1. 20
2. 25
3. 25
4. 30

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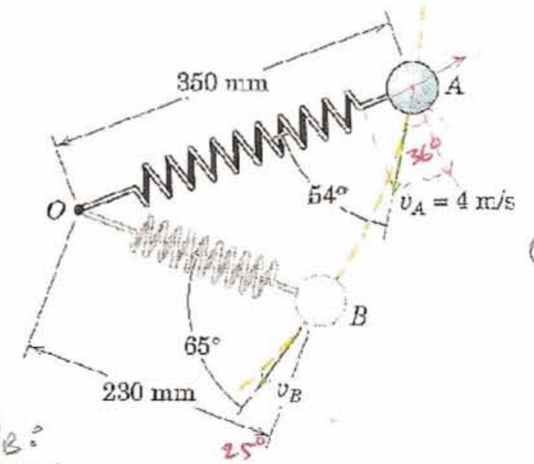
90 MINUTES OPEN BOOK QUIZ

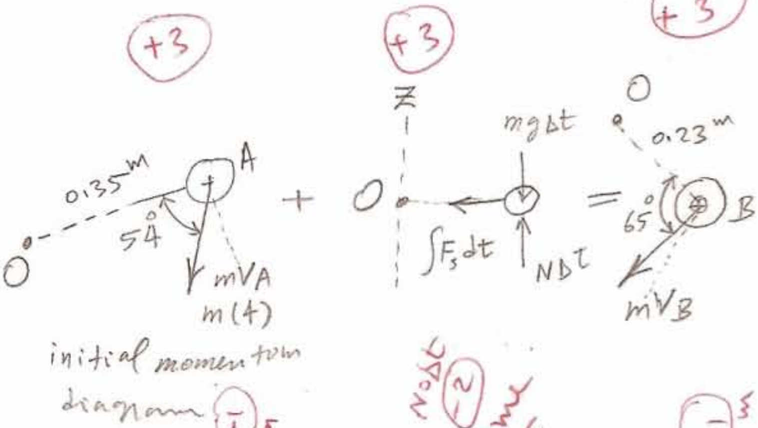
1- Solve the problems on this question booklet in the given space.
2- Use the scratch booklet before writing on the question booklet.
3- The scratch booklet will not be collected and will not be graded.
4- Neatness and clarity are important in grading.

PROBLEM 1: 20%

A particle of mass m moves with negligible friction on a horizontal surface and is connected to a light spring fastened at O. At position A the particle has the velocity $v_A = 4\ m/s$ and direction as shown. At position B the particle has the velocity v_B as shown. The spring constant is 120 N/m. (l_0 is not given)

1. Draw the impulse and momentum diagrams for the particle. (10 pts.)





2. Find V_B:

$\Sigma(\text{moments})_{z @ O}$:

$$H_{OA} = H_{OB} \ ; \ r_1 m V_\theta = r_2 m V_\theta$$

$$m(4)\sin 54^\circ (0.35)^m = m V_B (0.23)^m \sin 65^\circ$$

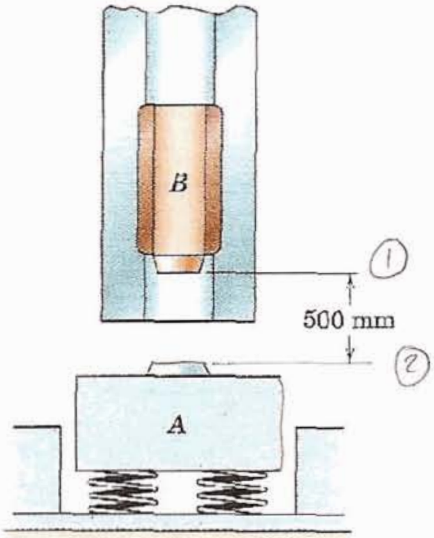
$$V_B = \frac{1.1326}{0.20845} = 5.433$$

$$\boxed{V_B = 5.433\ m/s}$$ Ans

$$V_B = \frac{4 \sin 54^\circ \ 0.35}{0.23 \ \sin 65^\circ}$$

PROBLEM 2: 25%

The 3000-kg anvil A of the drop forge is mounted on a nest of heavy coil springs having a combined stiffness of $2.8 \times 10^6\ N/m$. The 600-kg hammer B falls 500 mm from rest and strikes the anvil, which suffers a maximum downward deflection of 24 mm from its equilibrium position.



