of activity J is delayed for 2 time periods and does not start until time

period 35. Because it takes 5 units of time to perform activity J, this activity should be complete at time period 40. Thus, activity L cannot start any sooner than time period 40. So does activity L still have 2 units of slack? The answer is *no*.

If the start of activity J is delayed by 2 time units, the start of activity L is also delayed by 2 time units. Delaying the start of activity J by 2 time units depletes the slack available not only for activity J but also for activity L. So, the amount of slack available at activity L depends on the amount of slack used at activity J.

You must be careful in interpreting how much slack is available for a given activity. To be precise, slack represents the amount of time by which the start of an activity can be delayed without delaying the entire project, *assuming that all predecessor activities start at their earliest start times*. If any activity on a noncritical path starts late, the slack available along the rest of the noncritical path is reduced. For this reason, it is safer to focus on the latest start times of each activity (rather than on slack) because if all activities start by their latest start times, the project should not be delayed.

Determining the Critical Path

- Critical activities have zero slack and cannot be delayed without delaying the completion of the project.
- Noncritical activities have some positive amount of slack that represents the amount of time by which the start times of these activities can be delayed without delaying the completion of the entire project, assuming that all predecessor activities start at their earliest start times.

15.7 Project Management Using Spreadsheets

We can use a spreadsheet in a variety of ways to manage a project. As shown in Figure 15.11 (and in the file Fig15-11.xlsm that accompanies this book), we can use a spreadsheet to perform all the calculations required to determine the earliest and latest start and finish times for the project activities in our example problem.

To create the spreadsheet in Figure 15.11, we first entered all the labels, predecessor relations, and the activity times shown in columns A through D. The earliest finish times in column F are calculated as the earliest start time for each activity plus the time required to perform the activity.

Formula for cell F5: = E5 + D5

(Copy to cells F6 through F17.)

The latest start times in column G are calculated as the latest finish time of each activity minus the time required to perform the activity.

Formula for cell G5: = H5 - D5

(Copy to cells G6 through G17.)

Calculating the earliest start times in column E and the latest finish times in column H is a bit tricky but can be accomplished easily using array formulas. An **array formula**

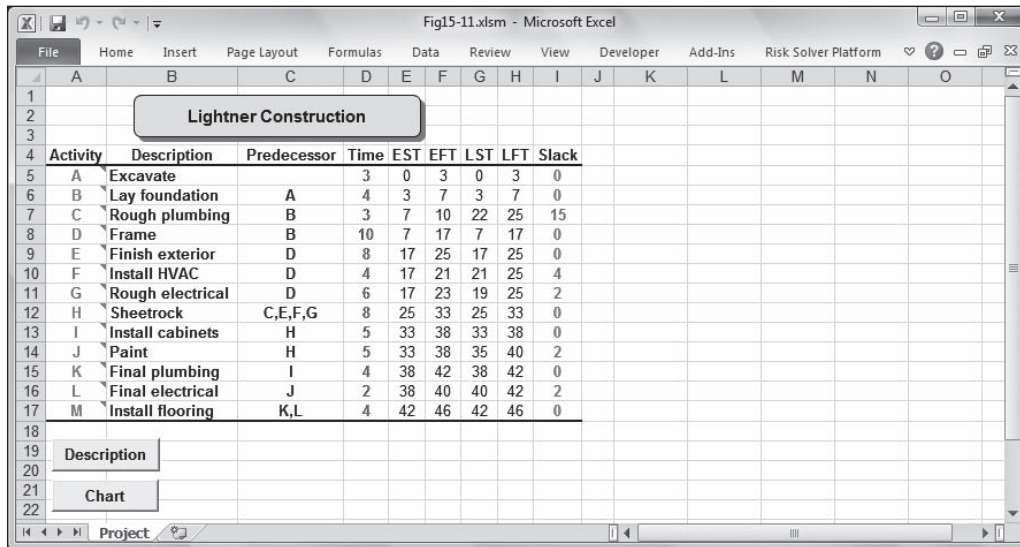


FIGURE 15.11

Spreadsheet calculations of start and finish times, slack, and critical activities

Key Cell Formulas		
Cell	Formula	Copied to
E5	=MAX(IF(ISERR(FIND(\$A\$5:\$A\$17,C5)),0,\$F\$5:\$F\$17)) (Press [Ctrl]+[Shift]+[Enter] to enter)	E6:E17
F5	=E5+D5	F6:F17
G5	=H5-D5	G6:G17
H5	=MIN(IF(ISERR(FIND(A5,\$C\$5:\$C\$17)),MAX(\$F\$5:\$F\$17),\$G\$5:\$G\$17)) (Press [Ctrl]+[Shift]+[Enter] to enter)	H6:H17
I5	=H5-F5	I6:I17

can perform multiple calculations using a range of cells and then return either a single result or multiple results. You create array formulas in the same way that you create other formulas, except that you press [Ctrl]+[Shift]+[Enter] to enter the formula.

Array Formulas

Array formulas typically perform operations on one or more ranges of cells. To help you understand array formulas a little better, consider the following examples that show how various array formulas can be used to perform the same operations as standard Excel functions.

Standard Excel Function

- =SUMPRODUCT(E5:E17,F5:F17)
- =SUMIF(E5:E17,">10",F5:F17)
- =COUNTIF(E5:E17,">0")

Equivalent Array Formula

- =SUM(E5:E17*F5:F17)
- =SUM(IF(E5:E17>10,F5:F17))
- =SUM(IF(E5:E17>0,1))

(Continued)

```
=SUMXMY2(E5:E17,F5:F17)
```

```
=VARP(E5:E17)
```

```
=SUM((E5:E17-F5:F17)^2)
```

```
=AVERAGE
```

```
((E5:E17-AVERAGE(E5:E17))^2)
```

As these examples illustrate, array formulas tend to be more obscure than their Excel function counterparts. However, when no standard Excel function exists to carry out a desired operation on a range of cells, array functions can be quite useful. For example, in Figure 15.11, suppose you want to find the maximum value in cells F5 through F17 where the corresponding value in cells E5 through E17 is less than 20. Your instincts may tell you to use the Excel MAXIF() function, but no such function currently exists! However, we can calculate the desired result using the array formula:

```
=MAX(IF(E5:E17<20,F5:F17))
```

Similarly, suppose you want to find the minimum value in cells F5 through F17 where the corresponding value in cells E5 through E17 is greater than 20. There is no MINIF() function in Excel, but we can calculate the desired result using the array formula:

```
=MIN(IF(E5:E17>20,F5:F17))
```

Remember that you must press [Ctrl]+[Shift]+[Enter] to enter array formulas. Also note that when you enter an array formula, Excel automatically inserts curly brackets (“{”}) around the formula. You should not attempt to type the curly brackets as part of your array formula.

The array formulas used here create circular references in the workbook. A **circular reference** occurs when the value in a cell depends on the value in another cell that, in turn, is dependent on the value in the original cell. Usually, a circular reference in a workbook indicates that a formula contains an error—and Excel displays a dialog box telling you so! However, there are occasions when a circular reference is exactly what is needed to accomplish a particular task. This is such an occasion. So, to tell Excel that we intend to use circular references, follow these steps:

1. Click File, Options, Formulas.
2. Select Enable Iterative Calculation.
3. Click OK.

Selecting the Iteration Option Automatically

The file Fig15-11.xlsm that accompanies this book contains a macro that automatically selects the Enable Iterative Calculation option. If you are familiar with macros in Excel and are curious, this macro may be found in the Workbook_Open event of the ThisWorkbook object in the VBA editor.

Now, each cell in column E must implement the logic of the forward pass described earlier (and summarized in Figure 15.7) to calculate the earliest start times. For each activity, this involves determining the maximum earliest finish time (EFT) for the activities that precede it. This is accomplished by the following formula.

Array Formula for cell E5: =MAX(IF(ISERR(FIND(\$A\$5:\$A\$17,C5)),0,\$F\$5:\$F\$17))

(Press [Ctrl]+[Shift]+[Enter] to enter; then copy to cells E6 through E17.)

Be sure to enter this (and any) array formula using [Ctrl]+[Shift]+[Enter] or it will not work properly. Also note that when you enter an array formula, Excel automatically inserts curly brackets (“{}”) around the formula.

For the earliest start time for activity A, this formula for cell E5 first uses the FIND() function to look for activities in the range from A5 through A17 that appear in cell C5 as predecessors of activity A. Because activity A has no predecessors, the FIND() function returns an error value, the ISERR() function returns the value True, and the IF() function returns the value zero as activity A’s EST value. When the formula is copied down the EST column, the relative reference to cell C5 changes accordingly to implement the correct forward pass logic for each of the remaining activities.

Note that when the FIND() function does find activities in the range from A5 through A17 that appear as predecessor activities in column C, this array formula returns the maximum of the corresponding EFT values from the range F5 through F17. For example, the array formula in cell E12 computes the EST for activity H as the maximum EFT of activities C, E, F, and G (that is, MAX(F7,F9,F10,F11) = 25). So for each activity, this array formula returns the maximum EFT for the activities that precede it, or the value zero if the activity has no predecessors.

Each cell in column H must implement the logic of the backward pass described earlier (and summarized in Figure 15.9) to calculate the latest finish times. For each activity, this involves determining the minimum latest start time (LST) for the activities that follow it. This is accomplished by the following formula.

Array Formula for cell H5:

=MIN(IF(ISERR(FIND(A5,\$C\$5:\$C\$17)),MAX(\$F\$5:\$F\$17),\$G\$5:\$G\$17))

(Press [Ctrl]+[Shift]+[Enter] to enter; then copy to cells H6 through H17.)

Again, be sure to enter this formula using [Ctrl]+[Shift]+[Enter] or it will not work properly.

For the latest finish time for activity A, this formula for cell H5 first uses the FIND() function to locate activities that are successors of activity A. That is, the FIND() function looks for activities in the range from C5 through C17 where activity A (cell A5) appears as a predecessor. Because activity A is (only) found as a predecessor of activity B, the ISERR() function returns the value False, and the IF() function returns activity B’s LST value of 3 (cell G6) as activity A’s LFT value. When the formula is copied down the LFT column, the relative reference to cell A5 changes accordingly to implement the correct backward pass logic for each of the remaining activities.

Note that any activity in column A that does not appear as a predecessor activity in the range C5 through C17 must be a terminal node for the project. In this case, the FIND() function will not find the value from column A in the range C5 through C17, the ISERR() function returns the value True, and the IF() function returns the maximum of all the EFT values from the range F5 through F17. For example, the array formula in cell H17 computes the LFT for activity M (which has no successors) as the maximum EFT of all activities (that is, MAX(F5:F17) = 46). So for each activity, this array formula returns the minimum LST for the activities that succeed it, or the maximum of all EFTs if the activity has no successors.

Finally, the slack values in column I are calculated as the difference between the latest and earliest start times for each activity as follows:

$$\text{Formula for cell I5:} \quad =\text{H5} - \text{F5}$$

(Copy to cells I6 through I17.)

The final results in Figure 15.11 are identical to those we calculated manually and summarized earlier in Figure 15.10. More importantly, if we need to change any of the predecessor relations in column C or the time estimates in column D, the spreadsheet in Figure 15.11 automatically adapts the project network and updates the results to reflect these changes. This spreadsheet also makes use of the conditional formatting capabilities of Excel to highlight critical and noncritical activities using red and blue font colors, respectively.

Conditional Formatting

Conditional formatting allows you to dynamically change the font, border, and pattern attributes of a range of cells depending on the value of the cell or a condition you specify. You can explore the Conditional Formatting option on the Home tab in Excel.

15.7.1 IMPORTANT IMPLEMENTATION ISSUE

The technique presented in Figure 15.11 relies on the (case-sensitive) `FIND()` function to identify immediate predecessor and successor activities when computing ESTs and LFTs for each activity. The function `FIND(x, y)` attempts to find the text string denoted by x within the text string denoted by y . As a result, it is critically important to use activity labels that are unique and do *not* appear as substrings within other activity labels.

For example, the 26 letters of the English alphabet may be used to uniquely identify up to 26 activities in a project. However, using the strings “A1” and “A11” as activity labels would not work in the technique presented in Figure 15.11 because the `FIND()` function would locate “A1” within “A11” (that is, `FIND(“A1”, “A11”)=1`), erroneously identifying activity A11 as a predecessor or successor of activity A1. Fortunately, use of the strings “A01” and “A11” as activity labels easily remedies this situation.

Similarly, if you want to use numbers rather than letters to identify activities, using the numbers 1, 2, 3, . . . , 9 as activity labels would create matching problems within activity labels 11, 12, 13, . . . , 19 (among others). However, this can be avoided easily by applying Excel’s Text format to cells containing activity labels and immediate predecessors (that is, columns A and C in Figure 15.11) and using two-digit numbers for all activity labels (for example, 01, 02, 03, . . . , 09, 10, 11, 12, 13, . . . , 99). If more than 100 numeric activity labels are needed, three-digit numbers (formatted as text) should be used.

