QUIZ 1

Fall 2002-2003

(Tuesday, November 19, 2002)

CIVE310 - MECHANICS OF MATERIALS

CLOSED BOOK, 1 ½ HOURS

Name: Key Key **ID#:** 3000-00000

NOTES

- 3 PROBLEMS – 11 PAGES.
- ALL YOUR ANSWERS SHOULD BE PROVIDED ON THE QUESTION SHEETS.
- ONE EXTRA SHEET IS PROVIDED AT THE END.
- ASK FOR ADDITIONAL SHEETS IF YOU NEED MORE SPACE.
- SOME ANSWERS MAY REQUIRE MUCH LESS THAN THE SPACE PROVIDED.
- ***DO NOT*** USE THE BACK OF THE SHEETS FOR ANSWERS.
- DRAFT BOOKLET WILL BE PROVIDED; BUT DO NOT USE FOR ANSWERS.
- BOTH QUESTION SHEETS AND DRAFT BOOKLET SHOULD BE RETURNED.

YOUR COMMENT(S)

_ _

_ _

DO NOT WRITE IN THE SPACE BELOW

MY COMMENT(S)

_ _

_ _

YOUR GRADE

Problem I: _ _ _ /40
Problem II: _ _ _ /30
Problem III: _ _ _ /30
Other: _ _ _

TOTAL: /100

Problem I: *(40 points)*

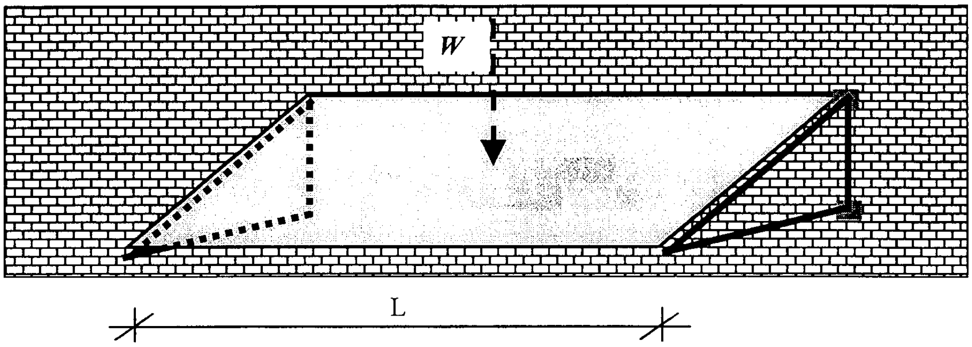


Figure I-a

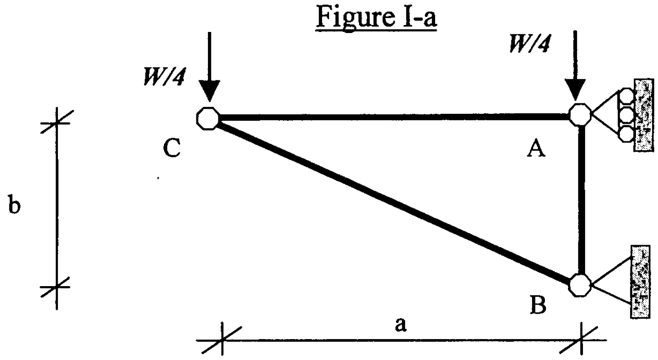


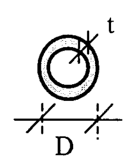
Figure I-b

The truss modeled and shown in Figure I-b is used to hold a flat horizontal cover of a shop-front (Figure I-a). The total weight of the cover W is carried by each of the two trusses as shown in Figure I-b. It is also possible for the cover to be loaded with snow.
Assume linear elastic behavior and ***be careful with units***.

The ***dimensions*** of the system are given as follows:

- $L = 4$ m $a = 2$ m $b = 1$ m
- Tubular steel cross-sections of truss bars

	Bar	AB	BC	AC
Outside Diameter	D (mm)	30	40	20
Thickness (same)	t (mm)	3	3	3



The ***properties*** of the system are given as follows:

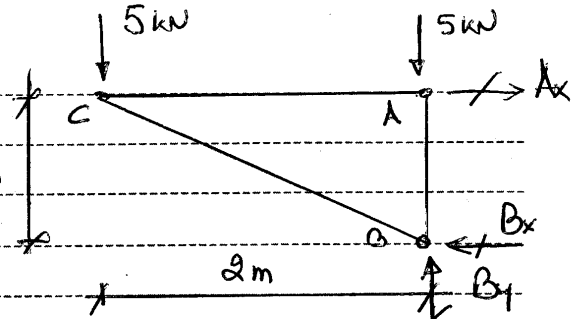
- $\sigma_Y = 300{,}000$ kPa (kN/m^2): Yield stress in tension and compression of steel tubes
- FS : Factor of safety

