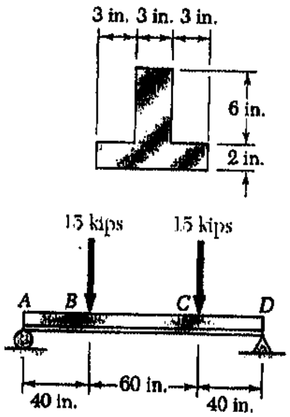
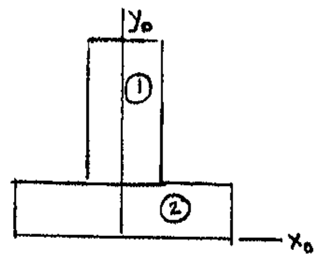
PROBLEM 4.10

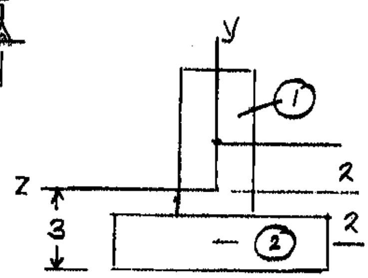
4.9 through 4.11 Two vertical forces are applied to a beam of the cross section shown. Determine the maximum tensile and compressive stresses in portion BC of the beam.



SOLUTION



	A	$\bar{y}_o$	$A\bar{y}_o$
①	18	5	90
②	18	1	18
Σ	36		108



$$\bar{Y}_o = \frac{108}{36} = 3 \text{ in}$$

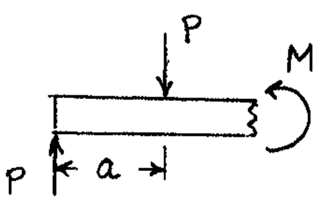
Neutral axis lies 3 in. above the base.

$$I_1 = \frac{1}{12} b_1 h_1^3 + A_1 d_1^2 = \frac{1}{12}(3)(6)^3 + (18)(2)^2 = 126 \text{ in}^4$$

$$I_2 = \frac{1}{12} b_2 h_2^3 + A_2 d_2^2 = \frac{1}{12}(9)(2)^3 + (18)(2)^2 = 78 \text{ in}^4$$

$$I = I_1 + I_2 = 126 + 78 = 204 \text{ in}^4$$

$$y_{top} = 5 \text{ in} \qquad y_{bot} = -3 \text{ in.}$$



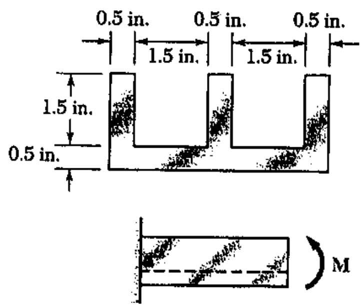
$$M - Pa = 0$$

$$M = Pa = (15)(40) = 600 \text{ kip}\cdot\text{in.}$$

$$\sigma_{top} = -\frac{M y_{top}}{I} = -\frac{(600)(5)}{204} = -14.71 \text{ ksi}$$ ◄

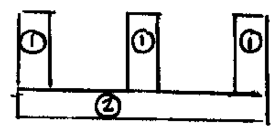
$$\sigma_{bot} = -\frac{M y_{bot}}{I} = -\frac{(600)(-3)}{204} = 8.82 \text{ ksi}$$ ◄

PROBLEM 4.21



4.21 Knowing that for the extruded beam shown the allowable stress is 12 ksi in tension and 16 ksi in compression, determine the largest couple **M** that can be applied.

SOLUTION



	A	$\bar{y}_o$	$A\bar{y}_o$
①	2.25	1.25	2.8125
②	2.25	0.25	0.5625
	4.50		3.375

$$\bar{Y} = \frac{3.375}{4.50} = 0.75 \text{ in}$$

The neutral axis lies 0.75 in. above bottom.