15.8 Gantt Charts

A **Gantt chart** is a popular technique for portraying the schedule of a project’s activities over time. Figure 15.12 shows a Gantt chart that displays the earliest start and finish times and the slack for each activity in our example problem. (Click the Chart button in

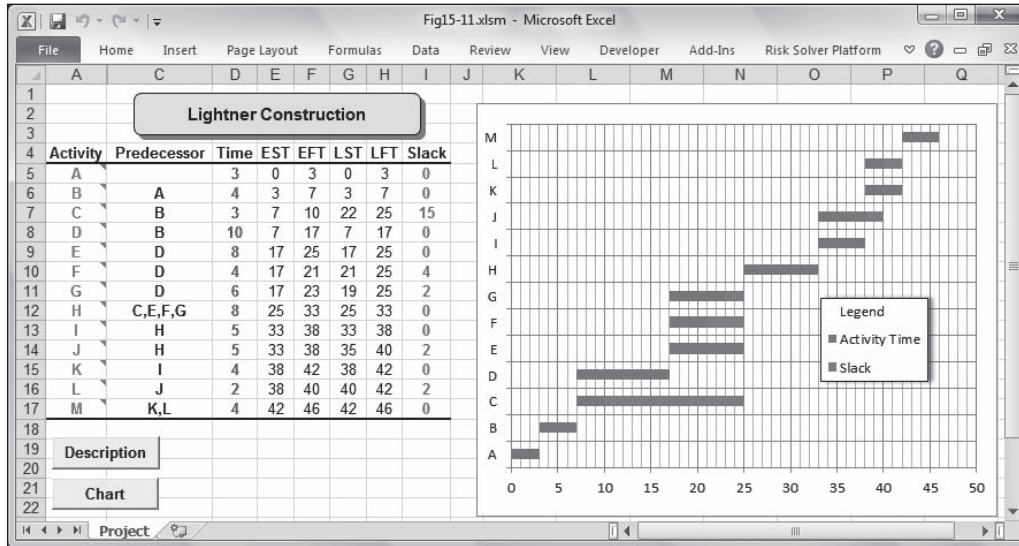


FIGURE 15.12

Gantt chart for the example problem

the worksheet to display this graph.) This type of chart is useful in helping a project manager see when activities can begin and end, and keep track of which activities are underway at any point in time.

Creating Gantt Charts in Excel

The Gantt chart in Figure 15.12 is a stacked horizontal bar chart that plots each activity's EST, activity time, and slack. However, the bars for the EST for each activity are formatted so that they do not appear on the graph—giving the illusion that only the activity times and slack are being graphed. To create a Gantt chart like the one shown in Figure 15.12, follow these steps:

1. Click cell A5. While pressing the left mouse button on cell A5, press and hold down the Ctrl key, and select the ranges A5:A17, D5:E17, and I5:I17.
2. Click Insert, Bar.
3. Select the 2-D Stacked Bar chart. Excel then displays an initial stacked bar chart on the screen.
4. Right-click the graph, and choose Select Data.
5. Choose Series2, and click the up arrow icon.
6. Edit Series2, and change its name to Legend.
7. Edit Series1, and change its name to Activity Time.
8. Edit Series3, and change its name to Slack.
9. Click OK.
10. Right-click any of the bars labeled Legend (representing the EST of the activities), and select Format Data Series.
11. Click Fill, and select the No Fill option.
12. Click Border Color, and select the No Line option.
13. Click Close.

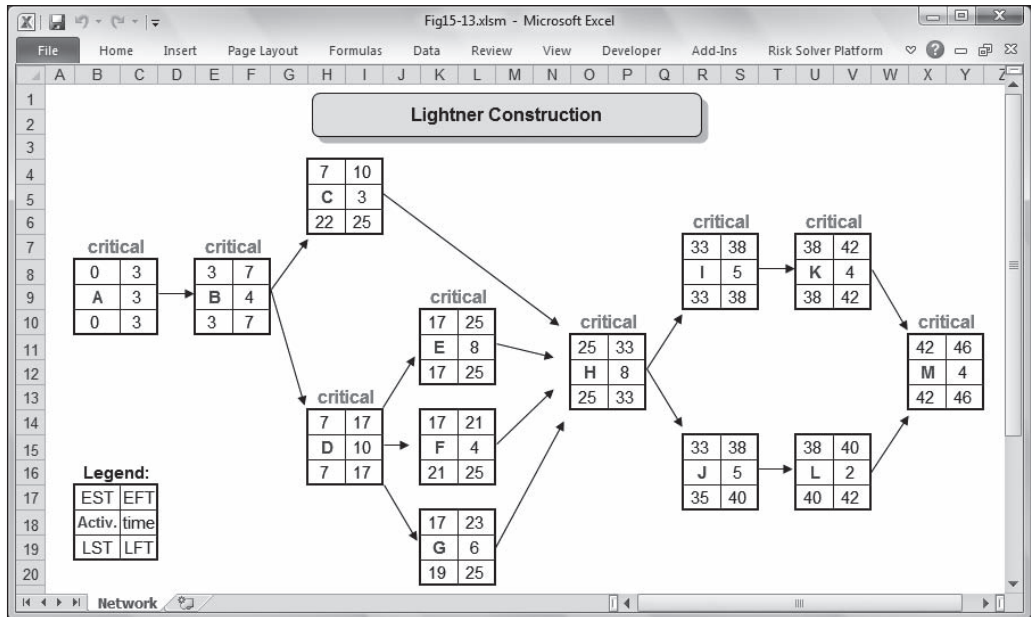
The basic Gantt chart is now complete. You can continue to customize the chart by right-clicking any element and specifying options in the resulting dialog boxes.

A weakness of the Gantt chart shown in Figure 15.12 is that it does not show the precedence relationships between the activities in a project. For example, the Gantt chart shows that the painting and cabinetry activities should both end at time period 38, and that the final electrical and final plumbing activities should begin at that point. But what happens if the painting activity is delayed? Would this prevent both the final electrical and final plumbing activities from starting on time? To answer this question, we need to know if painting is a predecessor for both the final plumbing and final electrical activities. The Gantt chart does not provide this information. Thus, although the Gantt chart is a valuable monitoring tool, it does not tell the whole story about the precedence relationships among project activities.

As shown in Figure 15.13 (and the file Fig15-13.xlsm that accompanies this book), we can display the network structure of a project in a spreadsheet. Here, we used the relationships in Figures 15.7 and 15.9 to calculate the earliest and latest start and finish times for each activity. Then, we used the Excel drawing tool to draw arrows indicating the precedence relationships between activities. This approach is feasible for small projects but become cumbersome for larger projects.

FIGURE 15.13

Project network implemented in a spreadsheet



15.9 Project Crashing

With the CPM technique, we can use the times required to perform each activity to determine the least amount of time required to complete the entire project. However, the time required to complete an activity can often be shortened, or crashed. For example, suppose that the amount of time it takes to paint a house is *normally* five days. But, by hiring more painters or asking the existing painters to work overtime, it might be possible to paint the same house in two days. Thus, activities can have normal times and crash times.

The **normal time** of an activity represents the ordinary amount of time required to complete the activity with the usual amount of resources and effort. The **crash time** of

an activity represents the least amount of time in which an activity can be accomplished if extra effort and resources are applied to the activity. The extra effort and resources required to crash certain activities usually increase the cost of performing these activities.

In managing projects, we often need to evaluate the trade-offs between the cost of crashing certain activities and the benefit of accomplishing these activities in less than normal time. For example, in our problem, we determined that it takes 46 working days to complete the construction of the house. But what if the buyer wants the house built in 35 days? We need some way to determine if this can be done, and, if so, how much additional cost will be involved. Similarly, it is not unusual for some critical activities in a project to take more than their normal time. After all, projects are not immune to Murphy's Law, which states that if something can go wrong, it will go wrong. When a critical activity is delayed, the completion of the project will be delayed unless management chooses to crash some of the remaining critical activities.

Cashing in on a Project Crash

In early 1973, TRW (a large government defense contractor) was in trouble. TRW held a contract to develop software for several DEC PDP-10 computers to analyze the power needs of the Northwest region and control the amount of electricity generated by hydroelectric dams on the Columbia River. But the PDP-10 software was infested with bugs, and the project soon fell behind schedule. If TRW could not get the bugs out quickly, it would have to pay substantial contract penalties, so it was time to crash the project. An urgent request from TRW's headquarters to DEC for bug-hunting experts led to a surprising suggestion—a high school senior named Bill Gates at Seattle's exclusive Lakeside prep school. Bill and his friend, Paul Allen, had debugged DEC software for Seattle's Computer Center Corporation in exchange for free time to "play" with the computers. But TRW offered to compensate Gates and Allen with real money—\$165 per week. Gates and Allen took the job. Two years later, Bill Gates and Paul Allen teamed up again on a slightly more profitable software venture—the founding of the Microsoft Corporation.

Source: James Wallace and Jim Erickson, *HARDDRIVE*, HarperBusiness, 1992.

15.9.1 AN LP APPROACH TO CRASHING

We can use Solver to help determine the least costly way of crashing a project to meet certain deadlines. Although it is tempting to apply Solver directly to the spreadsheet model shown in Figure 15.11, that model contains numerous nonlinearities that would force us to use Solver's evolutionary algorithm. Although this can work, the solution process is slow, and the results may be local rather than global optimal solutions. As it turns out, we can also solve project-crashing problems using an LP formulation that is easy for Solver to solve and guarantees we obtain global optimal solutions.

We will illustrate the LP approach to crashing here. To do this, we first must determine the normal time, normal cost, crash time, and crash cost for each project activity. Suppose that Tom asked his subcontractors to estimate crash times and costs for activities involved in our example problem. This information is summarized in Figure 15.14 (and in the file Fig15-14.xlsm that accompanies this book), along with the normal times and normal costs that Tom originally estimated for each activity.

FIGURE 15.14

Summary of normal and crash times and costs for the home-building project

The screenshot shows an Excel spreadsheet titled 'Fig15-14.xlsm - Microsoft Excel'. The main data table is titled 'Project Crashing Data' and is located in the range G5:M17. The table has columns for Activity, Normal Time, Normal Cost, Crash Time, Crash Cost, Allowable Crash Days, and Crash Cost Per Day. Below the main table is a section titled 'Key Cell Formulas' with columns for Cell, Formula, and Copied to.

Project Crashing Data						
Activity	Normal		Crash		Allowable Crash Days	Crash Cost Per Day
	Time	Cost	Time	Cost		
A	3	\$5,000	2	\$6,000	1	\$1,000
B	4	\$12,000	3	\$15,000	1	\$3,000
C	3	\$3,000	2	\$3,500	1	\$500
D	10	\$20,000	6	\$25,000	4	\$1,250
E	8	\$8,000	5	\$10,000	3	\$667
F	4	\$11,000	3	\$12,000	1	\$1,000
G	6	\$3,500	4	\$4,500	2	\$500
H	8	\$5,000	5	\$6,500	3	\$500
I	5	\$8,000	3	\$9,500	2	\$750
J	5	\$4,000	2	\$5,500	3	\$500
K	4	\$7,000	2	\$8,500	2	\$750
L	2	\$2,000	1	\$2,500	1	\$500
M	4	\$10,000	2	\$12,000	2	\$1,000

Key Cell Formulas		
Cell	Formula	Copied to
G5	=C5-E5	G6:G17
H5	=(F5-D5)/G5	H6:H17

As shown in Figure 15.14, in his original plan, Tom estimated that the framing activity (activity D) would take 10 days and cost \$20,000. However, the subcontractor indicated that if this activity were crashed by the maximum amount, it could be completed in only 6 days at a cost of \$25,000. For planning purposes, Tom might assume that the time required to complete the framing activity can be reduced from its normal time of 10 days by no more than 4 days ($10 - 6 = 4$). This value is indicated in Figure 15.14 in the Allowable Crash Days column. Tom might also assume that each day this activity is crashed will increase the cost of the activity by approximately \$1,250 ($(\$25,000 - \$20,000)/4 = \$1,250$). This value is shown for the framing activity in the Crash Cost Per Day column. The allowable number of crash days and crash costs per day for the remaining activities are determined in a similar way and are shown in Figure 15.14.

15.9.2 DETERMINING THE EARLIEST CRASH COMPLETION TIME

Tom might want to know the earliest possible date that the project could be completed and by what amount each activity would need to be crashed in order to achieve this date. An LP formulation of this problem is represented as:

$$\begin{array}{ll}
 \text{MIN:} & T_M + t_m - C_M \\
 \text{Subject to:} & T_B - T_A \geq t_A - C_A \\
 & T_C - T_B \geq t_B - C_B \\
 & T_D - T_B \geq t_B - C_B \\
 & T_E - T_D \geq t_D - C_D \\
 & T_F - T_D \geq t_D - C_D
 \end{array}$$

$$\begin{aligned}
T_G - T_D &\geq t_D - C_D \\
T_H - T_C &\geq t_C - C_C \\
T_H - T_E &\geq t_E - C_E \\
T_H - T_F &\geq t_F - C_F \\
T_H - T_G &\geq t_G - C_G \\
T_I - T_H &\geq t_H - C_H \\
T_J - T_H &\geq t_H - C_H \\
T_K - T_I &\geq t_I - C_I \\
T_L - T_J &\geq t_J - C_J \\
T_M - T_K &\geq t_K - C_K \\
T_M - T_L &\geq t_L - C_L \\
C_i &\leq \text{allowable crash days for activity } i \\
T_i, C_i &\geq 0, \text{ for all } i
\end{aligned}$$

where

T_i = the time at which activity i begins

t_i = the normal activity time of activity i

C_i = amount of time by which activity i is crashed

Each of the constraints in this LP model corresponds to one of the precedence relations in the project network. For example, the first time constraint ($T_B - T_A \geq t_A - C_A$) indicates that the difference between the start times of activities A and B must be greater than or equal to the normal time for activity A (t_A) minus the amount by which activity A is crashed (C_A). Equivalently, this constraint may be stated as $T_B \geq T_A + t_A - C_A$. That is, the start time for activity B must be greater than or equal to the start time for activity A plus the normal time for activity A minus the amount by which activity A is crashed.

The logic in the rest of the time constraints can be interpreted in a similar way. Also note that this model places appropriate upper limits on the amount of time by which each activity can be crashed. Finally, the objective function in this problem attempts to minimize the final completion time of the project (which is equal to the start time for activity M, plus its normal activity time, minus the amount by which this activity is crashed). Thus, the solution to this model should indicate the earliest possible time at which the project can be completed if activities are crashed.