The ***loading/weight*** on the system are given as follows:

- $W = 20$ kN : Total weight of cover (about 2 Tons)
- $\gamma = 5$ kN/m³ : Weight density of snow (about ½ of water)
- h : Height of snow uniformly distributed above cover
- ***Own weight of truss bars is negligible***

1. Assume ***NO*** snow on cover.
   Discuss the safety of the truss system and briefly comment, assuming that the cover itself is safe. *(15 points)*
   Are the bars properly arranged (in terms of cross-sections), and, if not, suggest a more proper arrangement of cross-sections, and briefly justify your choice. *(10 points)*

Calculations and/or Diagrams:

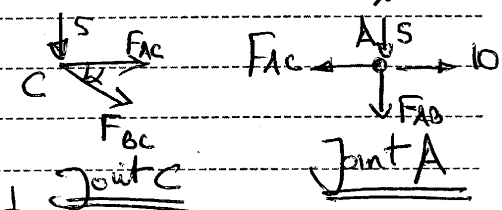


$W = 20\,kN$

$B_y = 10\,kN$ ↑

$B_x = 10\,kN$ ←

$A_x = 10\,kN$ →



Joint A: $F_{AC} = \oplus 10\,kN\ (T)$

$F_{AB} = \ominus 5\,kN\ (C)$

Joint C: $\cos\alpha = \frac{2}{\sqrt{5}}$ $\sin\alpha = \frac{1}{\sqrt{5}}$

$F_{BC} \sin\alpha = -5 \Rightarrow F_{BC} = \ominus 11.2\,kN\ (C)$

Areas: $A = \frac{\pi}{4}(D_{out}^2 - D_{in}^2)$

$D_{in} = D_{out} - 2t$

$A_{AB} = \frac{\pi}{4}(30^2 - 24^2) = 254.5\,mm^2$

$A_{BC} = \frac{\pi}{4}(40^2 - 34^2)\ 348.7\,mm^2$

$A_{AC} = \frac{\pi}{4}(20^2 - 14^2) = 160.2\,mm^2$

$\rightarrow \times 10^{-6}\,m^2$

Stresses $\sigma_{AB} = \frac{F_{AB}}{A_{AB}} = \frac{5}{254.5 \times 10^{-6}} = 19{,}646\,kPa$ (C)

$\sigma_{AC} = \frac{F_{AC}}{A_{AC}} = \frac{10}{160.2 \times 10^{-6}} = 62{,}422\,kPa$ (T)

$\sigma_{BC} = \frac{F_{BC}}{A_{BC}} = \frac{11.2}{348.7 \times 10^{-6}} = 32{,}119\,kPa$ (C)

Safety $FS_{AB} = \frac{\sigma_y}{\sigma_{AB}} = 15.2$ $FS_{AC} = \frac{\sigma_y}{\sigma_{AC}} = 4.81$ $FS_{BC} = 9.34$

Calculations and/or Diagrams (cont'd):

Safety is OK → more than enough
This should be expected under the own
weight of the system (only).

Arrangement: While safe, but safety varies
between bars → better be within same
range (4.81 → 15.2)

SO: Rearrange as follows:
Bigger Force ⇒ Bigger Area.

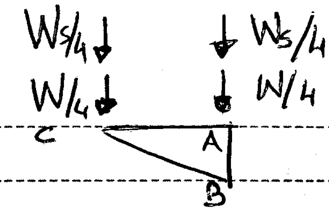
				Change with
Smallest AB →	20/3	⇒ FS =	9.61	(AC)
Medium AC →	30/3	⇒ FS =	7.63	(AB)
Largest BC	40/3	⇒ FS =	9.34	Same

Closer

7

2. The factor of safety is set at FS=1.5.
   With the original arrangement of bar cross-sections given on page 2, determine the height *h* of a uniformly distributed snow above the cover that the truss is allowed to carry, assuming that the cover remains safe. *(15 points)*

Calculations and/or Diagrams:



Snow on top of cover (uniform)
→ Total Weight W_S -

Most Critical Bar is AC (with Original Areas) from 1st question FS

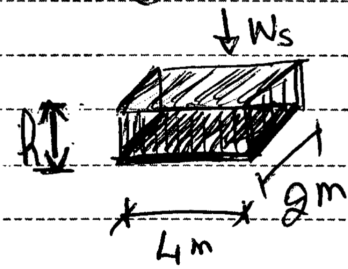
$F_{AC} = 10kN\,(T)$ for $W/4 = 5kN$ → $\sigma_{AC} = 62,422$
$F_{AC} = ?$ for $(W/4 + W_s/4)$ → $\sigma_{AC} = ?$

$$\Rightarrow \sigma_{AC} = 62,422 \times \frac{(5 + W_s/4)}{5}$$

$$FS = 1.5 = \frac{300,000}{62,422\left(1 + \frac{W_s}{20}\right)} \Rightarrow W_s = 44.1\ kN$$ (Total Snow)