$y_{top} = 2.0 - 0.75 = 1.25$ in, $\quad y_{bot} = -0.75$ in.

$$I_1 = \frac{1}{12} b_1 h_1^3 + A_1 d_1^2 = \frac{1}{12}(1.5)(1.5)^3 + (2.25)(0.5)^2 = 0.984375 \text{ in}^4$$

$$I_2 = \frac{1}{12} b_2 h_2^3 + A_2 d_2^2 = \frac{1}{12}(4.5)(0.5)^3 + (2.25)(0.5)^2 = 0.609375 \text{ in}^4$$

$$I = I_1 + I_2 = 1.59375 \text{ in}^4$$

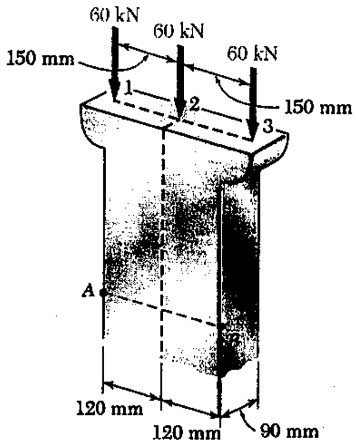
$$|\sigma| = \left|\frac{My}{I}\right| \qquad M = \left|\frac{\sigma I}{y}\right|$$

Top: compression $\quad M = \dfrac{(16)(1.59375)}{1.25} = 20.4$ kip·in

Bottom: tension $\quad M = \dfrac{(12)(1.59375)}{0.75} = 25.5$ kip·in

Choose the smaller as M_{all} $\quad M_{all} = 20.4$ kip·in. ◄

PROBLEM 4.116



4.116 **Determine the stress at points** ***A*** **and** ***B***, **(*a*) for the loading shown, (*b*) if the 60-kN loads are applied at points 2 and 3 are removed.**

SOLUTION

(a) Loading is centric

$$P = 180 \text{ kN} = 180 \times 10^3 \text{ N}$$

$$A = (90)(240) = 21.6 \times 10^3 \text{ mm}^2 = 21.6 \times 10^{-3} \text{ m}^2$$

At A and B $\quad \sigma = -\frac{P}{A} = -\frac{180 \times 10^3}{21.6 \times 10^{-3}} = -8.33 \times 10^6 \text{ Pa}$

$$= -8.33 \text{ MPa}$$

(b) Eccentic loading

$$P = 160 \text{ kN} = 60 \times 10^3 \text{ N}$$

$$M = (60 \times 10^3)(150 \times 10^{-3}) = 9.0 \times 10^3 \text{ N} \cdot \text{m}$$

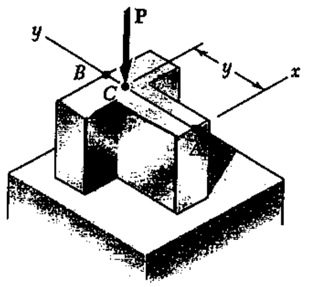
$$I = \frac{1}{12} b h^3 = \frac{1}{12}(90)(240)^3 = 103.68 \times 10^6 \text{ mm}^4 = 103.68 \times 10^{-6} \text{ m}^4$$

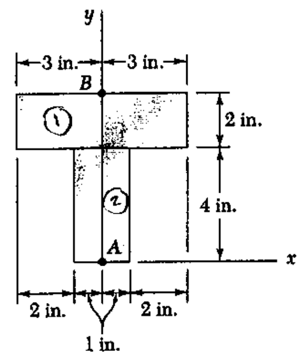
$$c = 120 \text{ mm} = 0.120 \text{ m}$$

At A $\quad \sigma_A = -\frac{P}{A} - \frac{Mc}{I} = -\frac{60 \times 10^3}{21.6 \times 10^{-3}} - \frac{(9.0 \times 10^3)(0.120)}{103.68 \times 10^{-6}} = -13.19 \times 10^6 \text{ Pa} = -13.19 \text{ MPa}$

At B $\quad \sigma_B = -\frac{P}{A} + \frac{Mc}{I} = -\frac{60 \times 10^3}{21.6 \times 10^{-3}} + \frac{(9.0 \times 10^3)(0.120)}{103.68 \times 10^{-6}} = 7.64 \times 10^6 \text{ Pa} = 7.64 \text{ MPa}$

PROBLEM 4.134





(*b*)

4.134 A vertical force **P** of magnitude 20 kips is applied at a point *C* located on the line of symmetry of the cross section of a short column. Knowing that $y = 5$ in., determine (*a*) the stress at point *A*, (*b*) the stress at point *B*, (*c*) the location of the neutral axis.