Using LP to Identify Earliest and Latest Start Times

With minor modifications, the LP model used here for crashing projects can also be used to determine earliest start times and latest start times for all the activities in the project. To do this, first solve for the earliest start times by changing the objective to minimize the sum of all the activity start times (that is, MIN: $\sum T_i$), and do not allow crashing (that is, set all $C_i = 0$). Next, solve for the latest start times by changing the objective to maximize the sum of all the activity start times (that is, MAX: $\sum T_i$), force T_M to equal its earliest start time identified in the previous step (that is, set $T_M = 42$), and do not allow crashing (that is, set all $C_i = 0$).

15.9.3 IMPLEMENTING THE MODEL

One approach to implementing this model in a spreadsheet is shown on the Crash Model worksheet in Figure 15.15. In this spreadsheet, the values in column D represent the start time of each activity (the T_i), and the values in column E correspond to the amount by which each activity is crashed (the C_i).

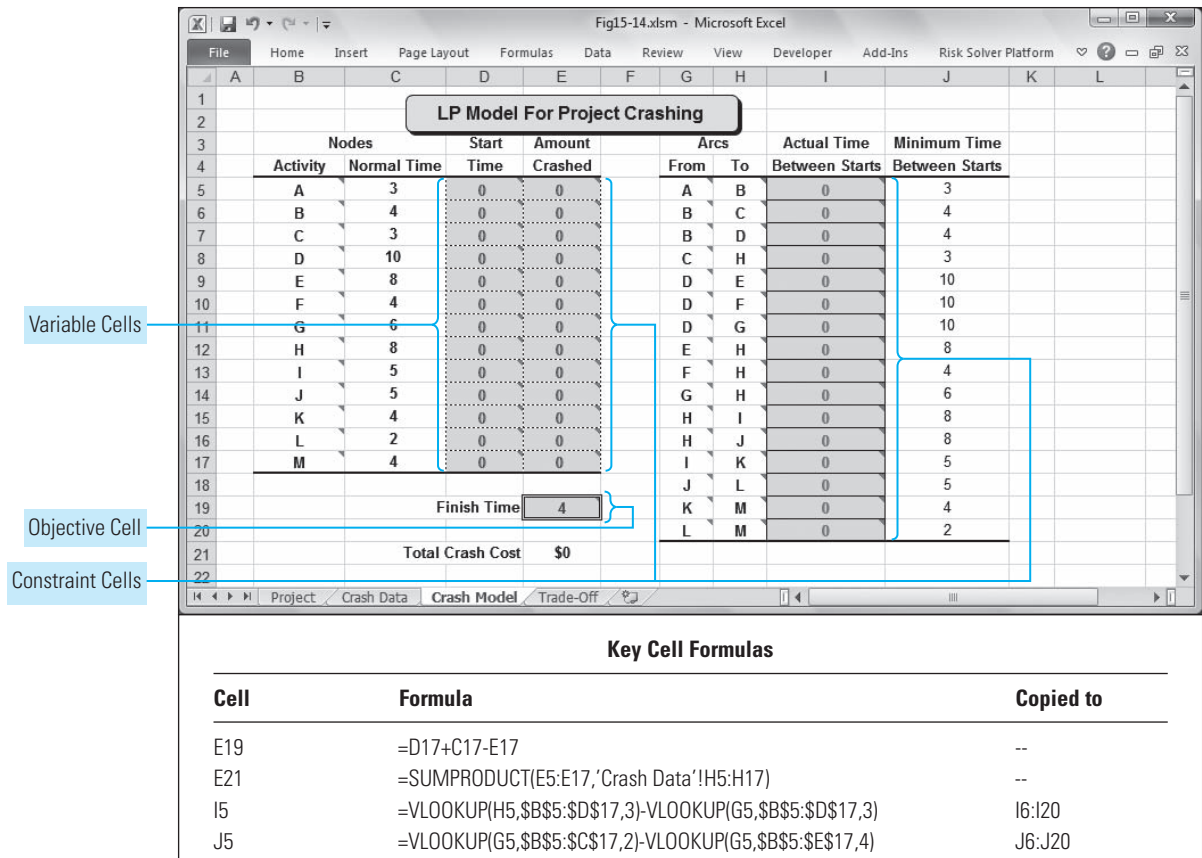
The LHS formulas for the time constraints are implemented in column I in the same manner as described in the previous section. Specifically, cell I5 contains the following formula, which is copied to cells I6 through I20:

$$\text{Formula for cell I5: } =\text{VLOOKUP}(H5,\$B\$5:\$D\$17,3)-\text{VLOOKUP}(G5,\$B\$5:\$D\$17,3)$$

(Copy to I6 though I20.)

The first VLOOKUP() function in this formula corresponds to the value of T_B . The first VLOOKUP() function takes the contents of cell H5 (the letter B) and looks for its match in the first column of the range $\$B\$5:\$D\17 (the match is in cell B6), and then returns the value in the third column of the range—or the value in cell D6. Cell D6 corresponds to T_B . The second VLOOKUP() function works in a similar way to return the appropriate value for T_A . The difference between the values of these two VLOOKUP() functions represents the amount of time between the start of activities A and B.

FIGURE 15.15 Spreadsheet implementation of the crashing model to determine the earliest possible finish time



The RHS values of the time constraints are implemented in column J as:

$$\begin{array}{ll} \text{Formula for cell J5:} & =\text{VLOOKUP}(G5,\$B\$5:\$C\$17,2)- \\ \text{(Copy to J6 through J20.)} & \text{VLOOKUP}(G5,\$B\$5:\$E\$17,4) \end{array}$$

This formula first finds the cell in column C corresponding to the normal time for activity A, and then subtracts the cell in column E corresponding to the amount by which activity A will be crashed.

The objective function for this problem is implemented in cell E19 with the following formula:

$$\text{Formula for cell E19:} \quad =D17+C17-E17$$

This value represents the time at which the project will be completed. Finally, for informational purposes (and future reference), in cell E21, we implement the following formula to calculate the total cost associated with crashing the project:

$$\text{Formula for cell E21:} \quad =\text{SUMPRODUCT}(E5:E17,'Crash Data'!H5:H17)$$

This formula calculates the sum of products between the crash cost per day for each activity (calculated in cells H5 through H17 on the Crash Data sheet shown in Figure 15.14) and the amount by which each activity is crashed (represented by cells E5 through E17 on the Crash Model sheet shown in Figure 15.15).

15.9.4 SOLVING THE MODEL

Figure 15.16 shows the Solver parameters and options used to solve this model. In defining the constraints for this problem, the values of cells E5 through E17 (the C_i) are required to be less than or equal to their maximum allowable values calculated earlier on the Crash Data sheet shown in Figure 15.14.

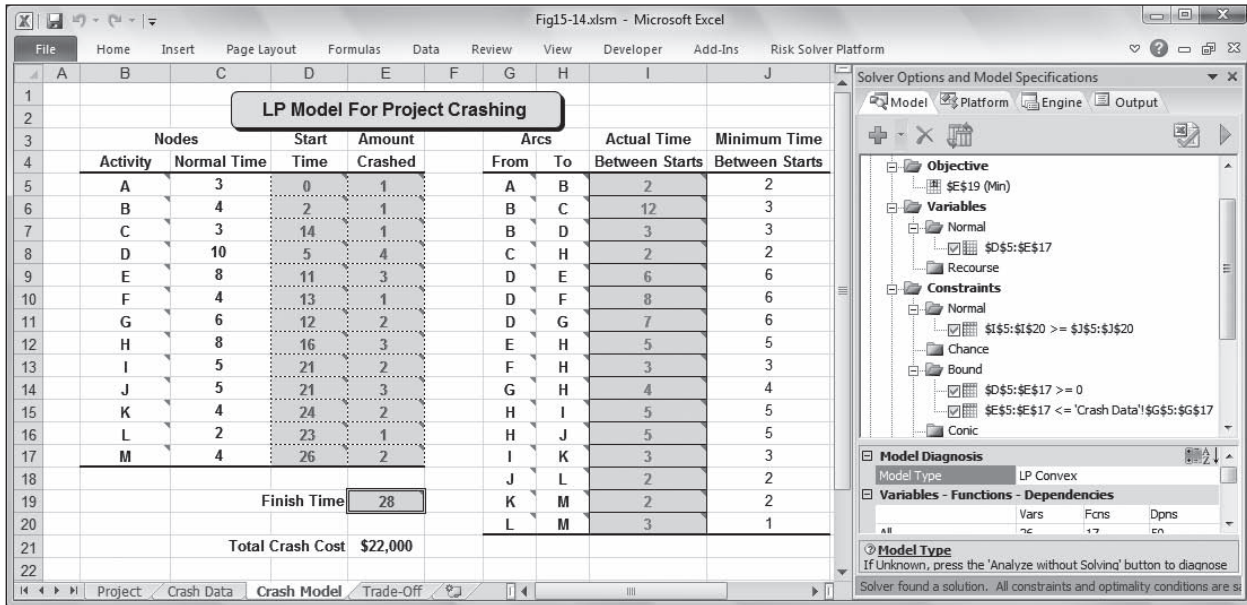
Solver Settings:
Objective: E19 (Min)
Variable cells: D5:E17
Constraints:
I5:I20 >= J5:J20
E5:E17 <= 'Crash Data'!G5:G17
D5:E17 >= 0
Solver Options:
Standard LP/Quadratic Engine (Simplex LP)

FIGURE 15.16

Solver settings and options for the earliest possible finish times

The optimal solution for this problem, shown in Figure 15.17, indicates that by crashing various activities, the project could be completed within 28 days rather than 46. However, the Total Crash Cost figure indicates this would raise the cost of the project by \$22,000. (Note that, in general, the total crash cost obtained when solving this type of model is not necessarily the minimum crash cost because the model minimizes completion time, not crash costs. Because alternate optimal solutions can occur, another crash schedule might also allow the project to be completed within 28 days but at a lower total crash cost. Thus, to determine if the minimum total crash cost is \$22,000, we need to add a constraint to hold the finish time at 28 days and attempt to minimize the total crash cost. In this case, it turns out that another solution (not shown) allows us to finish the project in 28 days at a minimum total crash cost of \$19,000.)

FIGURE 15.17 Optimal solution for the earliest possible finish times



15.9.5 DETERMINING A LEAST COSTLY CRASH SCHEDULE

Another use of a crashing model is to determine the least costly way to complete a project at some time earlier than would be possible using normal activity times. For example, if the home buyer insists on the house being built within 35 days, we might want to determine the least costly way of crashing the schedule to meet this deadline.

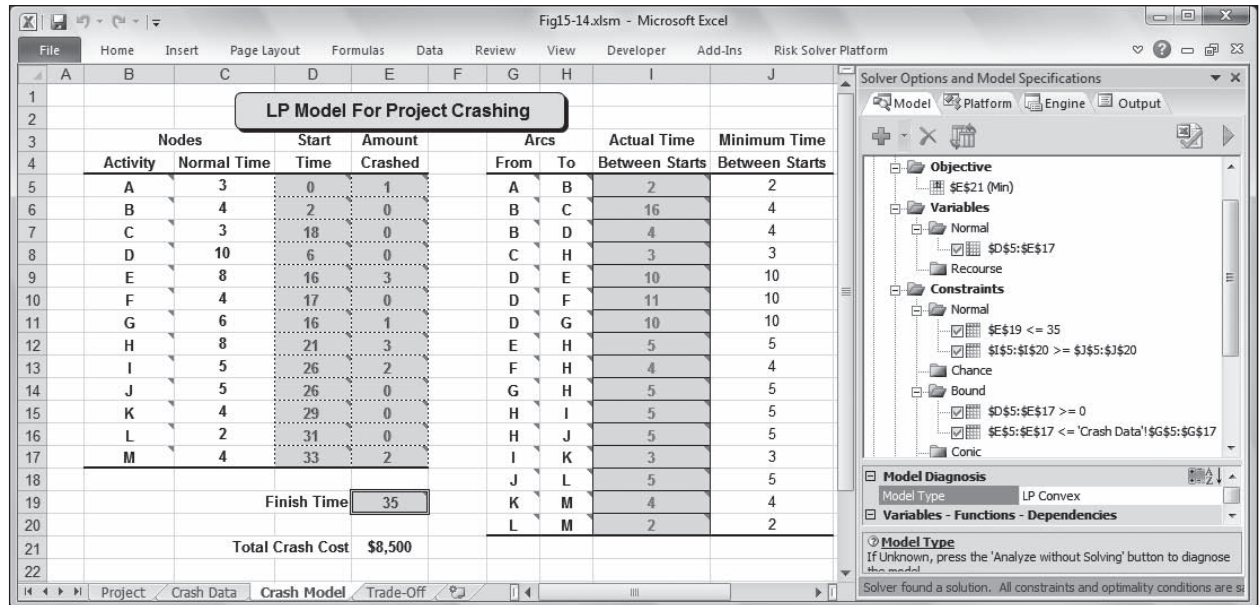
We can solve this type of problem easily using the existing model. In this case, we would simply add a constraint to hold the project completion time at 35 days and attempt to minimize the total crash cost. The Solver parameters and options required to accomplish this are shown in Figure 15.18, and the optimal solution is shown in Figure 15.19.

FIGURE 15.18

Solver settings and options for the least cost crash schedule to complete the project by time period 35

<p>Solver Settings:</p> <p>Objective: E21 (Min) Variable cells: D5:E17 Constraints: E19 <= 35 I5:I20 >= J5:J20 E5:E17 <= 'Crash Data'!G5:G17 D5:E17 >= 0</p> <p>Solver Options: Standard LP/Quadratic Engine (Simplex LP)</p>
--

FIGURE 15.19 Optimal solution for the least cost crash schedule to complete the project by time period 35



The solution shown in Figure 15.19 indicates that it would cost an additional \$8,500 to crash the project so that it could be completed within 35 days. The values in the Amount Crashed column indicate the amounts by which each activity must be crashed in order to implement this schedule.