1. Find the initial deflection of the springs at equilibrium due to anvil weight.

$F_s = k\Delta \ell$.

$\Delta \ell = \frac{F_s}{k} = \frac{3000(9.81)\ N}{2.8 \times 10^6\ N/m} = 10.5107$ mm

$\Delta \ell = 0.0105107$ m

2.Find the velocity of hammer B just before impact.(7 pts.)

Hammer:

$T_1 + V_1 = T_2 + V_2$ (rest, datum)

$m_h g \Delta Y = \frac{1}{2} m_h V_2^2$

$V = \sqrt{2 g \Delta Y} = \sqrt{2(9.81)(0.5)}$

$V_B = V_h = 3.1321$ m/s Ans.

+7 −5

3. Find the velocity of anvil A just after impact.(8 pts.)

ANVIL:

$T_1 + V_1 = T_2 + V_2$ (0 stop)

$\frac{1}{2} m_A V_A'^2 + m_A g \Delta Y + \frac{1}{2} k s_1^2 = 0 + 0 + \frac{1}{2} k s_2^2$ (stop; $V_{g2} + V_{e2}$)

Anvil: 24 mm, V_A' ↓, stop, datum

$\frac{1}{2}(3000) V_A'^2 + 3000(9.81)\left(\frac{24}{1000}\right) + \frac{1}{2}(2.8\times10^6)(0.0105107)^2 = \frac{1}{2}(2.8\times10^6)\left(\frac{34.5107}{1000}\right)^2$

$1500 V_A'^2 + 706.32 + 154.6647 = 1667.384$ (1512.72)

total deflection = 10.5107 mm + 24 mm = 34.5107 mm

$1500 V_A'^2 + 860.9847 = 1667.384$

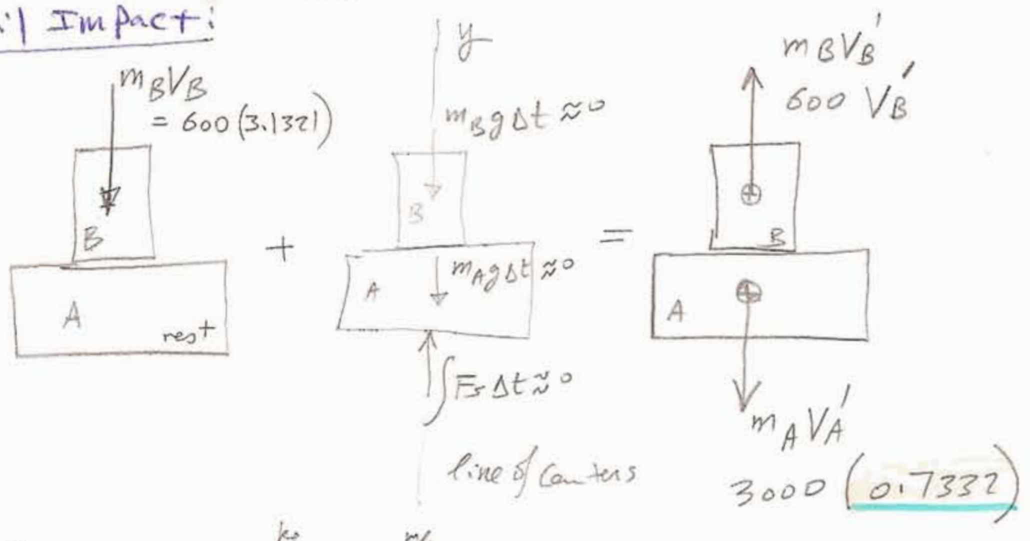
$1500 V_A'^2 = 806.3993$; $V_A'^2 = 0.5376$