SOLUTION

Locate centroid

Part	A, in^2	$\bar{y}$, in	$A\bar{y}$, in^3
1	12	5	60
2	8	2	16
Σ	20		76

$$\bar{y} = \frac{\Sigma A_i \bar{y}_i}{\Sigma A_i} = \frac{76}{20} = 3.8 \text{ in}$$

Eccentricity of load $e = 5 - 3.8 = 1.2$ in.

$$I_1 = \frac{1}{12}(6)(2)^3 + (12)(1.2)^2 = 21.28 \text{ in}^4$$

$$I_2 = \frac{1}{12}(2)(4)^3 + (8)(1.8)^2 = 36.587 \text{ in}^4$$

$$I = I_1 + I_2 = 57.867 \text{ in}^4$$

(a) Stress at A $\quad c_A = 3.8$ in

$$\sigma_A = -\frac{P}{A} + \frac{Pec_A}{I} = -\frac{20}{20} + \frac{(20)(1.2)(3.8)}{57.867} = 0.576 \text{ ksi}$$

(b) Stress at B $\quad c_B = 6 - 3.8 = 2.2$ in

$$\sigma_B = -\frac{P}{A} - \frac{Pec_B}{I} = -\frac{20}{20} - \frac{(20)(1.2)(2.2)}{57.867} = -1.912 \text{ ksi}$$

(c) Location of neutral axis: $\quad \sigma = 0$

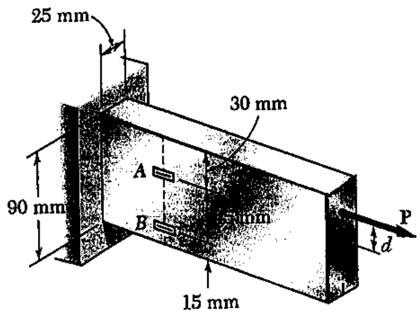
$$\sigma = -\frac{P}{A} + \frac{Pea}{I} = 0 \quad \therefore \quad \frac{ea}{I} = \frac{1}{A}$$

$$a = \frac{I}{Ae} = \frac{57.867}{(20)(1.2)} = 2.411 \text{ in}$$

Neutral axis lies 2.411 in. below centroid or $3.8 - 2.411$
$= 1.389$ in above point A.

Answer 1.389 in from point A

PROBLEM 4.140



4.140 An eccentric axial force **P** is applied as shown to a steel bar of 25 × 90-mm cross section. The strains at A and B have been measured and found to be

$\epsilon_A = +350\ \mu \qquad \epsilon_B = -70\ \mu$

Knowing that $E = 200$ GPa, determine (*a*) the distance d, (*b*) the magnitude of the force **P**.

SOLUTION

$h = 15 + 45 + 30 = 90$ mm

$b = 25$ mm $\qquad c = \frac{1}{2}h = 45 \text{mm} = 0.045$ m

$A = bh = (25)(90) = 2.25\times10^3$ mm^2 $= 2.25\times10^{-3}$ m^2

$I = \frac{1}{12}bh^3 = \frac{1}{12}(25)(90)^3 = 1.51875\times10^6$ mm^4

$= 1.51875\times10^{-6}$ m^4

$y_A = 60 - 45 = 15$ mm $= 0.015$ m , $y_B = 15 - 45 = -30$ mm $= -0.030$ m

Stresses from strain gages at A and B

$\sigma_A = E\varepsilon_A = (200\times10^9)(350\times10^{-6}) = 70\times10^6$ Pa

