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AMERICAN UNIVERSITY OF BEIRUT
FACULTY OF ENGINEERING AND ARCHITECTURE
MECH230 DYNAMICS- QUIZ 2

W. NAJEM
J. KASAMANI

MAY 2, 2009

NAME: KEY ID#: KEY

90 MINUTES CLOSED BOOK AND NOTES QUIZ

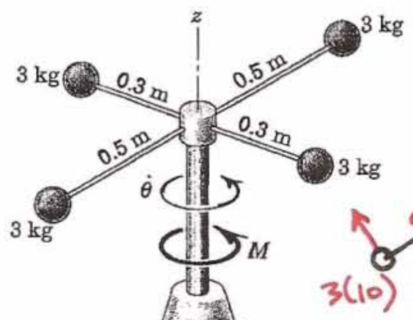
1. ANSWER ALL THE FOLLOWING QUESTIONS IN THE GIVEN SPACE ON THIS QUESTION BOOKLET.
2. THE SCRATCH BOOKLET WILL NOT BE COLLECTED AND HENCE WILL NOT BE GRADED.
3. CLARITY AND NEATNESS ARE IMPORTANT IN GRADING.

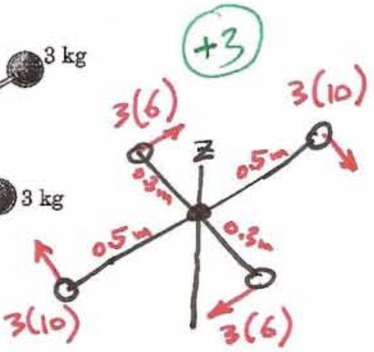
PROBLEM 1: 20%

The four 3-kg balls are rigidly mounted to the rotating frame and shaft, which are initially rotating freely about the vertical z-axis at the angular rate of 20 rad/s clockwise when viewed from above. If a constant torque M= 30 N.m is applied to the shaft, **calculate the time t to reverse the direction of rotation and reach an angular velocity $\dot{\theta}$=20 rad/s in the same sense as M. (10 pts.)**

Note first draw impulse and momentum diagram (10 pts.)

$V_\theta = r\dot{\theta} = 0.3(20) = 6$ m/s
$= 0.5(20) = 10$ m/s





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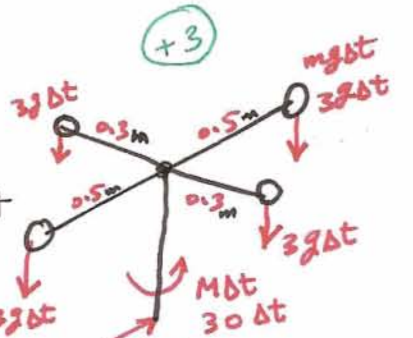

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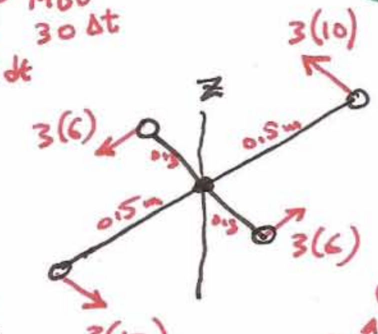

mV → (-1)

↑ ΣMoments : (+3) (+3) (+3)

$-2[0.3(3)(6)] - 2[0.5(3)(10)] + 30\Delta t = 2[0.3(3)(6)] + 2[0.5(3)(10)]$

(rmV, rmV, MΔt, rmV')

$-10.8 - 30 + 30\Delta t = 10.8 + 30$

$-40.8 \qquad\qquad 40.8$

$30\Delta t = 40.8 + 40.8 = 81.6$

$\Delta t = \frac{81.6}{30} = 2.72$ s

(+1)

$\boxed{\Delta t = 2.72 \text{ s}}$ Ans.