$V_A' = 0.7332$ m/s ↓

+8 −5

4. Find the height h of rebound of the hammer and the coefficient of restitution e which applies. (10 pts. @ 5 pts. each)

Hammer & Anvil Impact:



$+\uparrow \Sigma(\text{vectors})_y: \quad -600(3.1321) = 600 V_B' - 3000(0.7332)$

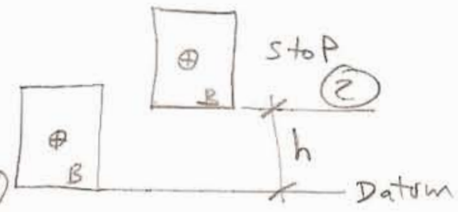
$-1879.26 = 600 V_B' - 2199.6$

$600 V_B' = 320.34 ; \quad V_B' = \frac{320.34}{600} = 0.5339 \text{ m/s} \uparrow$ Rebound velocity of B

Hammer after Impact: $T_1 + V_1 = T_2 + V_2$

$\frac{1}{2} m_B V_B'^2 + 0 \text{ (datum)} = 0 \text{ (stop)} + m_B g h$

$h = \frac{V_B'^2}{2g} = \frac{1}{2}(0.5339)^2 / 9.81 = 0.0145285 \text{ m}$



+5 $h = 14.529 \text{ mm}$ Ans. 0.0145 m

$+\uparrow e = \frac{V_A' - V_B'}{V_B - V_A} ; \quad +\uparrow e = \frac{-0.7332 - 0.5339}{-3.1321 - 0 \text{ (rest)}} = \frac{1.2671}{3.1321} = 0.4046$

$e = 0.405$ Ans.

+5

PROBLEM 3: 25%

PART I (12 pts.@ 2 pts. each)

A car mechanic 'walks' two wheel/tire units across a horizontal floor as shown. He walks with constant speed v and keeps the tires in the configuration shown with the same position relative to his body. If there is no slipping at any interface and the radius of both tires is r, determine (in terms of v and r the following:

1. The angular velocity of the lower tire (magnitude and direction)

$V_A = V = \omega_\ell r_{A/IC}$; $\omega_\ell = \frac{V}{r}$ ↻ Ans. CW +2

2. The angular velocity of the upper tire (magnitude and direction)

D is the IC of zero vel. of upper tire

∴ $V_C = \omega_U r_{C/IC}$; $\omega_U = \frac{V}{r}$ ↺ Ans. CCW +2

3. The velocities of points A, B, C, and D. (magnitude and direction)

$V_A = V_C$

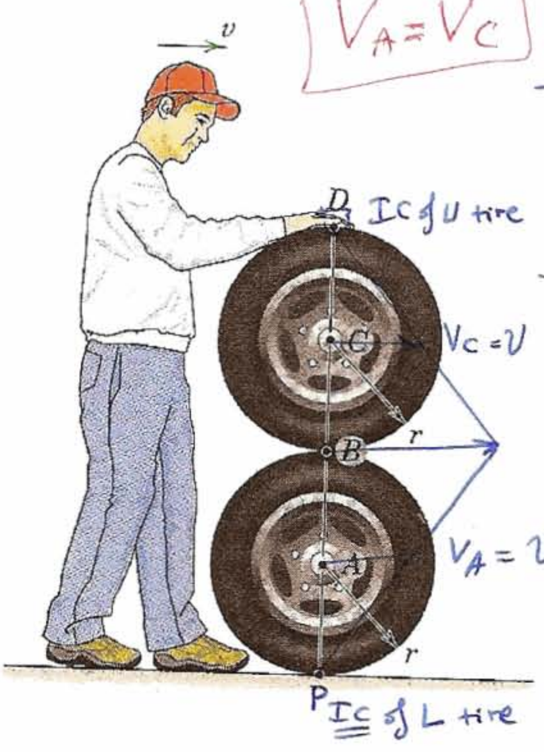
$V_A = V$ (right) → +2

$v_B = \omega r_{B/IC} = \omega(2r) = \frac{V}{r}(2r) = 2V$

$v_B = 2V$ → (right) +2

$V_C = V$ (right) → +2

$V_D = 0$ IC of zero vel. for upper tire +2



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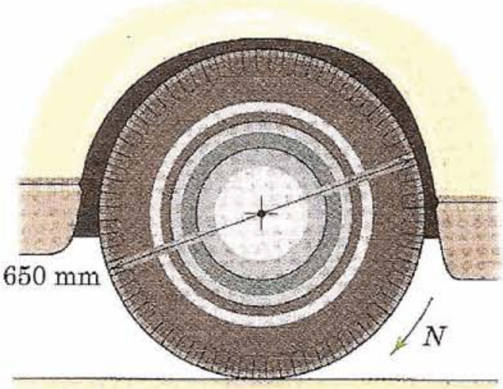
PROBLEM 3 CONTINUED:

PART II (13 pts.)

The rear driving wheel of a car has a diameter of 650 mm and has an angular speed N of 200 rev/min on an icy road. If the instantaneous center of zero velocity is 100 mm above the point of contact of the tire with the road:

$N = 200 \frac{rev.}{min} \times \frac{2\pi}{60}$

$\omega = 20.944$ rad/s



(−2) for wrong $r_{O/IC}$

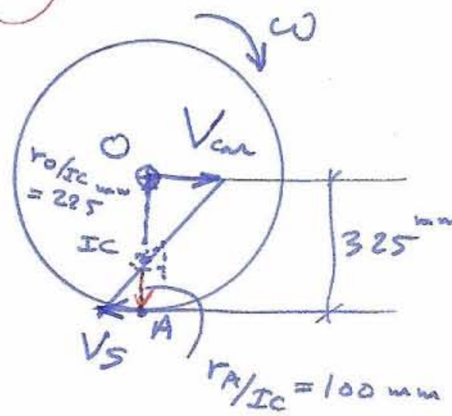
$r_{O/IC} = 0.225 \hat{j}$ m

1. Determine the velocity v of the car. (8 pts.)

$V_{car} = \omega r_{O/IC} = 20.944(0.225)$

$V_{car} = 4.712$ m/s → consistent with ω

$4.712 \hat{i}$ m/s Ans.



2. Determine the slipping velocity v_s of the tire on the ice. (5 pts.)

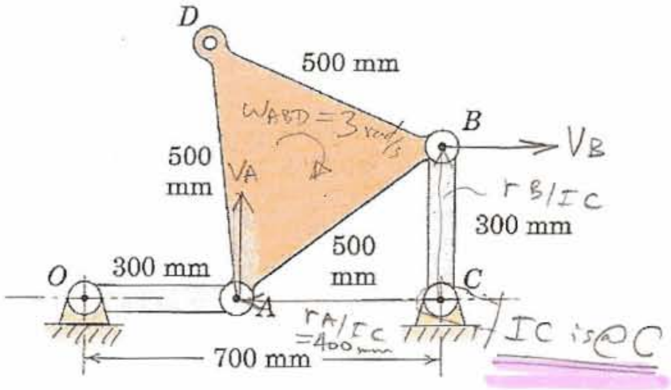
$V_s = \omega r_{A/IC}$

$V_s = 20.944\left(\frac{100}{1000}\right) = 2.094$ m/s ← Ans.

$-2.094 \hat{i}$ m/s Ans

PROBLEM 4: 30%

The triangular plate *ABD* has a clockwise angular velocity of 3 rad/s and link *OA* has zero angular acceleration for the instant represented.



1. On the given figure locate the I C of zero velocity of plate *ABD*.(5pts.) @ point C.
2. Find velocity of points *A and B*.(5pts.)

$V_A = \omega_{ABD} r_{A/IC} = 3(0.4) = 1.2\ m/s \uparrow$ Ans.