$\sigma_B = E\varepsilon_B = (200\times10^9)(-70\times10^{-6}) = -14\times10^6$ Pa

$$\sigma_A = \frac{P}{A} - \frac{My_A}{I} \qquad (1)$$

$$\sigma_B = \frac{P}{A} - \frac{My_B}{I} \qquad (2)$$

Subtracting $\quad \sigma_A - \sigma_B = -\frac{M(y_A - y_B)}{I}$

$$M = -\frac{I(\sigma_A - \sigma_B)}{y_A - y_B} = -\frac{(1.51875\times10^{-6})(84\times10^6)}{0.045} = -2835 \text{ N}\cdot\text{m}$$

Multiplying (2) by y_A and (1) by y_B and subtracting

$$y_A\sigma_B - y_B\sigma_A = (y_A - y_B)\frac{P}{A}$$

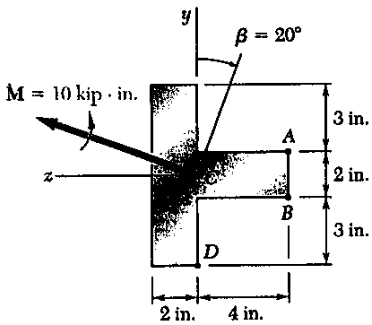
$$P = \frac{A(y_A\sigma_B - y_B\sigma_A)}{y_A - y_B} = \frac{(2.25\times10^{-3})[(0.015)(-14\times10^6) - (-0.030)(70\times10^6)]}{0.045}$$

$= 94.5\times10^3$ N

(a) $\quad M = -Pd \therefore \quad d = -\frac{M}{P} = -\frac{-2835}{94.5\times10^3} = 0.030$ m $= 30$ mm

(b) $\qquad P = 94.5$ kN·m

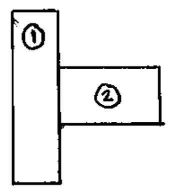
PROBLEM 4.148



4.148 through 4.150 The couple **M** is applied to a beam of the cross section shown in a plane forming an angle β with the vertical. Determine the stress at (*a*) point *A*, (*b*) point *B*, (*c*) point D.

SOLUTION

Locate centroid



	A, in^2	$\bar{z}$, in	$A\bar{z}$, in^3
①	16	-1	-16
②	8	2	16
Σ	24		0

The centroid lies at point C

$$I_z = \frac{1}{12}(2)(8)^3 + \frac{1}{12}(4)(2)^3 = 88 \text{ in}^4$$

$$I_y = \frac{1}{3}(8)(2)^3 + \frac{1}{3}(2)(4)^3 = 64 \text{ in}^4$$

$y_A = -y_B = 1$ in, $\quad y_D = -4$ in

$z_A = z_B = -4$ in, $\quad z_D = 0$

$M_z = 10 \cos 20° = 9.3969$ kip-in

$M_y = 10 \sin 20° = 3.4202$ kip. in.

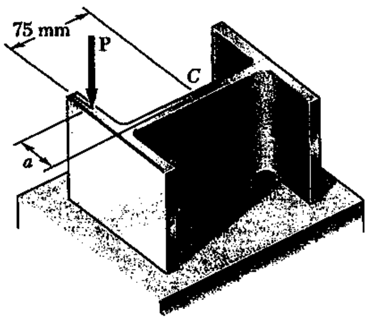
(a) $\sigma_A = -\dfrac{M_z y_A}{I_z} + \dfrac{M_y z_A}{I_y} = -\dfrac{(9.3969)(1)}{88} + \dfrac{(3.4202)(-4)}{64} = 0.321$ ksi

(b) $\sigma_B = -\dfrac{M_z y_B}{I_z} + \dfrac{M_y z_B}{I_y} = -\dfrac{(9.3969)(-1)}{88} + \dfrac{(3.4202)(-4)}{64} = -0.107$ ksi

(c) $\sigma_D = -\dfrac{M_z y_D}{I_z} + \dfrac{M_y z_D}{I_y} = -\dfrac{(9.3969)(-4)}{88} + \dfrac{(3.4202)(0)}{64} = 0.427$ ksi

PROBLEM 4.164

4.164 An axial load **P** of magnitude 50 kN is applied as shown to a short section of a W 150 × 24 rolled-steel member. Determine the largest distance *a* for which the maximum compressive stress does not exceed 90 MPa.



SOLUTION

Add y- and z- axes.