15.9.6 CRASHING AS AN MOLP

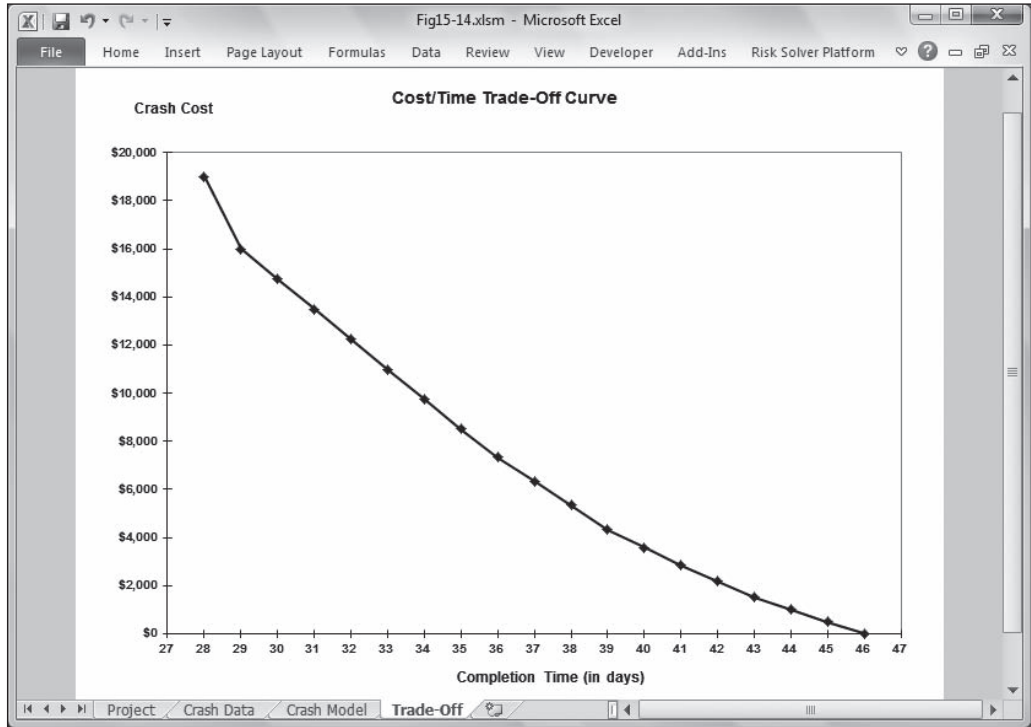
As we have seen, two objectives can be pursued in our crashing model. On the one hand, we might want to minimize the finish time of the project. On the other hand, we might want to minimize the cost of crashing the project. These two objectives are in conflict with one another because reducing the completion time of the project increases the crash costs. Thus, we could use the techniques for MOLPs described in Chapter 7 if we want to study the cost/time trade-offs more formally in this type of problem.

Another way to study the cost/time trade-offs is to re-solve the problem several times to determine the minimum crash cost for each possible completion time. (Risk Solver Platform's multiple parameterized optimization capability could be used for this purpose.) A graph showing the minimum crash cost for each possible completion time is useful in evaluating the trade-offs involved in crashing. Figure 15.20 shows such a graph for our example problem.

According to this graph, no crash costs will be incurred if the project is completed in its normal time of 46 days, but crash costs of \$19,000 will be incurred if we complete the project in the least amount of time (28 days). Although it is relatively inexpensive to crash the project by a few days, the graph shows that it becomes increasingly expensive to crash the project by additional amounts. Note that a rather substantial increase in cost is incurred to complete the project in 28 days rather than 29. So, this type of graph gives us a clear picture of the cost/time trade-offs involved in crashing a project.

FIGURE 15.20

Graph of the relationship between crash time and completion time



15.10 PERT: An Overview

Throughout our discussion of CPM, we assumed that the times required to complete project activities were known with certainty or could, at least, be estimated with a high degree of accuracy. This assumption does not hold for all projects. For example, consider the schedule of activities required to design and build the first space shuttle. Because no one had ever built a space shuttle before, no historical data were available for estimating how long it would take to perform many of the creative tasks required by this project. Even in projects in which the same or similar tasks have been performed before, the amount of time required to perform them might be different. PERT was developed as an attempt to deal with uncertainty in activity times.

In recent years, a number of problems have surfaced regarding the PERT technique, causing many to question the wisdom of using this technique at all. The following discussion of PERT focuses on illuminating these weaknesses. We will then briefly consider how to overcome some of these problems using the technique of simulation.

PERT differs from CPM in that it assumes the times required to perform the project activities are *not* known with certainty. PERT assumes that the activity times are random variables that have some mean, or expected, value and some variance. However, rather than specifying a mean and variance for each activity, we must give three time estimates for each activity in the project. Specifically, for each activity, PERT requires estimates of:

a_i = estimate of the duration of activity i assuming the most favorable conditions

b_i = estimate of the duration of activity i assuming the least favorable conditions

m_i = estimate of the most likely duration of activity i

We can think of a_i , b_i , and m_i as representing, respectively, the best case, worst case, and most likely times required to perform activity i . PERT uses these values to calculate the expected duration of each activity as:

$$t_i = \frac{a_i + 4m_i + b_i}{6}$$

and estimates the variance of each activity's duration as:

$$v_i = \frac{(b_i - a_i)^2}{36}$$

The preceding formulas are based on the assumption that the times required to perform each activity are random variables that follow the beta probability distribution. An in-depth understanding of the beta distribution is not necessary for our purposes. The beta distribution can be used to describe a wide variety of random variables and is a reasonable choice for describing the behavior of most activity times when their true distributions are unknown.

After calculating the expected times for each activity, the next step in PERT is to identify the critical path for the project. In most project networks, a variety of paths can be followed to get from the start activity to the finish activity. Each path represents a sequence of activities that must be performed in order to complete the project. Because PERT assumes that the times required to perform the activities along a path are random variables, the time required to complete a given path is also a random variable with some mean and variance.

The expected (or mean) time required to complete any path in the network is simply the sum of the expected times (the t_i) of the activities on the path. If we assume that the individual activity times in a project are independent of one another (as does PERT), we may also calculate the variance of the completion time for any path as the sum of the variances (the v_i) of the activities on the path. Because all the paths through a project network must be completed in order for the project to be completed, PERT deems the path with the largest expected completion time to be the most critical—the critical path.

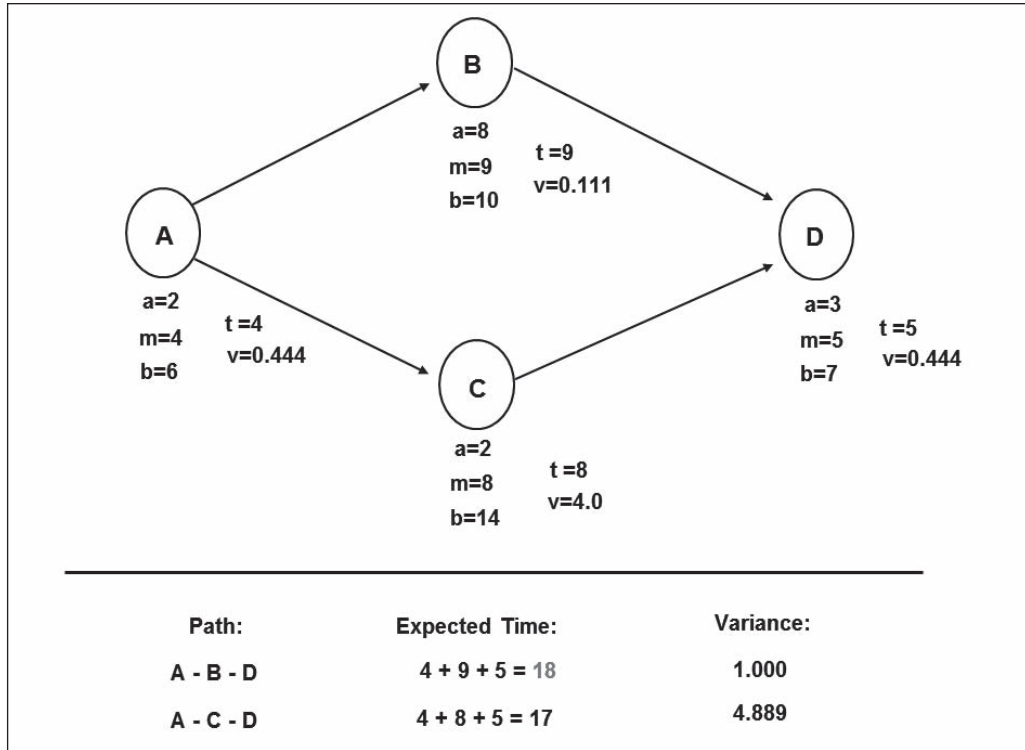
The expected completion time and variance for the critical path are used in PERT to estimate the probability of completing the project by various dates and to assist in negotiating and setting deadlines for the project's completion. Many projects impose financial penalties for each day, week, or month a project is late; therefore, it is often important to identify a deadline for a project that management is confident will not be exceeded. Thus, in addition to identifying a critical path for management to scrutinize, PERT also claims to offer assistance in estimating and setting deadlines for a project.

15.10.1 THE PROBLEMS WITH PERT

The PERT technique presents a number of problems that should cause us to approach it with skepticism. First, as indicated previously, PERT assumes that the time required to perform project activities are random variables that are independent of one another. Although this assumption makes it easy to complete the calculations in PERT, it is probably not a realistic assumption. For example, if one activity along the critical path (or any other path) runs a bit over its expected time, a diligent project manager will make sure that one or more of the subsequent activities run a bit under their expected times to catch up on the schedule. To the extent that these over- and under-runs offset one another, the expected completion time of the project should not be affected seriously. However, the variance of the completion times of the paths through the network will be reduced as a result.

FIGURE 15.21

Example of a PERT network



The more serious problem in PERT involves the identification of the critical path. As described earlier, PERT identifies the critical path as the path with the longest expected completion time. Thus, an unsuspecting project manager might focus on the activities on the critical path, believing that they are the activities most likely to delay the completion of the project. In reality, activities not on the critical path can pose a greater risk of delaying the project. Consider the activity network shown in Figure 15.21.

The network in Figure 15.21 represents a simple project consisting of only four activities. The three time estimates (a , b , and m) required by PERT are given for each activity. The expected time and variance (t and v) are calculated for each activity. Only two paths run through this network: A to B to D, and A to C to D. The expected times and variances for each path are calculated as shown in Figure 15.21. The path consisting of A-B-D has an expected time of 18 and variance of 1.0. The path consisting of A-C-D has an expected time of 17 and variance of 4.889. Thus, PERT would identify A-B-D as the critical path and use its expected value and variance to make estimates about when the project will be completed.

Earlier, we indicated that the time to complete any path in a PERT network is a random variable. We can view the time to complete path A-B-D and the time to complete path A-C-D as two separate random variables whose probability distributions are similar to those shown in Figure 15.22. Notice that the distribution of the critical A-B-D path is centered at its expected value of 18 and has a relatively small variance. The distribution of the noncritical A-C-D path is centered at its expected value of 17 and has a much larger variance.

If we focus only on the distribution of the critical path shown in Figure 15.22, it appears that almost no chance exists that completing this path will take more than 21 time periods. Thus, if we apply the PERT technique to this project, we would conclude

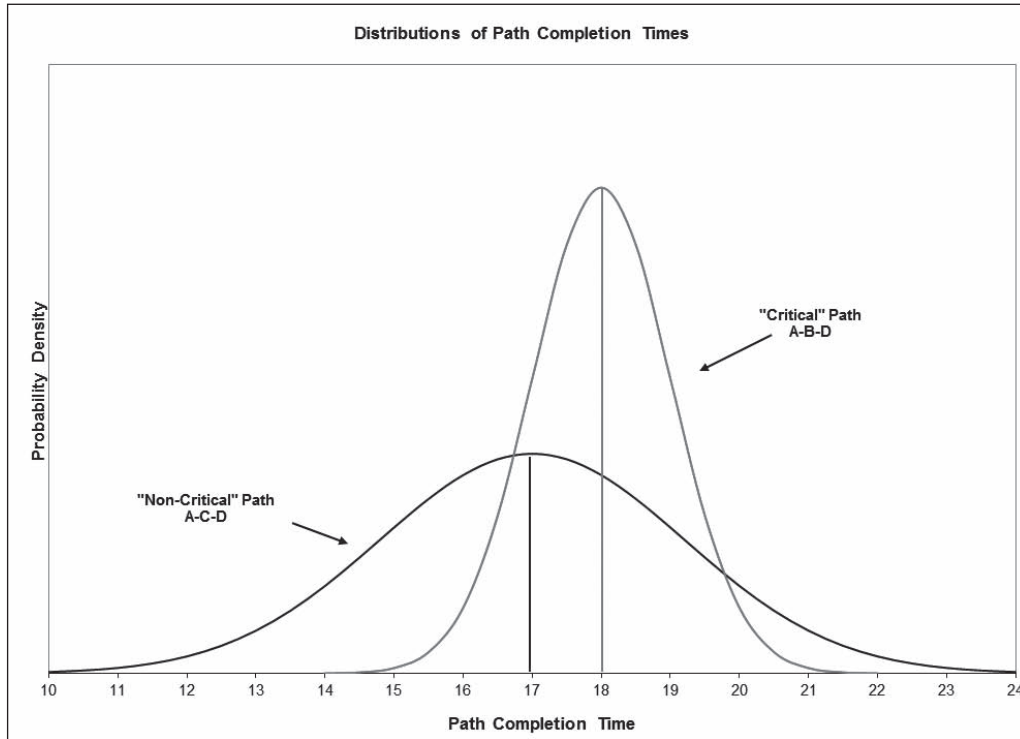


FIGURE 15.22

Graph of the distributions of completion times for each path through the PERT network

that the project can almost certainly be completed within 21 time units. However, the distribution of times for path A-C-D indicates a fairly significant probability that this noncritical path will take more than 21 days. So, which path is really the critical path—the one with the longest expected completion time or the one with the smallest probability of completion by the desired deadline? Furthermore, which activity in this project is really more critical—B or C? The fact that PERT causes us to ask such questions, and fails to answer them, should raise serious concerns about the wisdom of using this technique.

