12.47 (a) This portion of the problem requests that we compute the modulus of elasticity for nonporous TiC given that E = 310 GPa (45 x 10^6 psi) for a material having 5 vol% porosity. Thus, we solve Equation 12.9 for E_0, using P = 0.05, which gives

$$E_0 = \frac{E}{1 - 1.9P + 0.9P^2}$$

$$= \frac{310 \text{ GPa}}{1 - (1.9)(0.05) + (0.9)(0.05)^2} = 342 \text{ GPa} \quad (49.6 \text{ x } 10^6 \text{ psi})$$

(b) Now we are asked to compute the volume percent porosity at which the elastic modulus of TiC is 240 MPa (35 x 10^6 psi). Since from part (a), E_0 = 342 GPa, and using Equation 12.9 we get

$$\frac{E}{E_0} = \frac{240 \text{ MPa}}{342 \text{ MPa}} = 0.702 = 1 - 1.9P + 0.9P^2$$

Or

$$0.9P^2 - 1.9P + 0.298 = 0$$

Now, solving for the value of P using the quadratic equation solution yields

$$P = \frac{1.9 \pm \sqrt{(-1.9)^2 - (4)(0.9)(0.298)}}{(2)(0.9)}$$

The positive and negative roots are

$$P^+ = 1.94$$

$$P^- = 0.171$$

Obviously, only the negative root is physically meaningful, and therefore the value of the porosity to give the desired modulus of elasticity is 17.1 vol%.