$W_s = \underbrace{\gamma}_{5kN/m^3} \times 4 \times 2 \times h = 44.1 kN$

$\Rightarrow h = 1.1\ m$



Allowable $h_{snow} = 1.1\ m$

Problem II: *(30 points)*

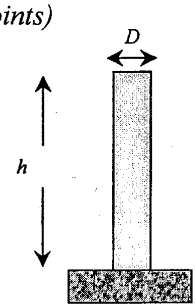
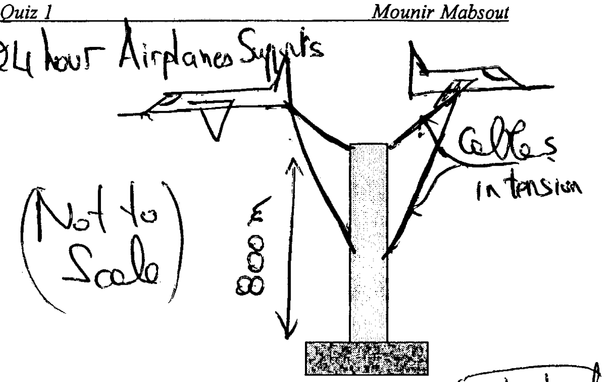


Figure II



Your Sketch (Use back of page if you wi

The free-standing circular solid column monument shown in Figure II has a height h and diameter D is made of concrete.

The ***properties*** of concrete are as follows:

- $E = 25 \times 10^6$ kPa (kN/m^2) : Modulus of elasticity
- $\gamma = 25$ kN/m^3 : Weight density of concrete (about 2.5 water)
- $\sigma_{uc} = 30{,}000$ kPa : Ultimate strength in compression
- $FS = 1.5$: Factor of safety

For D=1 m

Determine the allowable height h of the column so that it is safe under its own weight, and compute the corresponding top displacement. Does your result for height make sense? What problems do you foresee if you decide to build this column and suggest a solution to hold it (think of lateral stability/buckling, make a sketch (above), use your imagination - you may use fantasy and be UN-realistic in your solution).

What do you expect for h if the problem is turned upside down (much higher, higher, same, lower, much lower). Give an estimate and briefly explain. *(22 points)*

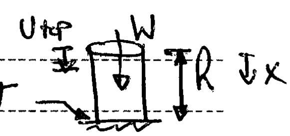
For D=100 m

How would your result for h and top displacement change if the diameter is 100 m? Briefly explain. Do you think you need your imaginary solution here? Briefly explain. *(8 points)*

Calculations and/or Diagrams:

$D = 1$ m

$\sigma_{max} = \dfrac{W}{A} = \dfrac{\gamma \cancel{A} h}{\cancel{A}} = \gamma h = \sigma_{allow}$ } Max σ



$h = \dfrac{\sigma_{uc}/FS}{\gamma} = \dfrac{30{,}000/1.5}{25} = 800$ m ; $\dfrac{D}{h} = \dfrac{1}{800}$

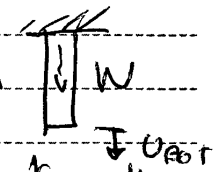
→ Column is very high but expected (own weight should have small effect)

→ Problems = Lateral Buckling will occur.

$U(x) - U_{top} = \int_0^h \dfrac{\gamma \cancel{A} x}{E \cancel{A}}\, dx = -\dfrac{\gamma h^2}{2E} \Rightarrow U_{top} = 0.32$ m (32 cm) ↓

Calculations and/or Diagrams (cont'd):

Upside Down: "Same problem" but Tension

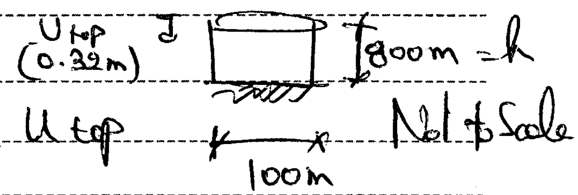


However I expect that ultimate strength in compression is much more than in tension

$|\sigma_{UC}| >> |\sigma_{UT}|$ (≃10 times larger)

So $h_{allowable}$ will be much smaller/lower under own weight (≃ 80 m)