15.10.2 IMPLICATIONS

The preceding discussion makes it clear that activities not on PERT’s critical path might, in fact, be more critical to the completion of a project than the activities PERT identifies as critical. Indeed, *every* activity in a project has some probability of being critical. For some activities, this probability might be near 0; for others, it might be closer to 1. A project manager should focus on the activities with the highest probability of being critical—regardless of whether they fall on the critical path. Unfortunately, PERT does little to help us identify the project activities that are truly the most critical.

15.11 Simulating Project Networks

Although the PERT technique has serious drawbacks, it does highlight an important point—activity times in a project are likely to be somewhat random. Although CPM ignores this point entirely, by calculating the earliest and latest start times for each activity,

it at least shows which activities are likely to be critical if the actual activity times deviate from their expected values.

The best way to evaluate the impact of variability in activity times on the critical path and the completion time of a project involves the technique of simulation described in Chapter 12. To simulate a project, we need to generate random times from appropriate probability distributions for each project activity. For the given set of random activity times, we can determine the critical path and the time required to complete the project. If we repeat this process many times, we can determine the frequency with which the project activities fall on the critical path. We could also analyze the resulting distribution project completion times to estimate more accurately the probability of completing the project within various time periods.

15.11.1 AN EXAMPLE

The following example, which is a modification of our earlier home-building example, illustrates the mechanics of simulating the duration of a project.

Tom has asked the subcontractors for each activity in the home-building process to supply a best-case, most-likely-case, and worst-case estimate of the amount of time required to complete each activity. Figure 15.23 shows these time estimates.

Tom wants to use these time estimates to conduct a simulation study of the project. In particular, he wants to determine how critical each activity is—that is, the probability of each activity falling on the critical path. He also wants to determine the number of days it will take to complete the entire project.

15.11.2 GENERATING RANDOM ACTIVITY TIMES

To simulate the duration of this project, we first must identify probability distributions that describe the behavior of the random activity times. Because the three time estimates for each activity correspond to the best-case, most-likely-case, and worst-case scenarios for each activity, we might assume that the random behavior of the

FIGURE 15.23

Summary of time estimates for activities in the home-building project

Activity	Time Estimates		
	<i>a</i> Best Case	<i>m</i> Most Likely Case	<i>b</i> Worst Case
A	2	3	5
B	3	4	6
C	2	3	7
D	8	10	14
E	6	8	10
F	3	4	5
G	4	6	8
H	7	8	10
I	4	5	7
J	3	5	9
K	3	4	6
L	1	2	4
M	3	4	6

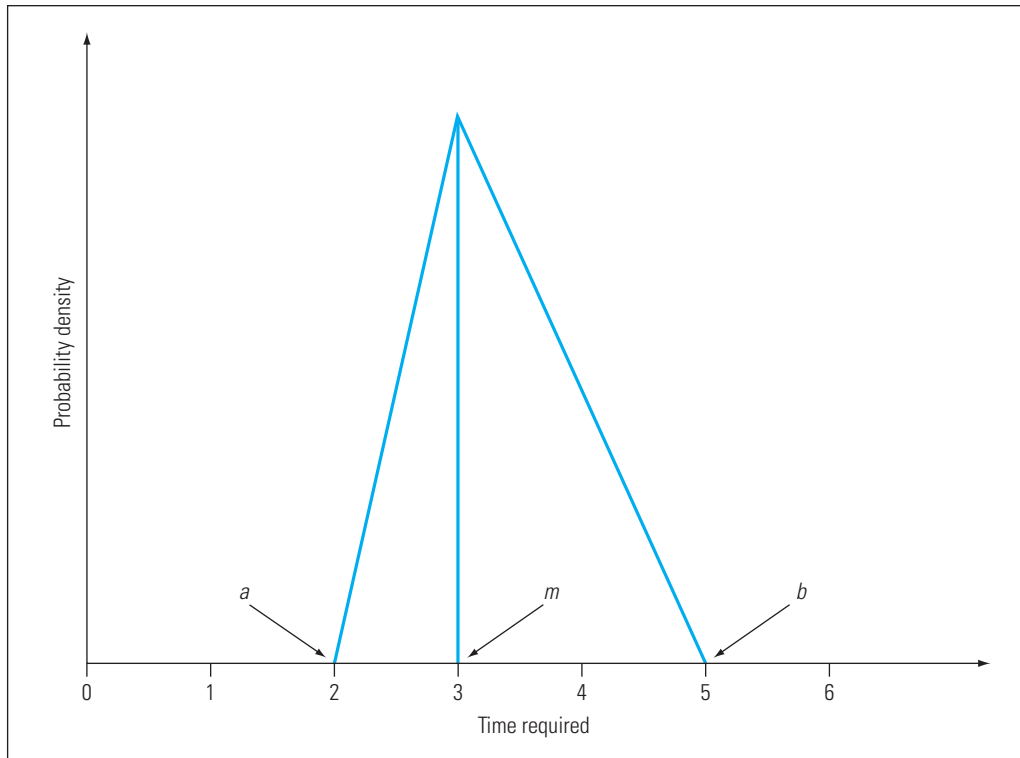


FIGURE 15.24

Example of a triangular distribution of activity times

activity times can be approximated by a triangular probability distribution such as the one shown in Figure 15.24 for activity A. Notice that the shape of the triangular distribution varies depending on the parameters a , m , and b , which correspond (respectively) to the best-case, most-likely-case, and worst-case time estimates for a given activity.

15.11.3 IMPLEMENTING THE MODEL

A spreadsheet model for this problem is shown in Figure 15.25 (and in the file Fig15-25.xlsm that accompanies this book).

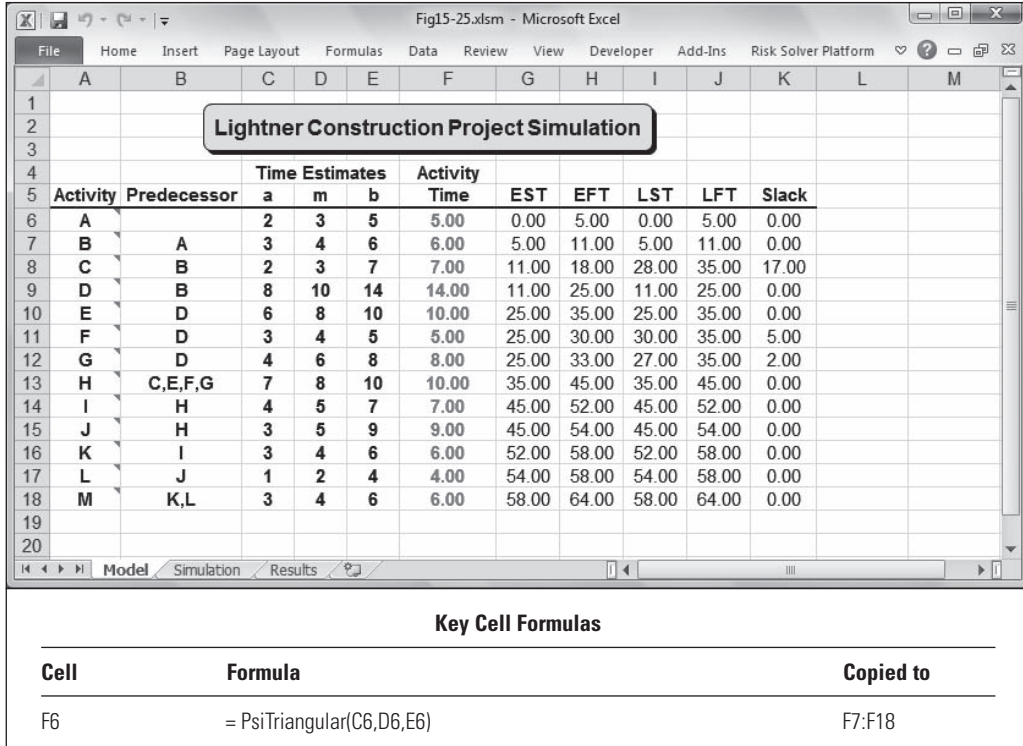
The three times for each project activity are entered in columns C, D, and E. The formula to generate the random values representing the time required for activities in column A are implemented in column F as follows:

Formula for cell F6: =PsiTriangular(C6,D6,E6)
 (Copy to F7 through F18.)

(Note: In order to use the PsiTriangular() function, Risk Solver Platform must be installed as described in Chapter 3.) Using the randomly generated activity times, we can create formulas to calculate the earliest and latest start and finish times and the amount of slack available for each activity. We would accomplish this in the same manner as described earlier with respect to Figure 15.11. The activities on the critical path are indicated by slack values of 0.

FIGURE 15.25

Spreadsheet model for the home-building project with random activity times



15.11.4 RUNNING THE SIMULATION

We can use the Data Table command to replicate this model repeatedly. To do so, we must identify the values in the model that we want to track. One objective in our problem is to determine how critical each activity is. Because critical activities are indicated by slack values of 0, we want to track the values in the Slack column for each activity in our model. However, because all paths go through activities A, B, H, and M, we know that these activities will always be critical, so there is no point in tracking their slack values. Thus, the slack value for activity C in the Model sheet in Figure 15.25 is repeated in cell B3 of the Simulation sheet shown in Figure 15.26 using the following formula:

Formula for cell B3: = VLOOKUP(B2,Model!\$A\$6:\$K\$18,11)
 (Copy to C3 through J3.)

This formula is copied to cells C3 through J3 to provide the slack values for activities D, E, F, G, I, J, K, and L, respectively.

A second objective in our problem is to estimate the amount of time required to complete the project—or the finish time of activity M. This value is tracked in column K on the Simulation sheet shown in Figure 15.26 through the following formula in cell K3:

Formula for cell K3: =Model!J18

The remainder of the spreadsheet shown in Figure 15.26 is completed using the Data Table command as follows:

1. Select the range A3 through K102.
2. Click Data, Data Tools, What-If Analysis, Data Table.
3. Specify a Column Input Cell of M1.
4. Click OK.

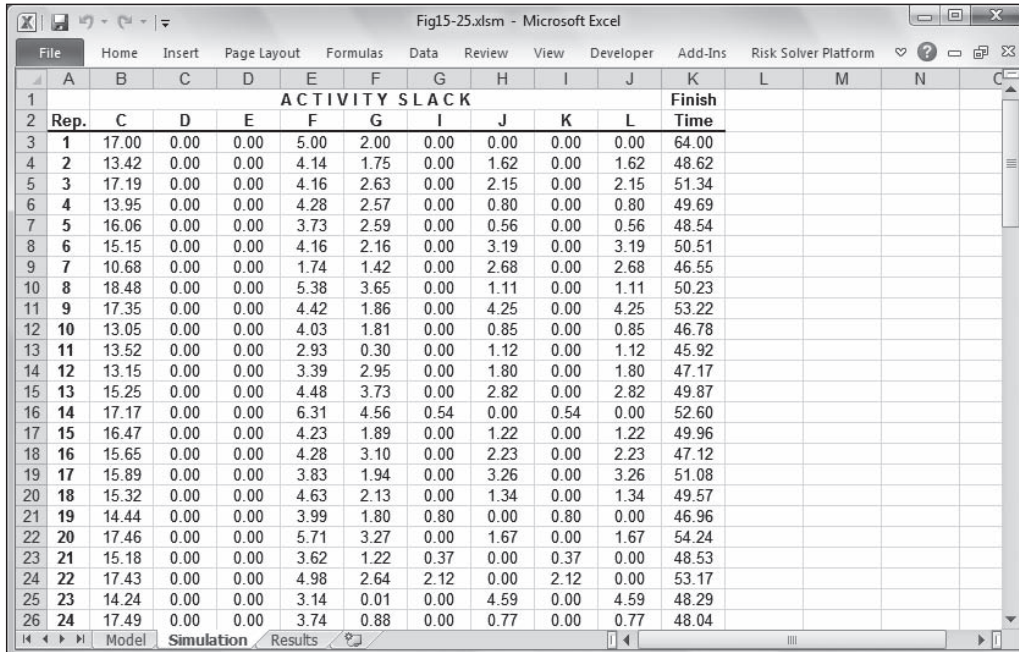


FIGURE 15.26

Simulation of the home-building project

Key Cell Formulas		
Cell	Formula	Copied to
B3	=VLOOKUP(B2,Model!\$A\$6:\$K\$18,11)	C3:J3
K3	=Model!J18	--

Each row in this table represents the amount of slack for the activities and the project completion times that were observed in each replication of our project activity model. The simulation shown in Figure 15.26 consists of 100 such replications.

