

12.40 This problem asks that we compute the crack tip radius ratio before and after etching. Let

$$\begin{aligned}\rho_t &= \text{original crack tip radius, and} \\ \rho'_t &= \text{etched crack tip radius}\end{aligned}$$

Also,

$$\sigma'_f = \sigma_f$$

$$a' = \frac{a}{2}$$

$$\sigma'_0 = 4\sigma_0$$

Solving for  $\frac{\rho'_t}{\rho_t}$  from the following

$$\sigma_f = 2\sigma_0 \left( \frac{a}{\rho_t} \right)^{1/2} = \sigma'_f = 2\sigma'_0 \left( \frac{a'}{\rho'_t} \right)^{1/2}$$

yields

$$\frac{\rho'_t}{\rho_t} = \left( \frac{\sigma'_0}{\sigma_0} \right)^2 \left( \frac{a'}{a} \right) = \left( \frac{4\sigma_0}{\sigma_0} \right)^2 \left( \frac{a/2}{a} \right) = 8$$