MECH 230
DYNAMICS

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PREVIOUS FINAL EXAM SOLUTION
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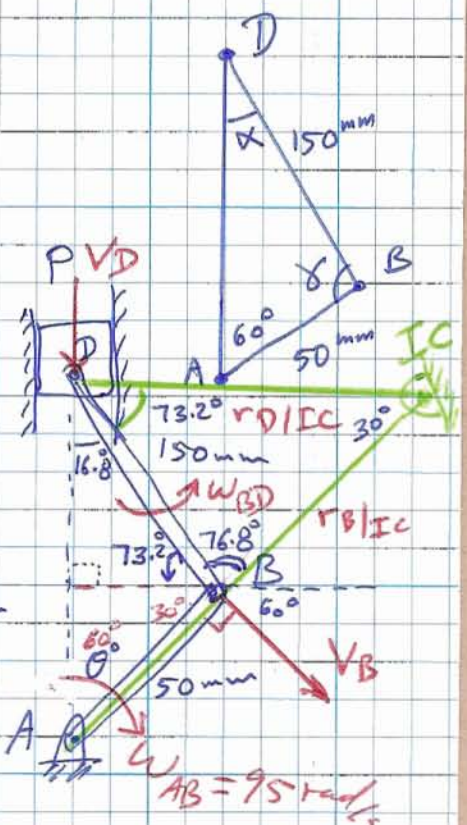
PROBLEM 1:

$\omega_{AB} = 95$ rad/s ↻ Constant $\Rightarrow \alpha_{AB} = 0$



1. $\frac{\sin\alpha}{50} = \frac{\sin 60°}{150}$; $\alpha = 16.78°$

2. Locate IC of zero velocity of BD:

$\overline{AD} = 150\cos 16.8° + 50\cos 60° = 168.598$ mm

$(50 + r_{B/IC})\sin 30° = 168.598$; $r_{B/IC} = 287.196$ mm

$\frac{r_{D/IC}}{\sin 76.8°} = \frac{287.196}{\sin 73.2°}$; $r_{D/IC} = 292.07$ mm

Link AB:

$V_B = \omega_{AB} r_{B/A} = 95 \frac{\text{rad}}{\text{s}}(50\text{ mm}) = 4750 \text{ mm/s} = 4.75 \text{ m/s}$

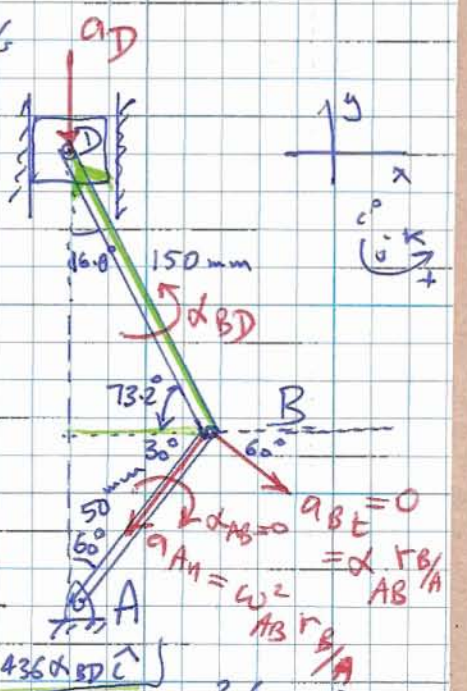
Link BD:

$\omega_{BD} r_{B/IC} = V_B$; $\omega_{BD} = \frac{V_B}{r_{B/IC}} = \frac{4750 \text{ mm/s}}{287.196 \text{ mm}}$

$\omega_{BD} = 16.539$ rad/s ↻ Ans.