Freezing Data Table Values

By default, Excel recalculates all data tables whenever a spreadsheet is recalculated. When using data tables for simulation purposes, it is desirable to freeze the values in a data table so this sample of observations can be analyzed without changes constantly occurring. To prevent a data table from recalculating, do the following:

1. Click Formula, Calculation Options.
2. Select Automatic Except for Data Tables.

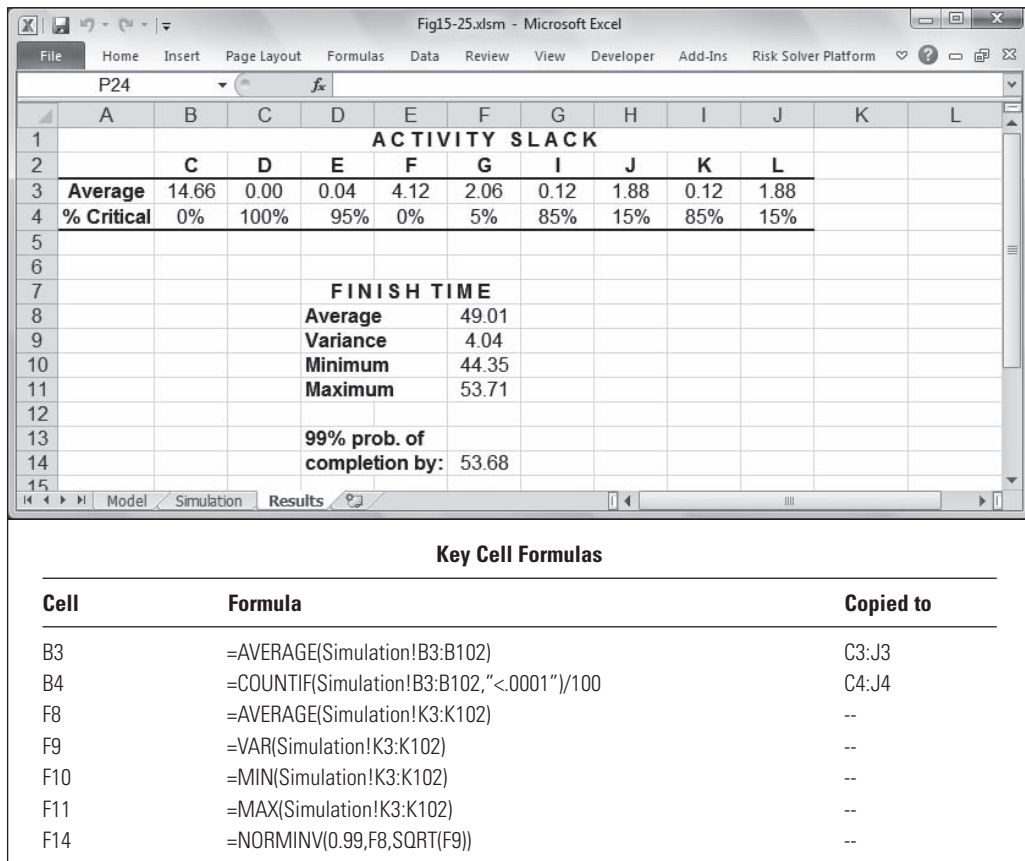
This tells Excel to recalculate the data tables only when you manually recalculate the spreadsheet by pressing the F9 function key.

15.11.5 ANALYZING THE RESULTS

The Results sheet shown in Figure 15.27 provides the results of the simulation. In the top portion of this sheet, we used the AVERAGE() function to calculate the average amount of slack observed for each activity during the simulation. We also calculated the percentage of time each activity was critical during the 100 replications performed on the model. For example, this sheet indicates that activity D was on the critical path (or had 0 slack) in 100% of the replications, activity E was critical in 95% of the replications, and so on. Thus, this sheet clearly shows which activities are most likely to be critical and warrant the most attention.

FIGURE 15.27

Results of the simulation



The average and variance of the finish times observed during the simulation are shown in cells F8 and F9, respectively. Assuming that the finish times are normally distributed, there is roughly a 50% chance that the actual project completion time will exceed 49.01 time periods. Thus, we might want to determine the amount of time within which it is virtually certain that the project will be completed. That is, we might want to determine the 99th percentile of the distribution of project completion times. This is calculated in cell F14 as:

$$\text{Formula in cell F14: } =\text{NORMINV}(0.99,\text{F8},\text{SQRT}(\text{F9}))$$

The result of this formula indicates a 99% chance exists that a normally distributed random variable with a mean of 49.01 and standard deviation of $\sqrt{4.04}$ will assume a

value less than or equal to 53.68. Notice that this is consistent with the maximum finish time (shown in cell F11) observed during the simulation. Thus, we could be reasonably confident that the project can be completed within 54 time periods.

15.12 Microsoft Project

Although spreadsheets can be used to track and manage projects, specialized project management software can greatly simplify much of this task. One of the most popular project management tools today is Microsoft Project (MS Project).

The first step to managing a project in MS Project is to enter the name and duration of each activity or task. This was done for the Lightner Construction project as shown in Figure 15.28 (and the file Fig15-28.mpp that accompanies this book). MS Project automatically inserts start and finish times for each task and draws a horizontal bar corresponding to the length of each task.

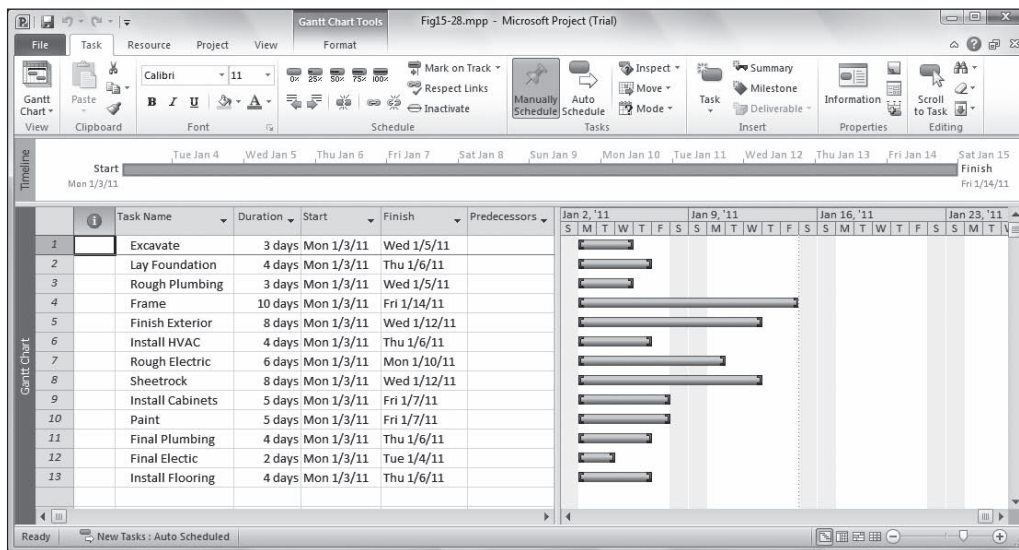


FIGURE 15.28

Listing of home construction tasks and durations in Microsoft Project

The next step is to indicate the precedence relations between the various tasks. This can be done in a variety of ways. You can simply type numbers in the Predecessors column that indicate the predecessors for each task. Alternatively, you can click on a task's horizontal bar and drag it to another bar to indicate the appropriate predecessor relations. Using either technique, the results for the Lightner Construction project (described in Figures 15.1 and 15.2) appear as shown in Figure 15.29. Note that the Gantt chart in Figure 15.29 is similar to the one shown earlier in Figure 15.12 except that the tasks are listed in reverse order, slack values for each activity are not shown, and the precedence relationships are explicitly shown on the graph via arrows.

After the tasks, durations, and precedence relations are entered, MS Project can easily create several different views of a project. For instance, you can select View, Network Diagram to create the chart shown in Figure 15.30. The Network Diagram shows the details about each task and also indicates the critical path.

Another useful view of a project is obtained by selecting View, Tracking Gantt. The resulting chart allows a project manager to keep track of when each task actually begins and track its progress toward completion. Figure 15.31 shows an example of this type of

FIGURE 15.29

Home construction project with precedence relations

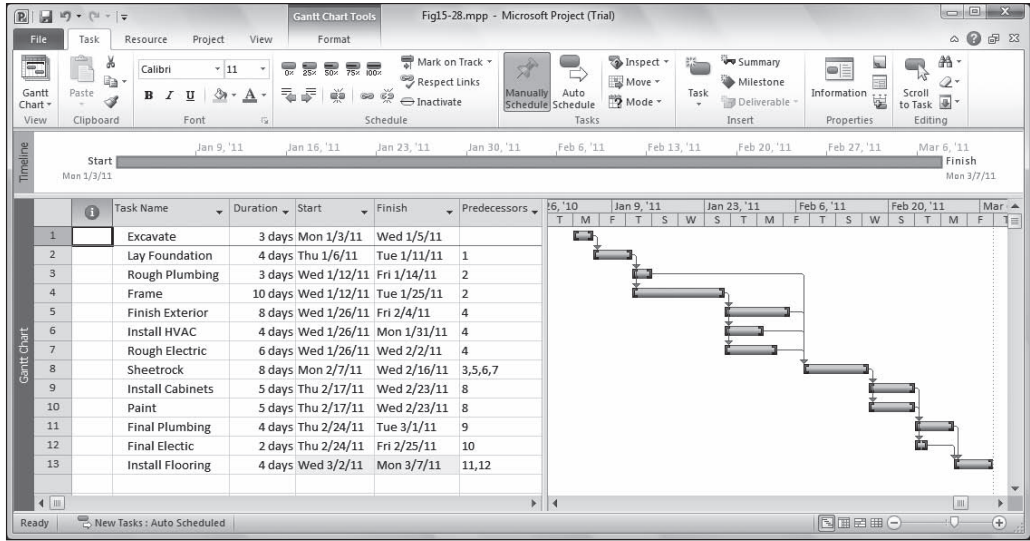


FIGURE 15.30

Network Chart view of the project

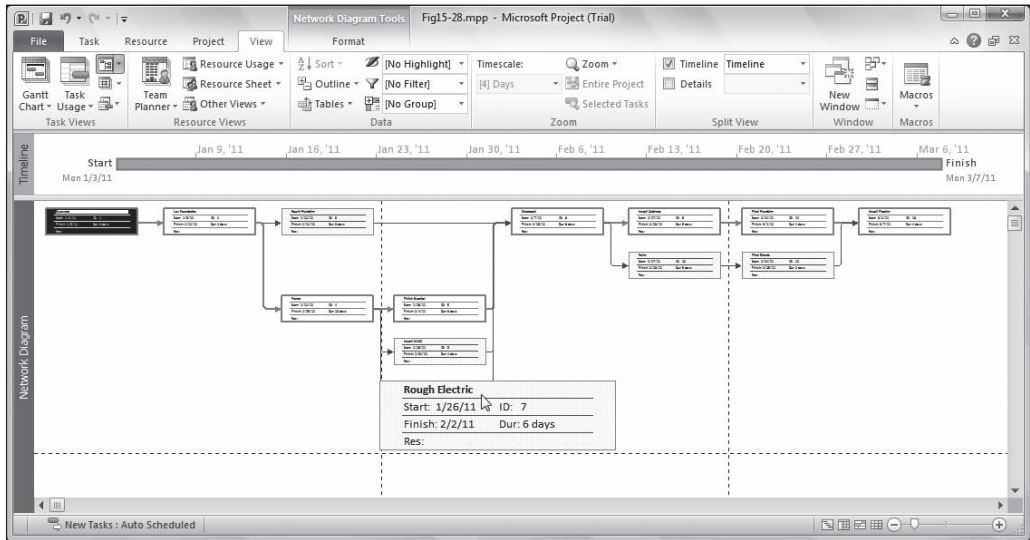


FIGURE 15.31

The Tracking Gantt view of the project

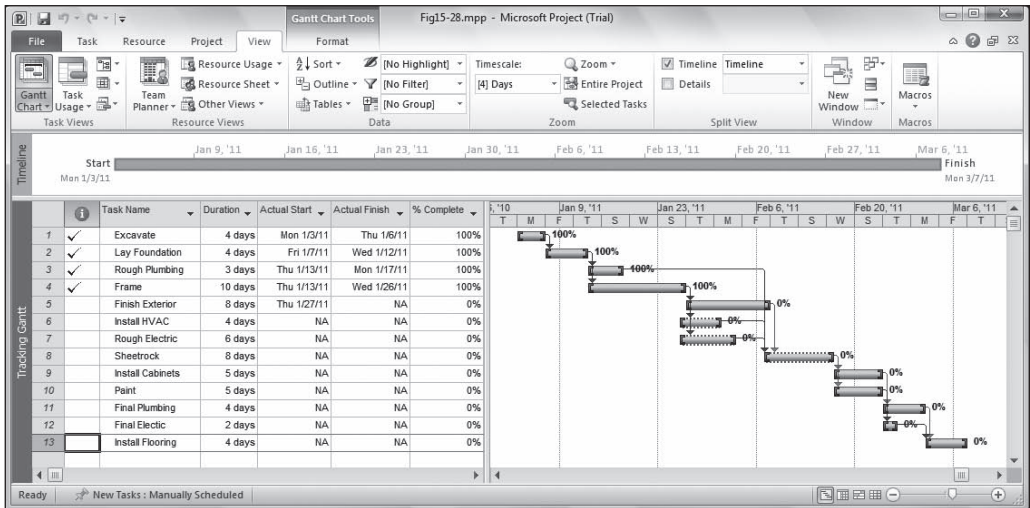


chart where hypothetical start and finish dates have been entered for several of the tasks.

MS Project offers many other features to assist with allocating and tracking costs and assigning resources to various activities in a project. A complete discussion of the capabilities of MS Project is beyond the intended scope of this book. However, this introduction should help you see that MS Project can be a very useful tool for project management.

15.13 Summary

This chapter presented a variety of techniques for managing projects by representing and analyzing a project in the form of a network. When the activity times in a project can be estimated with a reasonable amount of certainty, CPM can be applied to determine the earliest start and finish times for each activity and the earliest time for completing the entire project. CPM can also be used to determine the latest possible times that activities can start and finish without delaying the completion of the project. The slack in a project helps to distinguish between critical and noncritical activities. LP techniques can be used to analyze CPM networks and determine optimal crash schedules for projects that must be completed ahead of schedule.

When activity times cannot be estimated with certainty, they must be modeled as random variables. Although the PERT technique was designed to address these types of problems, it has some serious theoretical and practical shortcomings. However, simulation can be used to provide a more complete and accurate analysis of activity networks containing random activity times.

The techniques illustrated in this chapter merely scratch the surface of the issues one faces when managing projects. Entire books have been written on this topic covering such things as: selecting projects that are consistent with an organization's mission and objectives, the different organizational structures available for managing projects, obtaining and allocating needed resources for projects, managing project risks, managing international projects, monitoring and auditing project performance, and a host of other issues. The Project Management Institute (PMI) is an organization that offers education and professional certification for practicing project managers. Additional information about the field of project management is available on the PMI website at <http://www.pmi.org>.