absolutely

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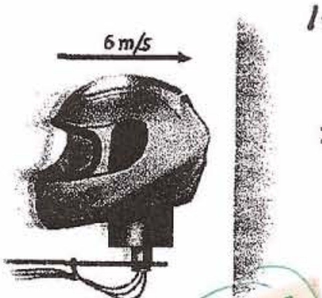
PROBLEM 2: 30%

A bioengineer studying helmet design uses an experimental apparatus that launches a 2.4-kg helmet containing a 2-kg model of the human head against a rigid surface at 6 m/s. The head, suspended within the helmet, is not immediately affected by the impact of the helmet with the surface and continues to move to the right at 6 m/s, so the head then undergoes an impact with the helmet.

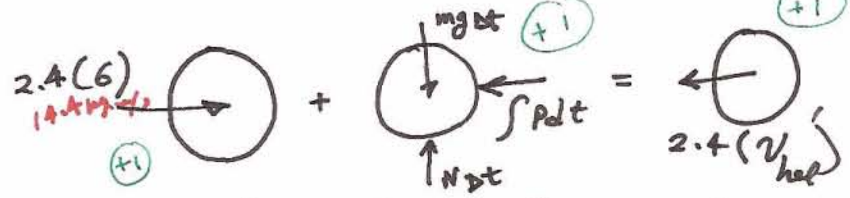
Part one:

(+5) If the coefficient of restitution of the helmet's impact with the surface is 0.85 and the coefficient of restitution of the subsequent impact of the head with the helmet is 0.15, **what is the velocity of the head after its initial impact with the helmet? (10 pts.)**

Note first draw the necessary impulse and momentum diagrams. (5 pts)



1. Consider helmet: $m_{hel} = 2.4\,kg$ $e = 0.85$ (+1)



$e = \frac{V'_{hel\,n} - V'_{wall\,n}}{V_{n\,wall} - V_{Hel\,n}}$; $0.85 = \frac{V'_{Hel\,n} - 0}{0 - (-6)}$

$V'_{Hel_n} = 0.85(6) = 5.1\ m/s$ ← (+3)

2. Consider Head & Helmet (+1) $m_{Head} = 2\,kg$ (+1)
$e = 0.15$

$\xrightarrow{+}$ $e = \frac{V'_{Hd} - V'_{hel}}{V_{hel} - V_{Hd}}$; $0.15 = \frac{V'_{Hd} - V'_{hel}}{-5.1 - 6}$

$0.15(-11.1) = V'_{Hd} - V'_{hel}$; $V'_{Hd} - V'_{hel} = -1.665$ (+3) eq. (1)

(-2 for diagram)

$\xrightarrow{+} \Sigma(\text{vectors})_x$: $2(6) - 2.4(5.1) = 2V'_{Hd} + 2.4V'_{hel}$

$-0.24 = 2V'_{Hd} + 2.4V'_{hel}$ (+3) eq. (2)

$$\left.\begin{aligned} 2V'_{Hd} + 2.4V'_{hel} &= -0.24 \\ V'_{Hd} - V'_{hel} &= -1.665 \end{aligned}\right] \text{ solve:}$$ (+1) (+10)

$V'_{Hd} = -0.9627\ m/s$ ⇒ $\boxed{V'_{Hd} = 0.9627\ m/s \leftarrow \text{ to the left}}$ Head

$V'_{hel} = 0.7023\ m/s$ →

Ans.

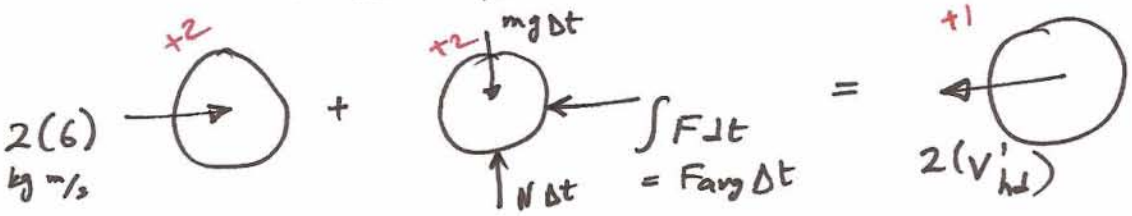
PROBLEM 2 CONTINUED

PART 2

Suppose that the simulated head alone strikes the rigid surface at **6** *m/s, the coefficient of restitution is* 0.5, and the duration of the impact is 0.002 s. **What is the magnitude of the average force exerted on the head by the impact?** (10 pts.)