3. $\vec{a}_D = \vec{a}_B + \vec{\alpha}_{BD} \times \vec{r}_{D/B} - \omega_{BD}^2 \vec{r}_{D/B}$

$-a_D\hat{j} = -451.25\cos 30°\,\hat{i} - 451.25\sin 30°\,\hat{j}$
$+ \alpha_{BD}\hat{k} \times (-0.15\cos 73.2°\,\hat{i} + 0.15\sin 73.2°\,\hat{j})$
$- (16.539)^2(-0.04335\,\hat{i} + 0.1436\,\hat{j})$



$-a_D\hat{j} = -390.794\,\hat{i} - 225.625\,\hat{j} - 0.04335\alpha_{BD}\hat{j} - 0.1436\alpha_{BD}\hat{i}$
$+ 11.858\,\hat{i} - 39.28\,\hat{j}$

$a_{A_n} = 95^2(0.05) = 451.25 \text{ m/s}^2$

$\hat{i}$: $0 = -390.794 - 0.1436\alpha_{BD} + 11.858$; $\alpha_{BD} = -2638.83$ rad/s²

$\hat{j}$: $-a_D = -225.625 - 0.04335\alpha_{BD} - 39.28$; $\alpha_{BD} = 2638.83$ rad/s² ↻

$-a_D = -225.625 - 0.04335(-2638.83) - 39.28$

$a_D = 150.512 \text{ m/s}^2$ Ans.

Ans.

PROBLEM 2 :

1. Given $m_{rod} = 4\ kg$

2. $\vec{a}_A = \vec{a}_G + \vec{\alpha}_{AD} \times \vec{r}_{A/G} - \omega^2_{AD} \vec{r}_{A/G}$ (O)

$-a_A \hat{j} = 3.6\hat{i} - 1.5\hat{j} + \alpha_{AD}\hat{k} \times (0.1\hat{i} - 0.1732\hat{j})$

$-a_A \hat{j} = 3.6\hat{i} - 1.5\hat{j} + 0.1\alpha_{AD}\hat{j} + 0.1732\alpha_{AD}\hat{i}$

$\hat{i}: \ 0 = 3.6 + 0.1732\alpha_{AD} \ ; \ \alpha_{AD} = -20.785\ rad/s^2$

$\hat{j}: \ -a_A = -1.5 + 0.1\alpha_{AD} \ ; \ -a_A = -1.5 - 0.1(20.785)$

$a_A = 3.5785\ m/s^2 \downarrow$

$\alpha_{AD} = 20.785\ rad/s^2 \circlearrowleft$ Ans.

$a_{BN} = \omega^2_{BC} r_{B/C} = 36(0.1) = 3.6\ m/s^2$

$\omega_{BC} = 6\ rad/s$

$\alpha_{BC} = 15\ rad/s^2$

$a_{Bt} = \alpha_{BC} r_{B/C} = 15(0.1) = 1.5\ m/s^2$

3. FBD of ABD

K.D of ABD

$4(9.81)\ N$, B_x, B_y, N_A

$I_G\alpha$, $ma_{Gx} = 4(3.6)\ N$, $ma_{Gy} = 4(1.5)\ N$

4. $\circlearrowleft \Sigma M_G = I_G\alpha$;