15.14 References

- Gido, J., and J. Clements. *Successful Project Management with Microsoft® Project*. Cincinnati: South-Western, 1999.
- Golenko, G. "On the Distribution of Activity Times in PERT." *Journal of the Operational Research Society*, vol. 39, August 1988.
- Kloppenborg, T., *Contemporary Project Management*, South-Western Cengage Learning, 2009.
- Levy, F., A. Thompson, and S. Weist. "The ABCs of the Critical Path Method." *Harvard Business Review*, vol. 41, September-October 1963.
- Meredith, J., and S. Mantel. *Project Management: A Managerial Approach*. New York: Wiley, 1985.
- Phillips, C., and E. Davis. *Project Management with CPM, PERT, and Precedence Diagramming*. New York: Van Nostrand Reinhold, 1983.
- Ragsdale, C. "The Current State of Network Simulation in Project Management Theory and Practice." *OMEGA*, vol. 17, no. 1, 1989.
- Ragsdale, C. "A New Approach to Implementing Project Networks in Spreadsheets." *INFORMS Transactions on Education*, vol. 3, no. 3, 2003.
- Schonberger, R. "Why Projects Are 'Always' Late: A Rationale Based on Manual Simulation of a PERT/CPM Network." *Interfaces*, vol. 11, no. 5, 1981.

THE WORLD OF MANAGEMENT SCIENCE

Food and Drug Administration Uses PERT to Control the Timeliness of Research and Development Projects

The National Center for Drug Analysis develops automated drug testing procedures for the U.S. Food and Drug Administration (FDA). The FDA uses these procedures to conduct production studies of drugs selected on the basis of therapeutic significance and how widely they are used.

To allocate personnel and facilities efficiently to drug testing studies, it is important to have a good estimate of when a new, automated testing procedure will be ready. If a method is not ready at the time samples are collected, either the samples must be analyzed manually, or existing schedules must be reworked, causing serious disruptions.

The FDA chose PERT to manage methods of development projects because it treats activity completion times as random variables. This is appropriate for research and development activities, many of which are being done for the first time.

The PERT network for a development project consists of 24 nodes and 31 arcs. The network is updated weekly, giving management access to probabilities of completion times, activities on the critical path and second most critical path, slack times for noncritical activities, a list of completed activities, and so on. This information makes it easy to review and adjust personnel assignments and to control completion times. The PERT plans and reports are also useful to the individual chemists, providing a more comprehensive procedure for planning and organizing their work.

The director of the center successfully implemented the use of PERT by developing appropriate forms and computer programs, thoroughly training staff members on how to use PERT, and then incrementally introducing the procedure in a series of pilot tests. The pilot test results suggested some improvements, which were incorporated as the center proceeded with full implementation.

Source: N K. Kwak and Lawrence Jones, "An Application of PERT to R&D Scheduling." *Information Processing and Management*, vol. 14, no. 2, 1978, pp. 121–131.

Questions and Problems

1. In this chapter, we stated that the longest path through an activity network is the critical path. Why isn't the completion time for a project defined by the shortest path through the network? Explain your answer.
2. Throughout this chapter, we assumed that a project manager identifies and organizes the activities in a project, estimates the time required for each activity, then determines how quickly the project can be completed. However, the deadlines for some projects are fixed before any planning begins. For example, consider the planning of a New Year's Eve celebration in the downtown area of a major city. Which, if any, of the techniques described in this chapter might be helpful in planning such an event?
3. In illustrating how to simulate an activity network, we used a triangular distribution based on three time estimates to generate random activity times. What problem might be encountered if we assume that activity times were normally distributed?

(Hint: Consider the values that a normally distributed random variable with a mean of 5 and variance of 3 might assume.)

4. Consider the CPM network for the following set of activities:

Activity	Predecessor Activities
A	—
B	A
C	A, B

Is it necessary to draw an arc from the node for activity A to the node for activity C? Why or why not?

5. Consider the following set of activities:

Activity	Days Required	Predecessor Activities
A	3	—
B	2	A
C	5	A
D	8	B, C
E	4	B, C
F	2	D, E

- Draw the CPM network for this problem.
 - Manually determine the earliest and latest start and finish times for each activity and the amount of slack for each activity.
 - What is the critical path?
 - How long should it take to complete the project?
 - Verify your answer to part c using Microsoft Project.
6. Consider the following set of activities:

Activity	Days Required	Predecessor Activities
A	3	—
B	2	—
C	5	A, B
D	8	A, B
E	4	D

- Draw the CPM network for this problem. (Remember that a CPM network requires unique start and finish nodes.)
 - Manually determine the earliest and latest start and finish times for each activity and the amount of slack for each activity.
 - What is the critical path?
 - How long should it take to complete the project?
 - Verify your answer to part c using Microsoft Project.
7. Consider the following set of activities:

Activity	Time Required	Predecessor Activities
A	5	—
B	4	A
C	5	B

(Continued)

Activity	Time Required	Predecessor Activities
D	8	B
E	9	C
F	6	C, D
G	8	D
H	3	E, F, G

- Draw the CPM network for this problem.
 - Identify each path through the network and its expected length.
 - What is the critical path?
 - Verify your answer to part c using Microsoft Project.
8. Consider the following set of activities:

Activity	Days Required	Predecessor Activities
A	3	—
B	4	A
C	4	A
D	3	A
E	3	D
F	4	B
G	6	B
H	5	F, C, E
I	6	G, H
J	4	F, C, E
K	2	D
L	6	I, J, K

- Draw the CPM network for this problem.
 - Create a spreadsheet to calculate and summarize the earliest and latest start and finish times, the slack for each activity, and the critical activities.
 - What is the earliest time that the project can be completed?
 - Use Microsoft Project to verify the critical path for this project.
9. The following data summarizes the per-day cost of crashing the activities in the problem described in the previous question.

Activity	Crash Cost per Day	Maximum Crash Days
A	\$50	2
B	60	3
C	57	1
D	45	2
E	25	2
F	30	2
G	65	5
H	55	2
I	28	4
J	33	3
K	40	1
L	37	2

- a. Create an LP model to determine the least costly way of crashing the project to complete it within 20 days.
 - b. By how much should each activity be crashed, and what is the total crashing cost?
10. The Runabout Boat Company is considering the purchase of several units of an expensive new machine for its production facilities around the country. Before committing to this purchase, Runabout wants to set up a test facility to test the machine and make sure it lives up to its manufacturer’s claims. The machine’s manufacturer has agreed to loan Runabout a machine for this purpose. The activities required to carry out this testing are summarized in the following table.

Activity	Description	Predecessor Activities	Days Required
A	Prep test facility	None	4
B	Complete paperwork	None	4
C	Transport machine	A, B	8
D	Hire repairmen	A, B	7
E	Hire operators	A, B	6
F	Train repairmen	C, D	15
G	Install machine	C	9
H	Train operators	E, G	12
I	Testing	F, H	20

- a. Draw the CPM network for this problem.
 - b. Create a spreadsheet to calculate and summarize the earliest and latest start and finish times, the slack for each activity, and the critical activities.
 - c. What is the earliest time that the project can be completed?
 - d. Use Microsoft Project to verify the critical path for this project.
11. Aerobic, Inc. custom designs and manufactures light aircraft. The activities involved in this process are summarized in the following table.

Activity	Description	Weeks Required	Predecessor Activities
A	Design	4	—
B	Order and receive materials	5	A
C	Construct frame	8	B
D	Prepare engine	6	B
E	Prepare avionics	5	B
F	Cover frame	4	C
G	Install engine	2	E, F
H	Paint	2	F
I	Install avionics	2	D, G, H
J	Test	1	I

- a. Draw the activity network for this problem.
 - b. Create a spreadsheet to calculate and summarize the earliest and latest start and finish times, the slack for each activity, and the critical activities.
 - c. What is the earliest this project can be finished?
 - d. Use Microsoft Project to verify the critical path for this project.
12. A software company is moving its helpdesk operations center from Newark, New Jersey to Plano, Texas. Some of its employees will be moved to Plano, and others will be terminated and replaced by new workers in Texas. The activities that must be carried out to implement the move are summarized in the following table.

Activity	Description	Predecessors	Weeks
A	Revise organizational structure	—	5
B	Select new office location	—	3
C	Determine personnel requirements	A	3
D	Design new call center	B, C	4
E	Renovate office space	D	8
F	Select employees to move	C	2
G	Hire new employees	F	4
H	Set up new administrative systems	A	5
I	Move office equipment and records	F	2
J	Train new employees	E, G, H, I	3

- Draw the CPM network for this problem.
 - Create a spreadsheet to calculate and summarize the earliest and latest start and finish times, the slack for each activity, and the critical activities.
 - What is the least amount of time required to complete the project?
 - Use Microsoft Project to verify the critical path for this project.
13. Linda Madison owns Creations, a business that provides hair and nail care services in a small college town. Recently, Linda has decided to expand her business to include tanning and massage services for her clients. In order to accommodate these new services and the additional clients Creations is expecting to attract, Linda is relocating her business to a larger facility. However, the new location will require some renovation before she can relocate there. Linda has identified the following activities that must be performed before she can open at her new location.

Activity	Description	Days Required	Predecessor Activities
A	Install new plumbing	10	—
B	Order and receive furniture	20	—
C	Order and receive flooring	15	—
D	Construct partitions	5	—
E	Paint and wallpaper	5	A, D
F	Install furniture	3	E, B
G	Install flooring	4	E, C
H	Move inventory and records	2	F, G
I	Clean old shop	2	H

- Draw the activity network for this problem.
 - Create a spreadsheet to calculate and summarize the earliest and latest start and finish times, the slack for each activity, and the critical activities.
 - What is the earliest this project can be finished?
 - Use Microsoft Project to verify the critical path for this project.
14. Refer to the previous question. Suppose that Linda’s lease on her current facility expires in 20 days. The normal and crash times and costs for each activity in her moving project are summarized in the following table. By what amount should each activity be crashed in order for her to complete the move within 20 days?

Activity	Normal		Crash	
	Time	Cost	Time	Cost
A	10	\$11,000	7	\$15,000
B	20	5,000	18	6,000
C	15	3,000	12	3,500

D	5	1,500	3	2,000
E	5	750	2	1,200
F	3	600	1	1,200
G	4	1,000	2	1,500
H	2	250	1	450
I	2	200	1	300

15. The activities required to service a car at Zippy-Lube can be viewed as a mini-project as summarized in the following table.

Activity	Description	Predecessor Activity	Time Required		
			Min	Most Likely	Max
A	Drain oil	—	2	3.5	5
B	Replace filter	A	1	2	3
C	Refill oil	B, E	2	2.5	4
D	Check tires	—	2	3	4
E	Wash windows	D	2	3	5
F	Fill fluids	E	2	3	4
G	Final test	C, F	0.5	1	1.5

The variability in the time required to perform each activity is due to the different types, sizes, and condition of the cars to be serviced.

- a. Draw the activity network for this problem.
 - b. Use PERT to identify the expected time and variance for each activity.
 - c. What is the critical path? What is the expected time and variance of the critical path?
16. Refer to the previous question. Create a spreadsheet model to simulate the completion times for servicing cars. Assume that activity times follow a triangular distribution, and perform 500 replications on the model.
- a. Estimate the probability of each activity falling on the critical path.
 - b. On average, how long should it take to finish servicing a car?
 - c. Suppose that the manager of this business wants to guarantee that your car will be ready within 10 minutes, or you will receive a coupon worth \$5 off your next visit. What proportion of customers will receive the coupon?
17. Koene Technology is a rapidly growing nanotechnology firm. The company is moving into a new building that will allow it more research lab space and improved computing facilities. The lease on the new facility has already been signed and architectural renovations plans approved by the local municipality. The activities required to complete the move are summarized as follows:

Activity	Description	Predecessors	Days
A	Hire computer services manager	None	10
B	Structural renovation	None	19
C	Enlarge parking lot	None	13
D	Hire computer services staff	A	8
E	Purchase computing equipment	A	14
F	Electrical modifications	B	4
G	HVAC modifications	B	3
H	Plumbing modifications	B	5
I	Exterior painting/finishing	B	5

(Continued)

Activity	Description	Predecessors	Days
J	Computer network installation	E, F, G	4
K	Sheetrock	F, G, H	6
L	Computer staff training	D, J	8
M	Interior painting/finishing	K	9
N	Landscaping	C, I	7

- Draw the activity network for this problem.
 - Create a spreadsheet to calculate and summarize the earliest and latest start and finish times, the slack for each activity, and the critical activities.
 - What is the earliest this project can be finished?
 - Use Microsoft Project to verify the critical path for this project.
18. Refer to the previous question. Suppose a partner in Keone Technology does not believe the time estimates for each activity can be specified with certainty. Instead, he prefers to specify a minimum, most likely, and maximum number of days for each activity. These estimates are given in the following table.

Activity	Minimum	Most Likely	Maximum
A	5	10	20
B	16	19	27
C	10	13	18
D	5	8	14
E	8	14	21
F	2	4	10
G	2	3	7
H	4	5	9
I	3	5	10
J	3	4	8
K	4	6	10
L	5	8	10
M	6	9	11
N	5	7	9

- Use PERT to identify the expected time and variance for each activity.
 - What is the critical path? What is the expected time and variance for this path?
 - Assume the distribution of the completion time for the critical path is normally distributed. What is the probability that it will take at least 40 days to complete the project?
19. Refer to the previous question. Create a spreadsheet model to simulate the completion times for Koene's move. Assume that activity times follow a triangular distribution, and perform 500 replications on the model.
- Estimate the probability of each activity falling on the critical path. Which activities are most critical?
 - On average, how long should it take to complete the project?
 - Based on your simulation results, estimate the probability that it will take at least 45 days to complete the project.
20. A national accounting firm has identified the following set of activities that must occur to carry out an audit for one of their major clients.

