Math 210, Introduction to Analysis- Fall 2016-2017 Quiz 1 , October 8.
Time 70 minutes

Question 1. (15 pts.)

(i) Let X be a metric space. Define what it means for a set $A \subset X$ to be compact.

(ii) Prove using the definition of limit(using ϵ and n_0) that $\lim \sqrt[n]{5} = 1$ as $n \longrightarrow \infty$ in $\mathbb{R}$ with the usual metric.

(iii) Let $A = \{1, 5, \sqrt{5}, \sqrt[2]{5}, \dots, \sqrt[n]{5}, \dots\}$. Prove that A is compact in $\mathbb{R}$ using the definition.

Quiestion 2. (10 pts) Find with justification $\limsup$ and $\liminf$ of the sequence defined by

$$x_n = (-1)^n + \sqrt[n]{3}.$$

Question 3. (15 pts)

a) Define what it means for an ordered set S to have the least upper bound property.

b) Let $A = [0, \sqrt{7}] \cap \mathbb{Q}$. Find, if it exists, $\sup A$ in $\mathbb{Q}$, and in $\mathbb{R}$. You need to justify your answers, whether the sup exists or not.

c) Does $\mathbb{Q}$ have the least upper bound property? Why?

Question 4. (15 pts) Find in $\mathbb{R}^2$ the limit of

$$\lim_{n \longrightarrow \infty} \left(\frac{\sin(n^2) + (-1)^n n}{n^3 + 1}, \frac{2\sqrt{n-3}}{\sqrt{n} + 4} \right)$$

(You may use Calculus to find the limit).Prove using the definition (ϵ, n_0) that this is indeed the limit in $(\mathbb{R}^2, d_\infty)$.

Question 5. (15 pts).

a) For a general metric space (X, d), define the following

(i) x is a Limit point of a set $E \subset X$.

(ii) The closure of E (i.e $\overline{E}$).

b) In $(\mathbb{R}^2, d_1)$, let $E = \{(x, y) \in \mathbb{R}^2 \text{ s.t } x < 1\}$. Find $\overline{E}$ with justification .