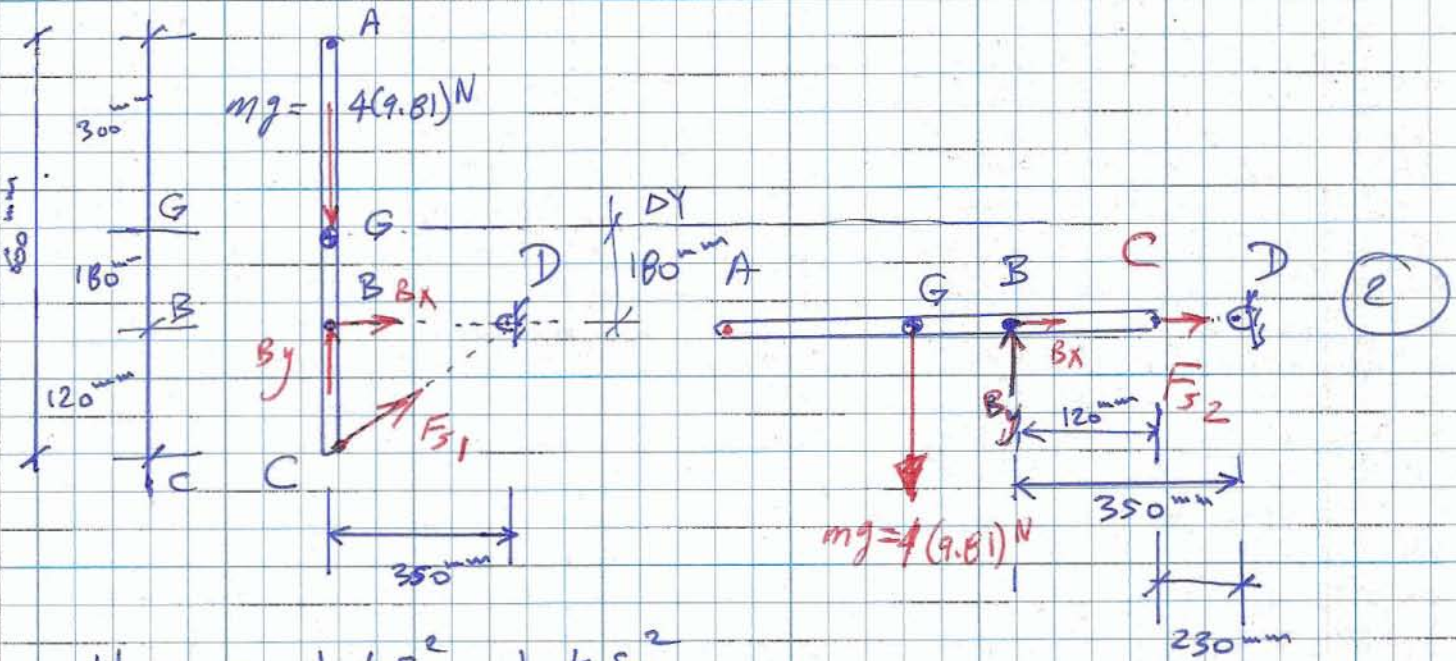
$-N_A(0.1732\ m) = -I_G\alpha_{ABD}$

$I_G = \frac{1}{12}mL^2 = \frac{1}{12}(4)(0.4)^2 = 0.053\ kg.m^2$

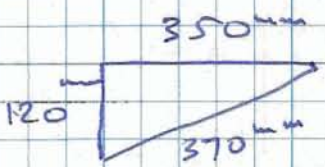
$N_A = \frac{0.053}{0.1732}(20.785) = 6.4\ N \leftarrow$ Ans.

PROBLEM 3 $m_{rod} = 4\,kg$; $K = 400\,N/m$; $l_o = 150\,mm$

1. ① rest $\Delta\theta = 90°$



2. $U_{1-2}_{spring} = \frac{1}{2}kS_1^2 - \frac{1}{2}kS_2^2$



$S_1 = \Delta l_1 = l_{f_1} - l_o =$

$= 370 - 150 = 220\,mm$

$S_2 = \Delta l_2 = l_{f_2} - l_o = 230 - 150 = 80\,mm$

$U_{1-2}_{spring} = \frac{1}{2}(400)(0.22)^2 - \frac{1}{2}400(0.08)^2 = 9.68 - 1.28$

$U_{1-2}_{spring} = 8.4\,J$ Ans.

3. $U_{1-2}_{W} = mg\Delta Y = 4(9.81)(0.18)$

$= 7.063\,J$ Ans.

4. Find ω_{rod}. $T_1 + \Sigma U_{1-2} = T_2$; $T_1 = 0$ rest ; $T_2 = \frac{1}{2}I_B\omega^2$

$I_B = I_G + md^2 = \frac{1}{12}ml^2 + md^2 = \frac{1}{12}(4)(0.6)^2 + 4(0.18)^2$

$I_B = 0.2496\,kg.m^2$

$0 + 8.4 + 7.063 = \frac{1}{2}(0.2496)\omega^2$; $\omega^2 = 123.902$

$\omega = 11.131\,rad/s$ ↻ Ans.

