12.22 This problem asks for us to calculate the atomic packing factor for aluminum oxide given values for the *a* and *c* lattice parameters, and the density. It first becomes necessary to determine the value of n' in Equation 12.1. This necessitates that we calculate the value of V_C, the unit cell volume. In Problem 3.6 it was shown that the area of the hexagonal base (AREA) is related to *a* as

$$\text{AREA} = 6\left(\frac{a}{2}\right)^2 \sqrt{3} = 1.5a^2\sqrt{3}$$

$$= (1.5)\left(4.759 \text{ x } 10^{-8} \text{ cm}\right)^2\left(\sqrt{3}\right) = 5.88 \text{ x } 10^{-15} \text{ cm}^2$$

The unit cell volume now is just

$$V_C = (\text{AREA})(c) = \left(5.88 \text{ x } 10^{-15} \text{ cm}^2\right)\left(1.2989 \text{ x } 10^{-7} \text{ cm}\right)$$

$$= 7.64 \text{ x } 10^{-22} \text{ cm}^3$$

Now, solving for n' (Equation 12.1) yields

$$n' = \frac{\rho N_A V_C}{\sum A_C + \sum A_A}$$

$$= \frac{\left(3.99 \text{ g/cm}^3\right)\left(6.023 \text{ x } 10^{23} \text{ formula units/mol}\right)\left(7.64 \text{ x } 10^{-22} \text{ cm}^3\text{/unit cell}\right)}{(2)(26.98 \text{ g/mol}) + (3)(16.00\text{g/mol})}$$

$$= 18.0 \text{ formula units/unit cell}$$

Or, there are 18 Al_2O_3 units per unit cell, or 36 Al^{3+} ions and 54 O^{2-} ions. From Table 12.3, the radii of these two ion types are 0.053 and 0.140 nm, respectively. Thus, the total sphere volume in Equation 3.2 (which we denote as V_S), is just

$$V_S = (36)\left(\frac{4}{3}\pi\right)(5.3 \text{ x } 10^{-9}\text{cm})^3 + (54)\left(\frac{4}{3}\pi\right)(1.4 \text{ x } 10^{-8}\text{cm})^3$$

$$= 6.43 \text{ x } 10^{-22} \text{ cm}^3$$

Finally, the APF is just