Midterm Exam: MATH 212

Professor: Mohammad El Smaily

Beirut, November 1, 2014

Duration: 60 minutes

Last Name: ______________________________

First Name: ______________________________

Student number: ______________________________

Lecture time (circle one of the following 3): 11:00 to 12:00 12:00 to 1:00 1:00 to 2:00

No Calculators or other aids are allowed
(cell phones must be turned off during this exam)
No credit will be given if the answers are not justified.
No books, no course notes, no cheat sheets are allowed

For marker's use only	
Question	Mark
1	/9
2	/4
3	/9
4	/3
Total	/25

[9=4+5 marks] **Problem 1. (the parts of this problem are not related)**
Let $u = u(t,x)$ be a smooth solution of the following PDE (where $x \in [0,1]$ and $t \geq 0$):

$$\begin{cases} u_t(t,x) = & u_{xx} + \lambda u_x, \quad \text{for all } t > 0, \quad 0 < x < 1 \\ u(t,1) = u(t,0) = & 0 \quad \text{for all } t > 0 \\ u(0,x) = & f(x) \end{cases} \tag{1}$$

where f is a random C^1, non constant, function which vanishes outside the interval $\left[\frac{1}{4}, \frac{3}{4}\right]$ and λ is a fixed real number. Define the quantity

$$m(t) := \int_0^1 (u(t,x))^2 \, dx.$$

1. (4 marks) Show that, if u is a smooth solution of the above problem, $\frac{dm}{dt}(t)$ remains strictly negative for all t (regardless of the sign of λ).

2. (5 marks) In this part, replace the conditions on $u(t,0)$ and $u(t,1)$ (in the PDE (1)) by the following:

$$u_x(t,1) = 0, \quad u(t,1) = 1 \text{ and } u(t,0) = 0.$$

(Notice the difference between the current conditions and the ones in part 1). Show that

$$\text{for all } t \geq 0, \quad m(t) \leq \lambda t + \int_0^1 (f(x))^2 \, dx.$$

(**Hint:** upon an integration by parts of the PDE multiplied by u, observe first that $m'(t)$ is less than or equal to λ.)

more space if needed

[4 marks] **Problem 2.** Let Ω be the **rectangular region** in the xy-plane with vertices $(0,0)$, $(1,0)$, $(1,\frac{1}{2})$, $(0,\frac{1}{2})$ and let u be a smooth solution to the following PDE:

$$-u_{xx} - u_{yy} = 3u \text{ for } (x,y) \in \Omega,$$

with the boundary condition

$$\frac{\partial u}{\partial n}(x,y) = 1 \text{ for } (x,y) \in \partial\Omega.$$

Use Green's identities and the PDE to show that

$$\iint_\Omega u(x,y)dxdy = -1.$$

[9 marks=5+4] **Problem 3.** Consider the linear wave equation

$$u_{tt}(t,x) = \mathbf{2}u_{xx}(t,x) \text{ for } x \in [0,5] \text{ and } t \geq 0,$$

with the initial data

$$u(0,x) = 11\sin(7\pi x) \text{ and } u_t(0,x) = g(x) = 2\sin 3\pi x$$

and boundary conditions

$$u(t,0) = u(t,5) = 0 \text{ for all } t > 0.$$

1. (5 marks) Using the method of separation of variables, find the explicit expression of the solution $u(t,x)$ (Give all necessary justifications).

2. (4 marks) Use D'Alembert's formula to compute the function $u(t, x)$ which solves the same wave equation, given in the previous page, with same initial data and boundary conditions.

[3 marks] **Problem 4.** Write down a **linear wave** equation **and a set of boundary-initial values** which are satisfied by the function

$$u(t,x) = \cos(2t)\sin(7\pi x).$$

(Justify your answer)

Extra space