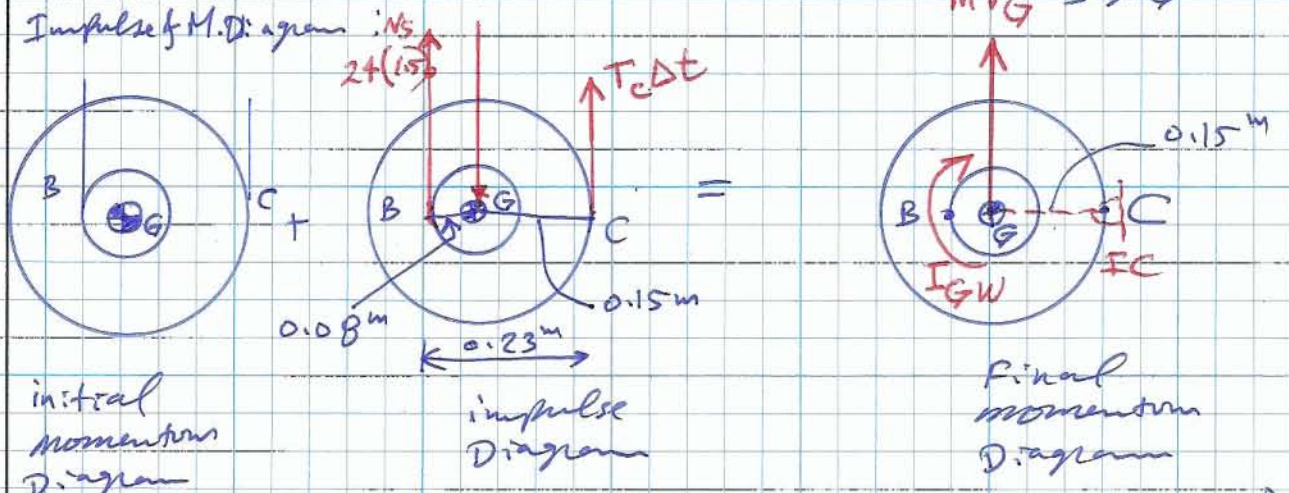
PROBLEM 4

$m = 3\,kg$; $k_G = 0.1\,m$; $P = 24\,N$

① rest

$mg\Delta t = 3(9.81)(1.5)^{N.s}$

1. Impulse of M.D. diagram :



initial momentum D.agram | impulse Diagram | Final momentum D.agram

$I_G = mk_G^2 = 3(0.1^2) = 0.03\ kg.m^2$

2. Find V_G @ $\Delta t = 1.5\,s$

$V_G = \omega r_{G/IC} = 0.15\omega$; $\omega = \frac{V_G}{0.15}$

$\overset{+}{\circlearrowleft} \Sigma(moment)_C$: $24^{N}(1.5)^{s}(0.23^{m}) - 3(9.81)^{N}(1.5^{s})(0.15^{m}) = 3V_G(0.15^{m}) + I_G\omega$

$8.28^{N.s.m} - 6.62175^{N.s.m} = 3(0.15)V_G + 0.03\left(\frac{V_G}{0.15}\right)^{kg.m^2\ m/s}$

$1.65825 = 0.45V_G + 0.2V_G$

$1.65825 = 0.65V_G$; $V_G = 2.551\ m/s \uparrow$ Ans.

3. Find T_C

$\overset{+}{\uparrow} \Sigma(vectors)_y$: $0 + T_C(1.5^{s}) + 24^{N.s}(1.5) - 3(9.81)^{N}(1.5)^{s} = 3^{kg.}(2.551)^{m/s}$

$T_C = 10.531\ N$ Ans.