6.13 This problem asks that we rank the magnitudes of the moduli of elasticity of the three hypothetical metals X, Y, and Z. From Problem 6.12, it was shown for materials in which the bonding energy is dependent on the interatomic distance r according to Equation 6.25, that the modulus of elasticity E is proportional to

$$E \propto -\frac{2A}{\left(\frac{A}{nB}\right)^{3/(1-n)}} + \frac{(n)(n+1)B}{\left(\frac{A}{nB}\right)^{(n+2)/(1-n)}}$$

For metal X, $A = 1.5$, $B = 7 \times 10^{-6}$, and $n = 8$. Therefore,

$$E \propto -\frac{(2)(1.5)}{\left[\frac{1.5}{(8)\left(7 \text{ x } 10^{-6}\right)}\right]^{3/(1-8)}} + \frac{(8)(8+1)\left(7 \text{ x } 10^{-6}\right)}{\left[\frac{1.5}{(8)\left(7 \text{ x } 10^{-6}\right)}\right]^{(8+2)/(1-8)}}$$

$$= 830$$

For metal Y, $A = 2.0$, $B = 1 \times 10^{-5}$, and $n = 9$. Hence

$$E \propto -\frac{(2)(2.0)}{\left[\frac{2.0}{(9)\left(1 \text{ x } 10^{-5}\right)}\right]^{3/(1-9)}} + \frac{(9)(9+1)\left(1 \text{ x } 10^{-5}\right)}{\left[\frac{2.0}{(9)\left(1 \text{ x } 10^{-5}\right)}\right]^{(9+2)/(1-9)}}$$

$$= 683$$

And, for metal Z, $A = 3.5$, $B = 4 \times 10^{-6}$, and $n = 7$. Thus

$$E \propto -\frac{(2)(3.5)}{\left[\frac{3.5}{(7)\left(4 \text{ x } 10^{-6}\right)}\right]^{3/(1-7)}} + \frac{(7)(7+1)\left(4 \text{ x } 10^{-6}\right)}{\left[\frac{3.5}{(7)\left(4 \text{ x } 10^{-6}\right)}\right]^{(7+2)/(1-7)}}$$

$$= 7425$$

Therefore, metal Z has the highest modulus of elasticity.