Quiz 1: MATH 212 (Introductory PDEs)

Instructor: Wael Mahboub

February 26, 2016

Duration: 60 minutes

Name (Last, First): _____

Student number: _____

For marker's use only	
Problem	Score
1	/20
2	/20
3	/20
4	/20
5	/20
Total	/100

[20 points=10+7+3] Problem 1. Consider the following PDE

$$u_{tt} - u_{xx} - \frac{4}{x}u_x - \frac{2}{x^2}u = 0. \tag{1}$$

The goal of this problem is to solve (1) with initial values $u(0,x) = x$, $u_t(0,x) = 0$.

(a) Let $v(t,x) = x^2 u(t,x)$. Show that $v(t,x)$ satisfy the wave equation

$$v_{tt} = v_{xx}. \tag{2}$$

(b) Solve the initial value problem given by (2) and suitable initial values.

(c) Deduce $u(t, x)$.

[20 points=6+7+7] Problem 2. Solve the following PDEs.

(a) $u_{tt}(t, x) + u_t(t, x) = 0$.

(b) $u_{tx}(t, x) = x$.

(c) $u_{tt}(t, x) + u(t, x) = sint$.

[20 points=10+10] Problem 3. Solve the following initial value problems.

(a) $u_{tt} = 4u_{xx}$, $u(0,x) = e^{-x^2}$, $u_t(0,x) = x$.

(b) $u_t - 2u_x + 3u = t$, $u(0,x) = sinx$.

[20 points] Problem 4. Solve the initial value problem

$$\begin{cases} u_{tt} - 6u_{tx} - 7u_{xx} = 0, \\ u(0,x) = x^2, \\ u_t(0,x) = e^x. \end{cases}$$

(Hint: Factor the associated linear differential operator.)

[20 points] Problem 5. Find all separable eigensolutions to the heat equation $u_t = u_{xx}$ on the interval $0 \leq x \leq \pi$, subject to Neumann boundary conditions $u_x(t, 0) = 0$, $u_x(t, \pi) = 0$.