Note first draw the impulse and momentum diagram. **(5 pts.)**

$e = 0.5$, $V_{Hd} = 6\,m/s \rightarrow$; $\Delta t = 0.002\,s$ Find F_{avg}.



$\overset{+}{\leftarrow}$ $e = \dfrac{V'_{hd} - V'_{wall}}{V_{wall} - V_{hd}}$; $0.5 = \dfrac{V'_{hd} - 0}{0 - (-6)}$; $V'_{hd} = 6(0.5)$

$V'_{hd} = 3\,m/s \leftarrow$

×5

$V'_{head} = 3\,m/s \leftarrow$

$\overset{+}{\leftarrow}$ $\Sigma(\text{vectors})_x: \ -2(6) + F_{avg}(0.002) = 2(3)$

$0.002\,F_{avg} = 6 + 12 = 18$

$F_{avg} = \dfrac{18}{0.002} = 9000\,N$

×5

$\therefore F_{avg} = 9\,kN$ Ans

$6\,m/s \times \dfrac{3600}{1000} = 21.6\,km/h$

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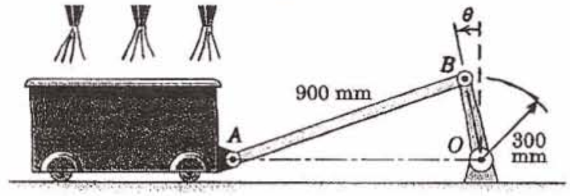
PROBLEM 3: 25%

In the design of a produce-processing plant, roller trays of produce are to be oscillated under water spray by the action of the connecting link AB and crank OB. For the instant when $\theta = 15°$, the angular velocity of AB is 0.086 rad/s clockwise.

Draw the velocity kinematic diagram of the mechanism. (5pts.)

Find the corresponding angular velocity of the crank and the velocity $\vec{v}_A$ of the tray. (20 pts.) method (+5)



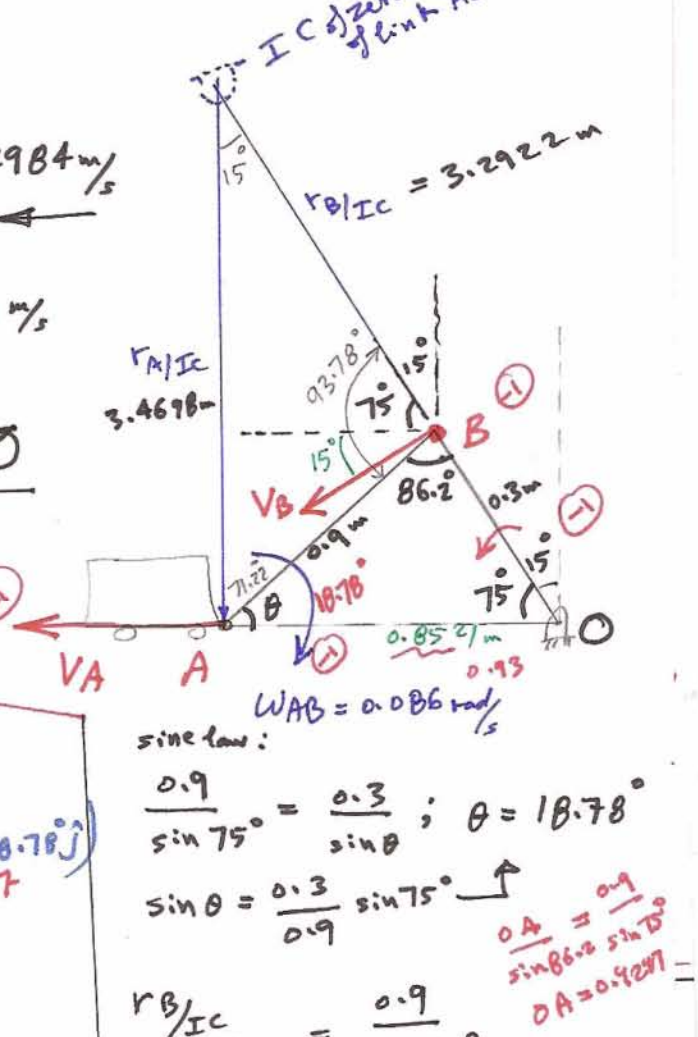
$V_A = \omega_{AB}\, r_{A/IC} = 0.086(3.4698) = 0.2984$ m/s

