

16.27 This problem asks that we derive a generalized expression analogous to Equation 16.16 for the transverse modulus of elasticity of an aligned hybrid composite consisting of two types of continuous fibers. Let us denote the subscripts  $f1$  and  $f2$  for the two fiber types, and  $m$ ,  $c$ , and  $t$  subscripts for the matrix, composite, and transverse direction, respectively. For the isostress state, the expressions analogous to Equations 16.12 and 16.13 are

$$\sigma_c = \sigma_m = \sigma_{f1} = \sigma_{f2}$$

And

$$\varepsilon_c = \varepsilon_m V_m + \varepsilon_{f1} V_{f1} + \varepsilon_{f2} V_{f2}$$

Since  $\varepsilon = \sigma/E$  (Equation 6.5), making substitutions of the form of this equation into the previous expression yields

$$\frac{\sigma}{E_{ct}} = \frac{\sigma}{E_m} V_m + \frac{\sigma}{E_{f1}} V_{f1} + \frac{\sigma}{E_{f2}} V_{f2}$$

Thus

$$\begin{aligned} \frac{1}{E_{ct}} &= \frac{V_m}{E_m} + \frac{V_{f1}}{E_{f1}} + \frac{V_{f2}}{E_{f2}} \\ &= \frac{V_m E_{f1} E_{f2} + V_{f1} E_m E_{f2} + V_{f2} E_m E_{f1}}{E_m E_{f1} E_{f2}} \end{aligned}$$

And, finally, taking the reciprocal of this equation leads to

$$E_{ct} = \frac{E_m E_{f1} E_{f2}}{V_m E_{f1} E_{f2} + V_{f1} E_m E_{f2} + V_{f2} E_m E_{f1}}$$