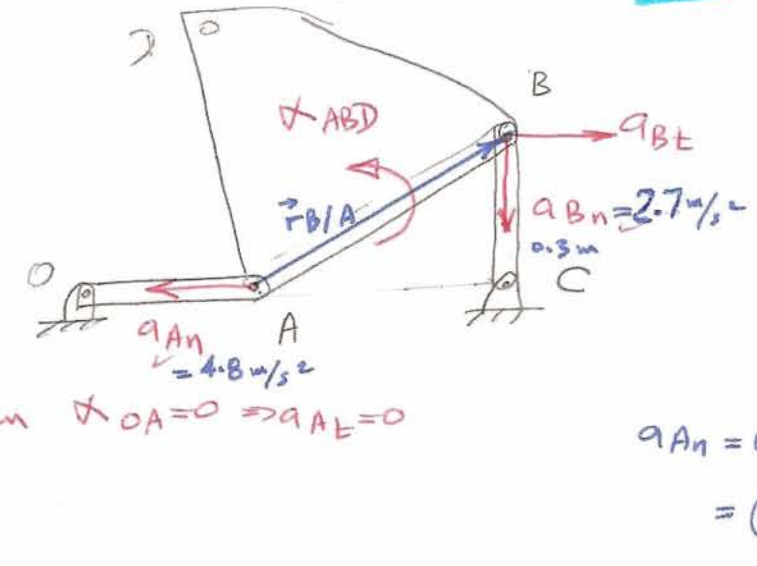
$V_B = \omega_{ABD} r_{B/IC} = 3(0.3) = 0.9\ m/s \rightarrow$ Ans.

3. Find the angular velocity of links *OA and BC*. (5pts.)

link OA: Fixed axis rotation @ O $\quad \omega_{OA} = \frac{V_A}{r_{A/O}} = \frac{1.2}{0.3} = 4\ rad/s$ Ans.

link BC: $\omega_{BC} = \frac{V_B}{r_{B/C}} = \frac{0.9}{0.3} = 3\ rad/s$ Ans.

4. Draw the absolute acceleration diagram for the mechanism(5pts.)



$a_{Bn} = \omega_{BC}^2 r_{B/C}$
$= 9(0.3)$
$a_{Bn} = 2.7\ m/s^2$

$a_{An} = \omega_{OA}^2 r_{A/O}$
$= (4^2)(0.3) = 4.8\ m/s^2$

5. Determine the angular acceleration of the plate *ABD* for this instant. (10pts.)

$\hat{i}\ \hat{j}\ \hat{k}$ (+)

$$\vec{a}_B = \vec{a}_A + \vec{\alpha}_{ABD} \times \vec{r}_{B/A} - \omega_{ABD}^2 \vec{r}_{B/A}$$

$$a_{Bt}\hat{i} - 2.7\hat{j} = -4.8\hat{i} + \alpha_{ABD}\hat{k} \times (0.4\hat{i} + 0.3\hat{j}) - 9(0.4\hat{i} + 0.3\hat{j})$$

$$= -4.8\hat{i} + 0.4\alpha_{ABD}\hat{j} - 0.3\alpha_{ABD}\hat{i} - 3.6\hat{i} - 2.7\hat{j}$$

$\hat{i}$: $a_{Bt} = -4.8 - 0.3\alpha_{ABD} - 3.6$ --- (1)

$\hat{j}$: $-2.7 = 0.4\alpha_{ABD} - 2.7$ --- (2)

solve eq. (2) $0.4\alpha_{ABD} = 2.7 - 2.7 = 0 \Rightarrow \alpha_{ABD} = 0$

$\alpha_{ABD} = 0$

$\vec{\alpha}_{ABD} = \vec{0}$ rad/s² Ans.

$a_{Bt} = -8.4$ m/s²

$\Rightarrow a_{Bt} = 8.4$ m/s² ←

$a_{Bt} = \alpha_{BC}\, r_{B/C\,fixed}$

$$\alpha_{BC} = \frac{8.4}{0.3} = 28 \text{ rad/s}^2 \circlearrowright$$

$\alpha_{BC} = 28$ rad/s² ↻