8.15   This problem asks that we determine the minimum allowable bar diameter to ensure that fatigue failure will not occur for a 1045 steel that is subjected to cyclic loading for a load amplitude of 66,700 N (15,000 lb$_f$). From Figure 8.34, the fatigue limit stress amplitude for this alloy is 310 MPa (45,000 psi). Stress is defined in Equation 6.1 as $\sigma = \dfrac{F}{A_0}$. For a cylindrical bar

$$A_0 = \pi \left(\frac{d_0}{2} \right)^2$$

Substitution for A_0 into the Equation 6.1 leads to

$$\sigma = \frac{F}{A_0} = \frac{F}{\pi \left(\dfrac{d_0}{2} \right)^2} = \frac{4F}{\pi d_0^2}$$

We now solve for d_0, taking stress as the fatigue limit divided by the factor of safety. Thus

$$d_0 = \sqrt{\frac{4F}{\pi \left(\dfrac{\sigma}{N} \right)}}$$

$$= \sqrt{\frac{(4)(66,700 \ \text{N})}{(\pi) \left(\dfrac{310 \ \text{x} \ 10^6 \ \text{N/m}^2}{2} \right)}} = 23.4 \ \text{x} \ 10^{-3} \ \text{m} = 23.4 \ \text{mm} \quad (0.92 \ \text{in.})$$