14.25 (a) We are asked to compute the densities of totally crystalline and totally amorphous poly(ethylene terephthalate) (ρ_c and ρ_a from Equation 14.8). From Equation 14.8 let $C = \frac{\% \text{ crystallinity}}{100}$, such that

$$C = \frac{\rho_c(\rho_s - \rho_a)}{\rho_s(\rho_c - \rho_a)}$$

Rearrangement of this expression leads to

$$\rho_c(C\rho_s - \rho_s) + \rho_c\rho_a - C\rho_s\rho_a = 0$$

in which ρ_c and ρ_a are the variables for which solutions are to be found. Since two values of ρ_s and C are specified in the problem statement, two equations may be constructed as follows:

$$\rho_c(C_1\rho_{s1} - \rho_{s1}) + \rho_c\rho_a - C_1\rho_{s1}\rho_a = 0$$

$$\rho_c(C_2\rho_{s2} - \rho_{s2}) + \rho_c\rho_a - C_2\rho_{s2}\rho_a = 0$$

In which $\rho_{s1} = 1.408\ \text{g/cm}^3$, $\rho_{s2} = 1.343\ \text{g/cm}^3$, $C_1 = 0.743$, and $C_2 = 0.312$. Solving the above two equations for ρ_a and ρ_c leads to

$$\rho_a = \frac{\rho_{s1}\rho_{s2}(C_1 - C_2)}{C_1\rho_{s1} - C_2\rho_{s2}}$$

$$= \frac{(1.408\ \text{g/cm}^3)(1.343\ \text{g/cm}^3)(0.743 - 0.312)}{(0.743)(1.408\ \text{g/cm}^3) - (0.312)(1.343\ \text{g/cm}^3)} = 1.300\ \text{g/cm}^3$$

And

$$\rho_c = \frac{\rho_{s1}\rho_{s2}(C_2 - C_1)}{\rho_{s2}(C_2 - 1) - \rho_{s1}(C_1 - 1)}$$

$$= \frac{(1.408\ \text{g/cm}^3)(1.343\ \text{g/cm}^3)(0.312 - 0.743)}{(1.343\ \text{g/cm}^3)(0.312 - 1.0) - (1.408\ \text{g/cm}^3)(0.743 - 1.0)} = 1.450\ \text{g/cm}^3$$

(b) Now we are to determine the % crystallinity for $\rho_s = 1.382\ \text{g/cm}^3$. Again, using Equation 14.8