$\vec{V}_A = -0.2984\ \hat{i}$ m/s Ans. (+10)

$V_B = \omega_{AB}\, r_{B/IC} = 0.086(3.2922) = 0.28313$ m/s

$\omega_{OB} = \dfrac{V_B}{r_{B/O}} = \dfrac{0.28313}{0.3} = 0.9438$ rad/s ↻

$\omega_{OB} = 0.9438\ \hat{K}$ rad/s Ans. (+10)



OR: $V_B = V_A + \vec{\omega}_{AB} \times \vec{r}_{B/A}$

$-V_B\cos 15°\,\hat{i} - V_B\sin 15°\,\hat{j} = -V_A\,\hat{i} + -0.086\,\hat{k} \times (0.9\cos 18.78°\,\hat{i} + 0.9\sin 18.78°\,\hat{j})$ [0.8521, 0.2897]

$= -V_A\,\hat{i} - 0.0733\,\hat{j} + 0.0249\,\hat{i}$

$\hat{j}$: $-V_B\sin 15° = -0.0733$; $V_B = 0.2832$ m/s

$\hat{i}$: $-V_B\cos 15° = -V_A + 0.0249$

$-0.2832\cos 15° - 0.0249 = -V_A$

$V_A = 0.29845$ m/s ← Ans.

$\omega_{OB} = \dfrac{V_B}{r_{B/O}} = \dfrac{0.2832}{0.3} = 0.944$ rad/s ↻ Ans.

sine law:

$\dfrac{0.9}{\sin 75°} = \dfrac{0.3}{\sin\theta}$; $\theta = 18.78°$

$\sin\theta = \dfrac{0.3}{0.9}\sin 75°$

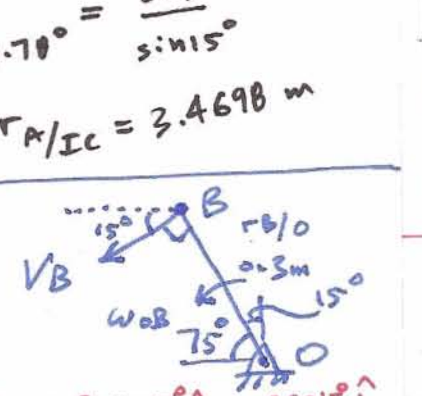
$\dfrac{OA}{\sin 86.2} = \dfrac{0.9}{\sin 75°}$; $OA = 0.9297$

