

6.D2 (a) This portion of the problem asks for us to compute the wall thickness of a thin-walled cylindrical Ni tube at 350°C through which hydrogen gas diffuses. The inside and outside pressures are, respectively, 0.658 and 0.0127 MPa, and the diffusion flux is to be no greater than  $1.25 \times 10^{-7}$  mol/m<sup>2</sup>-s. This is a steady-state diffusion problem, which necessitates that we employ Equation 5.3. The concentrations at the inside and outside wall faces may be determined using Equation 6.28, and, furthermore, the diffusion coefficient is computed using Equation 6.29. Solving for  $\Delta x$  (using Equation 5.3)

$$\begin{aligned} \Delta x &= - \frac{D \Delta C}{J} \\ &= - \frac{1}{1.25 \times 10^{-7} \text{ mol/m}^2\text{-s}} \times \\ &\quad (4.76 \times 10^{-7}) \exp\left(-\frac{39,560 \text{ J/mol}}{(8.31 \text{ J/mol-K})(350 + 273 \text{ K})}\right) \times \\ (30.8) \exp\left(-\frac{12,300 \text{ J/mol}}{(8.31 \text{ J/mol-K})(350 + 273 \text{ K})}\right) &\quad \left(\sqrt{0.0127 \text{ MPa}} - \sqrt{0.658 \text{ MPa}}\right) \\ &= 0.00366 \text{ m} = 3.66 \text{ mm} \end{aligned}$$

(b) Now we are asked to determine the circumferential stress:

$$\begin{aligned} \sigma &= \frac{r \Delta p}{4 \Delta x} \\ &= \frac{(0.125 \text{ m})(0.658 \text{ MPa} - 0.0127 \text{ MPa})}{(4)(0.00366 \text{ m})} \\ &= 5.50 \text{ MPa} \end{aligned}$$

(c) Now we are to compare this value of stress to the yield strength of Ni at 350°C, from which it is possible to determine whether or not the 3.66 mm wall thickness is suitable. From the information given in the problem, we may write an equation for the dependence of yield strength ( $\sigma_y$ ) on temperature ( $T$ ) as follows:

$$\sigma_y = 100 \text{ MPa} - \frac{5 \text{ MPa}}{50^\circ\text{C}} (T - T_r)$$

where  $T_r$  is room temperature and for temperature in degrees Celsius. Thus, at 350°C

$$\sigma_y = 100 \text{ MPa} - 0.1 \text{ MPa}/^\circ\text{C} (350^\circ\text{C} - 20^\circ\text{C}) = 67 \text{ MPa}$$

Inasmuch as the circumferential stress (5.50 MPa) is much less than the yield strength (67 MPa), this thickness is entirely suitable.

(d) And, finally, this part of the problem asks that we specify how much this thickness may be reduced and still retain a safe design. Let us use a working stress by dividing the yield stress by a factor of safety, according to Equation 6.24. On the basis of our experience, let us use a value of 2.0 for  $N$ . Thus

$$\sigma_w = \frac{\sigma_y}{N} = \frac{67 \text{ MPa}}{2} = 33.5 \text{ MPa}$$

Using this value for  $\sigma_w$  and Equation 6.30, we now compute the tube thickness as

$$\begin{aligned} \Delta x &= \frac{r \Delta p}{4 \sigma_w} \\ &= \frac{(0.125 \text{ m})(0.658 \text{ MPa} - 0.0127 \text{ MPa})}{4(33.5 \text{ MPa})} \\ &= 0.00060 \text{ m} = 0.60 \text{ mm} \end{aligned}$$

Substitution of this value into Fick's first law we calculate the diffusion flux as follows:

$$\begin{aligned} J &= -D \frac{\Delta C}{\Delta x} \\ &= - (4.76 \times 10^{-7}) \exp \left[ -\frac{39,560 \text{ J/mol}}{(8.31 \text{ J/mol-K})(350 + 273 \text{ K})} \right] \times \\ &\frac{(30.8) \exp \left[ -\frac{12,300 \text{ J/mol}}{(8.31 \text{ J/mol-K})(350 + 273 \text{ K})} \right] (\sqrt{0.0127 \text{ MPa}} - \sqrt{0.658 \text{ MPa}})}{0.0006 \text{ m}} \\ &= 7.62 \times 10^{-7} \text{ mol/m}^2\text{-s} \end{aligned}$$

Thus, the flux increases by approximately a factor of 6, from  $1.25 \times 10^{-7}$  to  $7.62 \times 10^{-7}$  mol/m<sup>2</sup>-s with this reduction in thickness.