

6.D3 This problem calls for the specification of a temperature and cylindrical tube wall thickness that will give a diffusion flux of $2.5 \times 10^{-8} \text{ mol/m}^2\text{-s}$ for the diffusion of hydrogen in nickel; the tube radius is 0.100 m and the inside and outside pressures are 1.015 and 0.01015 MPa, respectively. There are probably several different approaches that may be used; and, of course, there is not one unique solution. Let us employ the following procedure to solve this problem: (1) assume some wall thickness, and, then, using Fick's first law for diffusion (which also employs Equations 5.3 and 6.29), compute the temperature at which the diffusion flux is that required; (2) compute the yield strength of the nickel at this temperature using the dependence of yield strength on temperature as stated in Problem 6.D2; (3) calculate the circumferential stress on the tube walls using Equation 6.30; and (4) compare the yield strength and circumferential stress values--the yield strength should probably be at least twice the stress in order to make certain that no permanent deformation occurs. If this condition is not met then another iteration of the procedure should be conducted with a more educated choice of wall thickness.

As a starting point, let us arbitrarily choose a wall thickness of 2 mm ($2 \times 10^{-3} \text{ m}$). The steady-state diffusion equation, Equation 5.3, takes the form

$$\begin{aligned}
 J &= -D \frac{\Delta C}{\Delta x} \\
 &= 2.5 \times 10^{-8} \text{ mol/m}^2\text{-s} \\
 &= -(4.76 \times 10^{-7}) \exp \left[-\frac{39,560 \text{ J/mol}}{(8.31 \text{ J/mol-K})(T)} \right] \times \\
 &\quad \frac{(30.8) \exp \left[-\frac{12,300 \text{ J/mol}}{(8.31 \text{ J/mol-K})(T)} \right] (\sqrt{0.01015 \text{ MPa}} - \sqrt{1.015 \text{ MPa}})}{0.002 \text{ m}}
 \end{aligned}$$

Solving this expression for the temperature T gives $T = 500 \text{ K} = 227^\circ\text{C}$; this value is satisfactory inasmuch as it is less than the maximum allowable value (300°C).

The next step is to compute the stress on the wall using Equation 6.30; thus

$$\begin{aligned}
 \sigma &= \frac{r \Delta p}{4 \Delta x} \\
 &= \frac{(0.100 \text{ m})(1.015 \text{ MPa} - 0.01015 \text{ MPa})}{(4)(2 \times 10^{-3} \text{ m})} \\
 &= 12.6 \text{ MPa}
 \end{aligned}$$

Now, the yield strength (σ_y) of Ni at this temperature may be computed using the expression

$$\sigma_y = 100 \text{ MPa} - \frac{5 \text{ MPa}}{50^\circ\text{C}} (T - T_r)$$

where T_r is room temperature. Thus,

$$\sigma_y = 100 \text{ MPa} - 0.1 \text{ MPa}/^\circ\text{C} (227^\circ\text{C} - 20^\circ\text{C}) = 79.3 \text{ MPa}$$

Inasmuch as this yield strength is greater than twice the circumferential stress, wall thickness and temperature values of 2 mm and 227°C are satisfactory design parameters.