16.D5 In this problem, for an aligned and discontinuous glass fiber-epoxy matrix composite having a longitudinal tensile strength of 1200 MPa, we are asked to compute the required fiber fracture strength, given the following: the average fiber diameter (0.015 mm), the average fiber length (5.0 mm), the volume fraction of fibers (0.35), the fiber-matrix bond strength (80 MPa), and the matrix stress at fiber failure (6.55 MPa).

To begin, since the value of σ_f^* is unknown, calculation of the value of l_c in Equation 16.3 is not possible, and, therefore, we are not able to decide which of Equations 16.18 and 16.19 to use. Thus, it is necessary to substitute for l_c in Equation 16.3 into Equation 16.18, solve for the value of σ_f^*, then, using this value, solve for l_c from Equation 16.3. If $l > l_c$, we use Equation 16.18, otherwise Equation 16.19 must be used. *Note:* the σ_f^* parameters in Equations 16.18 and 16.3 are the same. Realizing this, and substituting for l_c in Equation 16.3 into Equation 16.18 leads to

$$\sigma_{cd}^* = \sigma_f^* V_f \left[1 - \frac{\sigma_f^* d}{4\tau_c l} \right] + \sigma_m'(1 - V_f)$$

$$= \sigma_f^* V_f - \frac{\sigma_f^{*2} V_f d}{4\tau_c l} + \sigma_m' - \sigma_m' V_f$$

This expression is a quadratic equation in which σ_f^* is the unknown. Rearrangement into a more convenient form leads to

$$\sigma_f^{*2} \left[\frac{V_f d}{4\tau_c l} \right] - \sigma_f^* (V_f) + \left[\sigma_{cd}^* - \sigma_m'(1 - V_f) \right] = 0$$

Or

$$a\sigma_f^{*2} + b\sigma_f^* + c = 0$$

where

$$a = \frac{V_f d}{4\tau_c l}$$