7.17 This problem asks for us to determine the tensile stress at which a BCC metal yields when the stress is applied along a [121] direction such that slip occurs on a (101) plane and in a $[\bar{1}11]$ direction; the critical resolved shear stress for this metal is 2.4 MPa. To solve this problem we use Equation 7.4; however it is first necessary to determine the values of ϕ and λ. These determinations are possible using Equation 7.6. Now, λ is the angle between [121] and $[\bar{1}11]$ directions. Therefore, relative to Equation 7.6 let us take $u_1 = 1$, $v_1 = 2$, and $w_1 = 1$, as well as $u_2 = -1$, $v_2 = 1$, and $w_2 = 1$. This leads to

$$\lambda = \cos^{-1}\left[\frac{u_1u_2 + v_1v_2 + w_1w_2}{\sqrt{\left(u_1^2 + v_1^2 + w_1^2\right)\left(u_2^2 + v_2^2 + w_2^2\right)}}\right]$$

$$= \cos^{-1}\left\{\frac{(1)(-1) + (2)(1) + (1)(1)}{\sqrt{\left[(1)^2 + (2)^2 + (1)^2\right]\left[(-1)^2 + (1)^2 + (1)^2\right]}}\right\}$$

$$= \cos^{-1}\left(\frac{2}{\sqrt{18}}\right) = 61.9°$$

Now for the determination of ϕ, the normal to the (101) slip plane is the [101] direction. Again using Equation 7.6, where we now take $u_1 = 1$, $v_1 = 2$, $w_1 = 1$ (for [121]), and $u_2 = 1$, $v_2 = 0$, $w_2 = 1$ (for [101]). Thus,

$$\phi = \cos^{-1}\left\{\frac{(1)(1) + (2)(0) + (1)(1)}{\sqrt{\left[(1)^2 + (2)^2 + (1)^2\right]\left[(1)^2 + (0)^2 + (1)^2\right]}}\right\}$$

$$= \cos^{-1}\left(\frac{2}{\sqrt{12}}\right) = 54.7°$$

It is now possible to compute the yield stress (using Equation 7.4) as

$$\sigma_y = \frac{\tau_{crss}}{\cos\phi\,\cos\lambda} = \frac{2.4\text{ MPa}}{\left(\frac{2}{\sqrt{12}}\right)\left(\frac{2}{\sqrt{18}}\right)} = 8.82\text{ MPa}$$