$\dfrac{r_{B/IC}}{\sin 71.22°} = \dfrac{0.9}{\sin 15°}$

$r_{B/IC} = 3.2922$ m

$\dfrac{r_{A/IC}}{\sin 93.78°} = \dfrac{0.9}{\sin 15°}$

$r_{A/IC} = 3.4698$ m



$\vec{r}_{B/O} = -0.3\sin 15°\,\hat{i} + 0.3\cos 15°\,\hat{j}$

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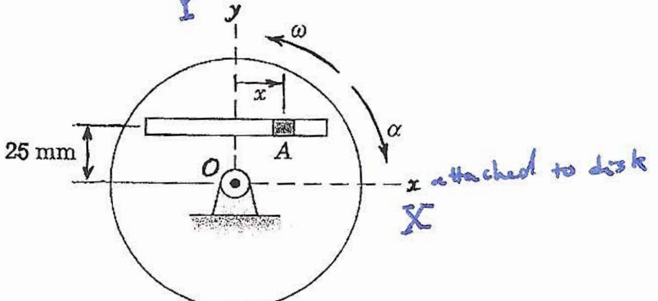
PROBLEM 4: 25%

The disk rotates about a fixed axis through O with angular velocity $\omega = 5\ rad/s$ ccw and angular acceleration $\alpha = 3\ rad/s^2$ *cw at the instant represented, in the directions shown. The slider A moves in* the straight slot.

a. **Determine the absolute velocity and acceleration of A for the same instant, when** $x = 36mm,\ \dot{x} = -100mm/s,\ \ddot{x} = 150\ mm/s^2$ **(20 pts.)**

b. **Find the Coriolis component of acceleration of slider A for the instant represented (5pts.)**



$$\vec{V}_A = \vec{V}_O + \vec{\Omega} \times \vec{r}_{A/O} + (\vec{V}_{A/O})_{xyz}$$

$$\vec{a}_A = \vec{a}_O + \dot{\vec{\Omega}} \times \vec{r}_{A/O} - \Omega^2 \vec{r}_{A/O} + 2\vec{\Omega} \times (\vec{V}_{A/O})_{xyz} + (\vec{a}_{A/O})_{xyz}$$

motion of rot. reference

$\vec{V}_O = \vec{0}$ Fixed

$\vec{a}_O = \vec{0}$ Fixed

$\vec{\Omega} = 5\hat{k}$ rad/s

$\dot{\vec{\Omega}} = -3\hat{k}$ rad/s²

motion of A w.r.t rot. reference

$\vec{r}_{A/O} = 36\hat{i} + 25\hat{j}$ mm — $0.036\hat{i} + 0.025\hat{j}$ m

$(\vec{V}_{A/O})_{xyz} = -100\hat{i}$ mm/s

$(\vec{a}_{A/O})_{xyz} = 150\hat{i}$ mm/s²

$\hat{i}$ $\hat{j}$ $\hat{k}$ +

$$\vec{V}_A = \vec{0} + 5\hat{k} \times (36\hat{i} + 25\hat{j}) - 100\hat{i} = -225\hat{i} + 180\hat{j}\ mm/s$$

$$= 180\hat{j} - 125\hat{i} - 100\hat{i}$$

$$\boxed{\vec{V}_A = -225\hat{i} + 180\hat{j}\ mm/s}$$ Ans.

$-0.225\hat{i} + 0.18\hat{j}$ m/s

+10 0.288

$$\vec{a}_A = \vec{0} + -3\hat{k} \times (36\hat{i} + 25\hat{j}) - 25(36\hat{i} + 25\hat{j}) + 2[5\hat{k} \times (-100\hat{i})] + 150\hat{i}$$

Coriolis

$$= -108\hat{j} + 75\hat{i} - 900\hat{i} - 625\hat{j} + -1000\hat{j} + 150\hat{i}$$

$$\boxed{\vec{a}_A = -675\hat{i} - 1733\hat{j}\ mm/s^2}$$ Ans.

$-0.675\hat{i} - 1.733\hat{j}$ m/s²

+10

calculation −1

$$\vec{a}_{A\ Coriolis} = 2\vec{\Omega} \times (\vec{V}_{A/O})_{xyz} = 2[5\hat{k} \times (-100\hat{i})]$$

$$\boxed{\vec{a}_{A\ Coriolis} = -1000\hat{j}\ mm/s^2}$$ Ans.

$-\hat{j}$ m/s²

+5