.

Quiz 2: MATH 212

Instructor: Mohammad El Smaily

Beirut, March 27, 2015

Duration: 75 minutes

Last Name: ______________________________

First Name: ______________________________

Student number: ______________________________

No Calculators or other aids are allowed
(cell phones must be turned off during this exam)
No credit will be given if the answers are not justified.
No books, no course notes, no cheat sheets are allowed

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Good Luck!

For marker's use only	
Question	Mark
1	/20
2	/25
3	/30
4	/25
Total	/100

[20 points=5+5+10] Problem 1.

a) Find the Fourier series of the 2π-periodic function $\phi(x) = \sin x \cos x$ **without** doing the integral computations of the coefficients a_k and b_k – Justify your answer.

b) Suppose that f is a piecewise continuous 2π-periodic function. Let $g(x) = \int_0^x f(t)dt$. Give a necessary and sufficient condition on f so that g is 2π-periodic.

c) Knowing that the Fourier series of the odd function $sign(x) := \begin{cases} -1 & \text{for } -\pi \le x < 0 \\ 0 & x = 0 \\ 1 & \text{otherwise} \end{cases}$

is given by $\dfrac{4}{\pi}\displaystyle\sum_{k=0}^{\infty} \frac{\sin(2k+1)\pi x}{2k+1}$, find (after justifications) the Fourier series of the 2π-periodic function defined by $g(x) = |x|$ for $x \in [-\pi, \pi]$.

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[25 points=20+ 5] Problem 2. Let $\tilde{f}$ be the 2π-periodic extension of the function f defined, over $[-\pi, \pi]$, by $f(x) = e^x$.

a) Compute the Fourier series of f.

b) What is the value of the Fourier series of f at the point $x = -\pi$? Is it $e^{-\pi}$?

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[30 points=10+10+5+5] Problem 3. Consider the following sequences of functions over the real line $\mathbb{R}$:

$$f_n(x) = xe^{-nx^2} \quad \text{and} \quad g_n(x) = nxe^{-nx^2}, \quad n = 1, 2, \cdots$$

1. Determine the pointwise limits of the the sequences of functions $\{f_n(x)\}_n$ and $\{g_n(x)\}_n$ for all $x \in \mathbb{R}$?
2. Compute $\max\limits_{x\in\mathbb{R}} |f_n(x)|$ and $\max\limits_{x\in\mathbb{R}} |g_n(x)|$.
3. Study the uniform convergence of $\{f_n(x)\}_n$ over $\mathbb{R}$. Justify your answer.
4. Study the uniform convergence of $\{g_n(x)\}_n$ over $\mathbb{R}$. Justify your answer.

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[25 points=10+5+5+5] Problem 4. Consider the following PDE whose unknown is $u(t,x)$

$$\frac{\partial^2 u}{\partial t^2}(t,x) = \frac{\partial^2 u}{\partial x^2} \quad \text{and} \quad u(0,x) = f(x), u_x(0,x) = g(x) \tag{1}$$

where $t \geq 0$ and $x \in \mathbb{R}$ and f, g are two give functions of class C^2 over $\mathbb{R}$. Let

$$r = x + t \text{ and } s = x - t \text{ and } u(t,x) = v(r,s) = v(x+t, x-t).$$

1. Use the cain rule to find $\frac{\partial^2 u}{\partial t^2}(t,x)$ and $\frac{\partial^2 u}{\partial x^2}$ in terms of the partial derivative v_s, v_s, v_{rr}, v_{rs} and v_{ss}.
2. Show that the PDE (1) is equivalent to the PDE

$$\frac{\partial^2 v}{\partial r \partial s} = 0. \tag{2}$$

3. Write down the general solution of (2) (**Help:** $\frac{\partial^2 v}{\partial r \partial s} = 0$ means that $\frac{\partial}{\partial r}\left(\frac{\partial v}{\partial s}\right) = 0$).
4. Use the initial data on $u(t,x)$ and part 3. to find the form of the solution of (1) interns of $f(x)$ and $g(x)$.

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Scratch paper