• D = 100m



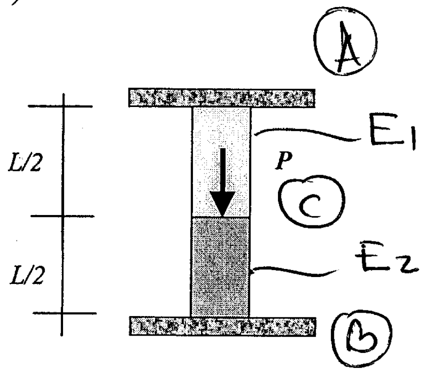
No effect on h and U_{top} since A "cancels".
(More Force on more area)

$$\frac{D}{h} = \frac{100}{800} = \frac{1}{8}$$ Reasonable ratio

"Stable" on its own, No need for "airplanes".

Calculations and/or Diagrams (cont'd):

Problem III: *(30 points)*



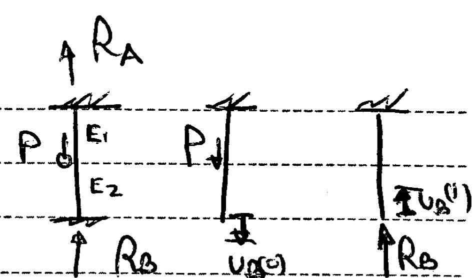
Same A

Figure III

The axial bar of constant cross-section area A. shown in Figure III, is assumed "weightless", and is made of two equal segments for a total length *L*. The upper and lower parts have a modulus of elasticity E_1 and E_2, respectively. The bar is loaded at mid height with a load *P*. Assume linear elastic behavior.

Compute the top and bottom reactions and the displacement at mid height and compare reactions for the case with case of $E_1 = E_2$ with the case when $E_1 > E_2$ and briefly comment. (20 points) What happens to the reactions and mid-height displacement when E_1 (upper part) becomes very large, i.e. $E_1 >>> E_2$. Briefly explain. (10 points)

Calculations and/or Diagrams:



Primary:

$u_B^{(0)} = \frac{P(L/2)}{E_1 A}$ ↓ $\quad$ (X↓)

~~$-u_A^{(0)}$~~

Secondary:

$u_B^{(1)} = -\frac{(R_B L/2)}{E_2 A} - \frac{R_B L/2}{E_1 A}$

~~$-u_A^{(1)}$~~

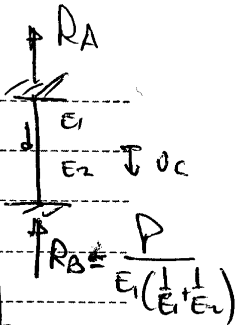
$\Rightarrow u_B^{(1)} = -\frac{R_B L}{2A}\left(\frac{1}{E_1} + \frac{1}{E_2}\right)$ ↑

$u_B^{(0)} + u_B^{(1)} = 0 \quad (\text{or } |u_B^{(0)}| = |u_B^{(1)}|)$

$\Rightarrow R_B = \frac{P}{\left(1 + \frac{E_1}{E_2}\right)}$ (↑) $\quad \Rightarrow R_A = \frac{P}{\left(1 + \frac{E_2}{E_1}\right)}$ (↑)

$= \frac{P}{E_1\left(\frac{1}{E_1} + \frac{1}{E_2}\right)} \quad = \frac{P}{E_2\left(\frac{1}{E_1} + \frac{1}{E_2}\right)}$

Calculations and/or Diagrams (cont'd):



Actual $\cancel{u_B}^{0} - v_C = \frac{FL/2}{E_2 A}$

$$F = \frac{P}{E_1(\frac{1}{E_1}+\frac{1}{E_2})}$$

$$u_C = + \frac{PL}{2E_1E_2(\frac{1}{E_1}+\frac{1}{E_2})A} = \frac{PL}{2(E_1+E_2)A} \downarrow$$

Compare
- $E_1 = E_2 \ (= E)$ $\quad R_A = \frac{P}{2} \ (\uparrow)$ $\quad R_B = \frac{P}{2} \ (\uparrow)$ $\quad \left(v_C = \frac{PL}{4EA}\right)$
- $E_1 > E_2$ $\quad R_A > R_B$ $\Rightarrow$ Stiffer top portion takes more reaction.

$E_1 \gg E_2$ $\quad R_B \simeq 0$ $\quad R_A \simeq P$

↓ Upper Portion Reaction carries all (P) the load

$v_C \simeq$ very small since E_1 (very large) will not allow it to move down. ($\simeq 0$).

EXTRA SHEET: Continued from page ______

Name: ____________________ **ID#:** ______________

Calculations and/or Diagrams: