6.12 This problem asks that we derive an expression for the dependence of the modulus of elasticity, E, on the parameters A, B, and n in Equation 6.25. It is first necessary to take dE_N/dr in order to obtain an expression for the force F; this is accomplished as follows:

$$F = \frac{dE_N}{dr} = \frac{d\left(-\frac{A}{r}\right)}{dr} + \frac{d\left(\frac{B}{r^n}\right)}{dr}$$

$$= \frac{A}{r^2} - \frac{nB}{r^{(n+1)}}$$

The second step is to set this dE_N/dr expression equal to zero and then solve for r (= r_0). The algebra for this procedure is carried out in Problem 2.14, with the result that

$$r_0 = \left(\frac{A}{nB}\right)^{1/(1-n)}$$

Next it becomes necessary to take the derivative of the force (dF/dr), which is accomplished as follows:

$$\frac{dF}{dr} = \frac{d\left(\frac{A}{r^2}\right)}{dr} + \frac{d\left(-\frac{nB}{r^{(n+1)}}\right)}{dr}$$

$$= -\frac{2A}{r^3} + \frac{(n)(n+1)B}{r^{(n+2)}}$$

Now, substitution of the above expression for r_0 into this equation yields

$$\left(\frac{dF}{dr}\right)_{r_0} = -\frac{2A}{\left(\frac{A}{nB}\right)^{3/(1-n)}} + \frac{(n)(n+1)B}{\left(\frac{A}{nB}\right)^{(n+2)/(1-n)}}$$

which is the expression to which the modulus of elasticity is proportional.