Activity	Days Required	Predecessor Activities	Activity	Days Required	Predecessor Activities
A	12	—	G	6	B, E
B	16	—	H	4	C, D, E
C	13	A	I	24	C, D, G
D	14	A	J	12	F, H
E	9	A	K	22	F, H
F	12	C	L	8	I, J

- a. Draw the CPM network for this problem.
 - b. Create a spreadsheet to calculate and summarize the earliest and latest start and finish times, the slack for each activity, and the critical activities.
 - c. What is the earliest time the audit can be completed?
 - d. Use Microsoft Project to verify the critical path for this project.
21. Refer to the previous question. Suppose that each of the audit activities can be crashed by the amounts indicated in the following table.

Activity	Crash Days	Cost Per Day	Activity	Crash Days	Cost Per Day
A	2	\$2,000	G	0	0
B	4	3,000	H	2	1,500
C	2	800	I	5	1,300
D	4	1,200	J	3	2,000
E	2	1,000	K	4	2,500
F	2	1,500	L	2	1,200

- a. What is the earliest the audit could be completed, and what is the incremental cost associated with this schedule?
 - b. How much incremental cost would be incurred if the client insisted that the audit be completed within 55 days?
 - c. Suppose the client wants the project completed within 55 days. Further suppose that it might be possible to crash activity G by 1 day. How much, if any, should the accounting firm be willing to pay to crash activity G by 1 day? Explain.
22. A partner in the national accounting firm mentioned in the previous questions does not believe the time estimates for each activity can be specified with certainty. Instead, she prefers to specify a minimum, most likely, and maximum time estimate for each task. These estimates are given in the following table.

Activity	Predecessor Activities	Min	Most Likely	Max
A	—	9	12	15
B	—	12	16	18
C	A	11	13	15
D	A	10	14	20
E	A	7	9	15
F	C	9	12	14
G	B, E	4	6	7
H	C, D, E	3	4	5
I	C, D, G	20	24	26
J	F, H	10	12	15
K	F, H	18	22	30
L	I, J	6	8	12

- a. Draw the activity network for this problem.
 - b. Use PERT to identify the expected time and variance for each activity.
 - c. What is the critical path? What is the expected time and variance for this path?
 - d. Assume the distribution of the completion time for the critical path is normally distributed. What is the probability that it will take at least 67 days to complete the project?
23. Refer to the previous question. Create a spreadsheet model to simulate the completion times for the auditing project. Assume that activity times follow a triangular distribution, and perform 500 replications on the model.
- a. Estimate the probability of each activity falling on the critical path. Which activities are most critical?
 - b. On average, how long should it take to complete the project?
 - c. Based on your simulation results, estimate the probability that it will take at least 67 days to complete the project.
24. The activities summarized in the following table must be performed by the promoter of a country music concert in Dallas.

Activity	Description	Predecessor Activity	Days Required		
			Min	Most Likely	Max
A	Find location	—	2	4	7
B	Hire opening act	A	3	6	10
C	Hire technicians	A	1	2	3
D	Set up radio ads	B	2	4	6
E	Distribute tickets	B	2	3	5
F	Rent sound and lights	B, C	3	4	6
G	Set up newspaper ads	B	1	2	4
H	Hire security	F	2	3	4
I	Conduct rehearsal	F, H	1	1.5	2
J	Final details	I	2	3	4

- a. Draw the activity network for this problem.
 - b. According to PERT, what is the expected length of time required to accomplish these activities?
 - c. Assume that the durations of the activities are random variables that follow triangular distributions. Create a spreadsheet model of the project, and use simulation to create 500 replications of the project. Rank the activities according to how critical they are.
 - d. When should the promoter begin working on the concert to have a 99% chance of completing the project by August 15? (Assume work is conducted seven days a week.)
25. A cosmetics company has identified a set of activities that must be performed in order to bring a new product to market. The activities, their precedence relations, and time estimates to complete each activity (in days) are shown in the following table.

Activity	Predecessors	Time Required		
		Min	Most Likely	Max
A	—	0	0	0
B	A	10	15	20
C	B	15	20	22
D	C	21	26	30
E	C	15	18	23

F	E	13	15	17
G	D, F	30	38	45
H	G	20	25	30
I	G	10	15	20
J	G, O	11	18	22
K	H, I, J	23	30	45
L	K	22	28	39
M	A	120	140	180
N	M	13	18	22
O	N	15	20	25
P	O	10	15	20
Q	L, P	30	33	44
R	Q	5	8	11
S	R	10	15	25
T	R	13	17	19
U	S, T	20	25	40

- Draw the activity network for this problem.
 - Assume that activity times follow a triangular distribution, and perform 500 replications on the model.
 - Estimate the probability of each activity falling on the critical path.
 - On average, how long should it take to finish the project?
 - Suppose a change can be made in activity J so that it no longer requires activity O to be completed before it begins. What effect would this have on the project completion time and the criticality of each activity?
26. Refer to the previous problem. Suppose that the maximum time listed for each activity is actually its normal completion time. Further assume that the maximum number of crash days per activity and cost per crash day are as follows.

	Maximum Crash Days	Crash Cost per Day
A	0	\$ 0.0
B	10	8.4
C	7	3.3
D	9	24.4
E	8	17.0
F	4	31.3
G	15	13.5
H	10	3.0
I	10	10.4
J	11	6.3
K	22	4.5
L	17	13.1
M	60	18.5
N	9	8.9
O	10	10.9
P	10	2.0
Q	14	11.6
R	6	3.8
S	15	6.3
T	6	18.0
U	20	7.2

- a. Create and solve an LP model to determine the earliest start time (with no crashing) for each activity.
- b. Modify your LP model and solve it to determine the latest start time (with no crashing) for each activity.
- c. Identify the EST, EFT, LST, LFT, and slack for each activity.
- d. What is the critical path?
- e. If crashing is allowed, how quickly could the project be completed? What is the optimal cost of completing the project in the least amount of time?
- f. What is the optimal cost of completing the project in 320 days?

CASE 15.1

Project Management at a Crossroad

Cal Hampton is on the board of directors of Crossroad Academy, a private, parent-run, K-12 school located in the town of Shawsville, VA. In an effort to increase enrollment at this school in the coming year, several months ago, Cal and other board members voted to give a 25% tuition discount to any new students from families who had never been affiliated with Crossroad before. As it turned out, this offer worked so well that Crossroad's enrollment swelled by 30%, and the school suddenly needed to build some new classrooms to accommodate this growth. And that's why Cal was in the town's building inspector's office on July 10th—being told that Crossroad's building plan for new classrooms are fine, but no classes may be held at the school until a fire sprinkler and alarm system have been installed.

The school was planning to use volunteer labor to build the new classrooms before classes started on August 23rd. But installing a fire sprinkler and alarm system throughout the building is quite another matter. So Cal and his fellow board members quickly got busy contacting sprinkler contractors and getting price quotes on this job. They also got busy lining up other places they can conduct classes until their building situation is resolved.

Cal quickly learned more than he every wanted to know about fire sprinkler systems. Because Crossroad's school building is a two-story wood frame structure, a dry pipe system has to be installed in the attic of the building to protect the upper floor against fire. In a dry pipe system, the pipes are filled with pressurized air to avert the risk of freezing in unheated spaces. If a fire is detected, one or more sprinkler heads open, releasing the pressurized air, which, in turn, allows water to flow through the pipes to the sprinkler heads. On the lower level of the building, a wet pipe system (where the pipes constantly hold water) would be more cost effective.

A multitude of other details have to be worked out: The wet and dry systems have to be designed and approved by the town, a new 6-inch water line has to be run to the building, new phone lines have to be installed to support the fire alarm monitoring service, various types of sprinkler heads have to be selected and ordered, and the whole thing needs to be installed and pass various tests required by the town and the local fire marshal.

Within a week, three different sprinkler contractors have visited the school and submitted proposals for completing the work at the school. One of the proposals is substantially more expensive than the other two and is immediately eliminated from further consideration. Now Cal and the rest of the board must decide on which of the other proposals to accept. One proposal is from a local company, Fire Protection Services (FPS), which has been in business for more than 20 years. The other proposal is from Safety Technology Group (STG), a fairly new company located in a large city about 40 miles from Shawsville.

Although STG is a new company, the people working for STG have many years of experience in the fire safety industry. Their project manager, Ron Brag, supervised the largest sprinkler installation job in the commonwealth of Virginia while working for a different sprinkler company. Additionally, Ron has told Cal that while STG has a number of jobs in progress, he has a crew who is just finishing a project and could start on the Crossroads job immediately. In fact, Ron has told Cal he thinks there is a good chance his company could get the job done before the start of school and, in the worst case, by the end of August. One of the families at Crossroad has used STG on another job and reported being pleased with their work.

FPS has held the service contract on the fire extinguishers at Crossroads for many years and has always done a good job. The project manager for FPS is a middle-aged man named Max Randall who is a slow-talking country boy. He has supervised a number of sprinkler installations in the area (FPS typically only does one or two jobs at a time) and knows the town building inspector and fire marshal well. He knows that Crossroad is desperate to get this job done quickly, but he tells Cal, “Well, to get your business, I could lie to you and tell you we’ll get this done before school starts. But the fact is, it will probably be the end of September before we or anyone else can get this done for you.”

The bids from FPS and STG are both for turn-key systems and are within a few thousand dollars of each other. So now Cal and the rest of Crossroad’s board members are trying to decide which bid to accept.

1. Describe how the various project-management techniques covered in this chapter might be relevant to this problem. When and to whom might CPM be useful? When and to whom might PERT be useful? When and to whom might the crashing technique be useful?
2. If you were Cal and wanted to get the project done as quickly as possible, what aspects of this job might be of greatest concern to you? How would you manage these issues?
3. What are some of the worst case outcomes that might arise in this project?
4. What special clauses, if any, might you want included in the contract with whatever sprinkler company is selected? Would any of the techniques discussed in this chapter be useful in determining or negotiating these clauses?

The World Trade Center Clean-Up

CASE 15.2

In the aftermath of the September 11, 2001, terrorist attacks on the World Trade Center, the New York City Department of Design and Construction was put in charge of recovery and clean-up operations at the site. The attack killed 2,823 people, destroyed a total of 8 buildings, and crushed 1,300 feet of subway tunnel. Ultimately, the clean-up operations involved 4 construction firms, 3,000 workers, and 3.1 million man hours. It took more than 100,000 truckloads to carry the 1.8 million tons of debris from the site. Amazingly, no workers were killed or critically injured during the clean-up effort, which officially ended on May 30, 2002, at a cost of \$650 million. (Source: <http://www.pbs.org/americanrebuilt>)

Obviously, a project of this magnitude could not be completed successfully without careful planning and coordination of many, many activities. Indeed, many clean-up “activities” could be considered subprojects of the entire clean-up operation.

Imagine that you were the person in charge of the clean-up operations at the World Trade Center site. No one has ever faced this task before, so no one can give you a nice, neat list of things to do with precise time estimates for each activity. It is an enormous task

requiring careful consideration of human emotion, dignity, and safety concerns in addition to the monumental logistics and financial issues. And the world is watching all you do.

Identify the key top-level activities you think would go into the clean-up project and prepare an AON project network summarizing the precedence relations among these activities. Also, describe how you would go about estimating the activity times and financial costs associated with these activities.

CASE 15.3

The Imagination Toy Corporation

Amy White is the director of marketing for the Imagination Toy Corporation (ITC). She just received a phone call from her boss indicating that the company's board of directors gave final approval for the production and marketing of the Mighty Morphin' Monkeys—a new product line of action play toys for ITC. Amy worked hard in developing the concept for this product line and is thrilled that her ideas will become reality. But this news also means that she must get busy developing the marketing and sales force training materials needed to launch this new product line successfully. Amy's boss wants to know how soon she can have the sales staff trained and equipped to handle the new line.

The development of marketing materials and training of the sales staff for the new product line constitute a project. Amy identifies 10 specific project tasks that need to be accomplished before she can roll out the marketing program for this product line. First, Amy needs to collect information about the details of the decisions made by the board of directors. She can start this task (task A) immediately, and she estimates that it will take five days to determine exactly which items and accessories will be included in the first offering of the product line. After she completes this task, she will request prototypes of all items from the engineering department (task B), which she expects will take 10 working days. While waiting for the prototypes, she can begin laying out the marketing program (task C). She expects this activity to take 8 days. After the prototypes (task B) are available, Amy estimates that it will take 7 days to prepare instructions (task D) on the operation and use of the items in the product line and 9 days to design its packaging (task E). When the marketing program (task C) is finished, it must be approved (task F) by the president of the company. Amy expects this approval to take 3 days.

Amy plans to hold a 2-day training course (task G) for the sales force after the operating instructions (task D) and the packaging design (task E) are completed. When the operating instructions (task D) are finished and the marketing plan is approved (task F), Amy will develop an information guide (task H) that the sales force can distribute to retailers. Amy expects to take 8 days to complete the information guide. Also, as part of the marketing plan, Amy wants to hire a number of actors to portray Mighty Morphin' Monkeys (task I) at various promotional events around the country. Hiring and training these actors is expected to take 8 days and can be done only after the marketing plan is approved (task F). Finally, after the marketing plan (task F) has been approved and the packaging for the product has been designed (task E), special point-of-sale display racks must be manufactured (task J). Amy expects this activity to take 12 working days.

1. Develop an AON network for this problem.
2. Create a spreadsheet to calculate and summarize the earliest and latest start and finish times, the slack for each activity, and the critical activities.
3. Identify the critical path.
4. If Amy starts working immediately, how long will it take her to complete this project?
5. Suppose that the engineering department can create the prototypes in only eight days if the engineers work overtime on this activity. Would this help reduce the length of time required to complete this project?