Math 210-1, Fall 2009

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Quiz 1, November 7, 40 points

1. (10 points) Let A and B be two bounded sets of real numbers, and assume that there exists an $\varepsilon > 0$ such that $b \geq \varepsilon$ for all $b \in B$. Define

$$\frac{A}{B} := \left\{ \frac{a}{b} \,\middle|\, a \in A,\, b \in B \right\}.$$

Show that

$$\sup \frac{A}{B} = \frac{\sup A}{\inf B}.$$

2. (10 points) Let $a_1 = a_2 = 5$ and

$$a_{n+1} = a_n + 6a_{n-1}, \quad (n \geq 2).$$

Show by induction that $a_n = 3^n - (-2)^n$ if $n \geq 1$.

3. (10 points) Prove that for arbitrary sets S_1, S_2 on $\mathbb{R}$,

$$\overline{S_1 \cup S_2} = \overline{S_1} \cup \overline{S_2}.$$

Hint: Use the *known* fact that for every set S of real numbers,

$$\overline{S} = S \cup \{x | x \text{ is a limit point of S }\}.$$

4. Find S^0, ∂S, and $\overline{S}$ for each one of the following sets:

(a) (5 points)

$$S = \bigcup_{n=1}^{+\infty} \left[\frac{1}{2n}, \frac{1}{2n-1}\right).$$

(b) (5 points) $S = [0, 1] \setminus \mathbb{Q}$.