For W 150 × 24 rolled-steel section

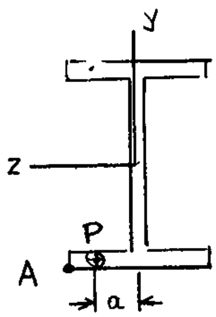
$A = 3060\ \text{mm}^2 = 3060 \times 10^{-6}\ \text{m}^2$

$I_z = 13.4 \times 10^6\ \text{mm}^4 = 13.4 \times 10^{-6}\ \text{m}^4$

$I_y = 1.83 \times 10^6\ \text{mm}^4 = 1.83 \times 10^{-6}\ \text{m}^4$

$d = 160\ \text{mm}, \qquad b_f = 102\ \text{mm}$

$y_A = -\frac{d}{2} = -80\ \text{mm}, \qquad z_A = \frac{b_f}{2} = 51\ \text{mm}.$



$P = 50 \times 10^3\ \text{N}$

$M_z = -(50 \times 10^3)(75 \times 10^{-3}) = -3.75 \times 10^3\ \text{N}\cdot\text{m}$

$M_y = -Pa$

$$\sigma_A = -\frac{P}{A} - \frac{M_z y_A}{I_z} + \frac{M_y z_A}{I_y} \qquad \sigma_A = -90 \times 10^6\ \text{Pa}$$

$$M_y = \frac{I_y}{z_A}\left\{\frac{M_z y_A}{I_z} + \frac{P}{A} + \sigma_A\right\}$$

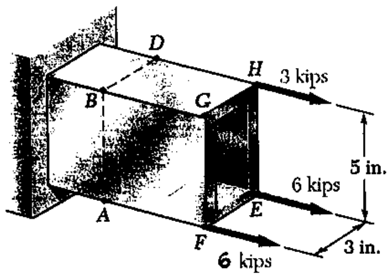
$$= \frac{1.83 \times 10^{-6}}{51 \times 10^{-3}}\left\{\frac{(-3.75 \times 10^3)(-80 \times 10^{-3})}{13.4 \times 10^{-6}} + \frac{50 \times 10^3}{3060 \times 10^{-6}} + (-90 \times 10^6)\right\}$$

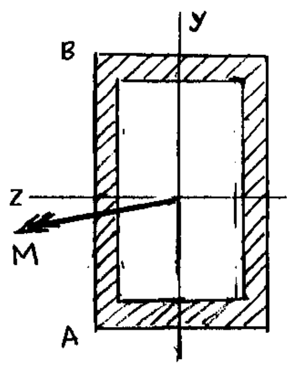
$$= \frac{1.83 \times 10^{-6}}{51 \times 10^{-3}}\left\{+22.388 + 16.340 - 90\right\} \times 10^6$$

$$= -1.8398 \times 10^3\ \text{N}\cdot\text{m}$$

$$a = -\frac{M_y}{P} = -\frac{-1.8398 \times 10^3}{50 \times 10^3} = 36.8 \times 10^{-3}\ \text{m} = 36.8\ \text{mm}$$

PROBLEM 4.162





4.162 The tube shown has a uniform wall thickness of 0.5 in. For the given loading, determine *(a)* the stress at points *A* and *B*, *(b)* the point where the neutral axis intersects line *ABD*.

SOLUTION

Add y- and z-axes as shown.

$$I_z = \frac{1}{12}(3)(5)^3 - \frac{1}{12}(2)(4)^3 = 20.583 \text{ in}^4$$

$$I_y = \frac{1}{12}(5)(3)^3 - \frac{1}{12}(4)(2)^3 = 8.5833 \text{ in}^4$$

$$A = (3)(5) - (2)(4) = 7.0 \text{ in}^2$$

Resultant force and bending couples

$$P = 3 + 6 + 6 = 15 \text{ kips}$$

$$M_z = -(2.5)(3) + (2.5)(6) + (2.5)(6) = 22.5 \text{ kip}\cdot\text{in.}$$

$$M_y = -(1.5)(3) - (1.5)(6) + (1.5)(6) = -4.5 \text{ kip}\cdot\text{in}$$

(a) $$\sigma_A = \frac{P}{A} - \frac{M_z y_A}{I_z} + \frac{M_y z_A}{I_y} = \frac{15}{7} - \frac{(22.5)(-2.5)}{20.583} + \frac{(-4.5)(1.5)}{8.5833} = 4.09 \text{ ksi}$$

$$\sigma_B = \frac{P}{A} - \frac{M_z y_B}{I_z} + \frac{M_y z_B}{I_y} = \frac{15}{7} - \frac{(22.5)(2.5)}{20.583} + \frac{(-4.5)(1.5)}{8.5833} = -1.376 \text{ ksi}$$

(b) Let point H be the point where the neutral axis intersects AB.

$$z_H = 1.5, \quad y_H = ?, \quad \sigma_H = 0$$

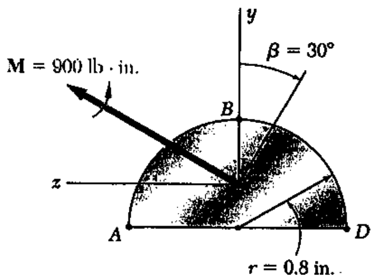
$$0 = \frac{P}{A} - \frac{M_z y_H}{I_z} + \frac{M_y z_H}{I_y}$$

$$y_H = \frac{I_z}{M_z}\left(\frac{P}{A} + \frac{M_y z_H}{I_y}\right) = \frac{20.583}{22.5}\left\{\frac{15}{7} + \frac{(-4.5)(1.5)}{8.5833}\right\} = 1.241 \text{ in.}$$

$$2.5 + 1.241 = 3.741 \text{ in.}$$

Answer: 3.741 in. above point A.

PROBLEM 4.150



4.148 through 4.150 The couple **M** is applied to a beam of the cross section shown in a plane forming an angle β with the vertical. Determine the stress at (*a*) point *A*, (*b*) point *B*, (*c*) point D.

SOLUTION

$$I_z = \frac{\pi}{8}r^4 - \left(\frac{\pi}{2}r^2\right)\left(\frac{4r}{3\pi}\right)^2 = \left(\frac{\pi}{8} - \frac{8}{9\pi}\right)r^4$$

$$= (0.109757)(0.8)^4 = 44.956 \times 10^{-3} \text{ in}^4$$

$$I_y = \frac{\pi}{8}r^4 = \frac{\pi}{8}(0.8)^4 = 160.85 \times 10^{-3} \text{ in}^4$$

$$y_A = y_D = -\frac{4r}{3\pi} = -\frac{(4)(0.8)}{3\pi} = -0.33953 \text{ in.}$$

$$y_B = 0.8 - 0.33953 = 0.46047 \text{ in.}$$

$$z_A = -z_D = 0.8 \text{ in.}, \qquad z_B = 0$$

$$M_y = 900 \sin 30° = 450 \text{ lb}\cdot\text{in}$$

$$M_z = 900 \cos 30° = 779.42 \text{ lb}\cdot\text{in.}$$

(a) $$\sigma_A = -\frac{M_z y_A}{I_z} + \frac{M_y z_A}{I_y} = -\frac{(779.42)(-0.33953)}{44.956 \times 10^{-3}} + \frac{(450)(0.8)}{160.85 \times 10^{-3}} = 8.12 \times 10^3 \text{ psi} = 8.12 \text{ ksi}$$

(b) $$\sigma_B = -\frac{M_z y_B}{I_z} + \frac{M_y z_B}{I_y} = -\frac{(779.42)(0.46047)}{44.956 \times 10^{-3}} + \frac{(450)(0)}{160.85 \times 10^{-3}} = -7.98 \times 10^3 \text{ psi} = -7.98 \text{ ksi}$$

(c) $$\sigma_D = -\frac{M_z y_D}{I_z} + \frac{M_y z_D}{I_y} = -\frac{(779.42)(-0.33953)}{44.956 \times 10^{-3}} + \frac{(450)(-0.8)}{160.85 \times 10^{-3}} = 3.65 \times 10^3 \text{ psi} = 3.65 \text{ ksi}$$