16.14 For a continuous and aligned fibrous composite, we are given its cross-sectional area (970 mm^2), the stresses sustained by the fiber and matrix phases (215 and 5.38 MPa), the force sustained by the fiber phase (76,800 N), and the total longitudinal strain (1.56 x 10^{-3}).

(a) For this portion of the problem we are asked to calculate the force sustained by the matrix phase. It is first necessary to compute the volume fraction of the matrix phase, V_m. This may be accomplished by first determining V_f and then V_m from $V_m = 1 - V_f$. The value of V_f may be calculated since, from the definition of stress (Equation 6.1), and realizing $V_f = A_f/A_c$ as

$$\sigma_f = \frac{F_f}{A_f} = \frac{F_f}{V_f A_c}$$

Or, solving for V_f

$$V_f = \frac{F_f}{\sigma_f A_c} = \frac{76{,}800 \text{ N}}{\left(215 \text{ x } 10^6 \text{ N/m}^2\right)(970 \text{ mm}^2)(1 \text{ m}/1000 \text{ mm})^2} = 0.369$$

Also

$$V_m = 1 - V_f = 1 - 0.369 = 0.631$$

And, an expression for σ_m analogous to the one for σ_f above is

$$\sigma_m = \frac{F_m}{A_m} = \frac{F_m}{V_m A_c}$$

From which

$$F_m = V_m \sigma_m A_c = (0.631)\left(5.38 \text{ x } 10^6 \text{ N/m}^2\right)\left(0.970 \text{ x } 10^{-3} \text{ m}^2\right) = 3290 \text{ N} \quad (738 \text{ lb}_\text{f})$$

(b) We are now asked to calculate the modulus of elasticity in the longitudinal direction. This is possible realizing that $E_c = \dfrac{\sigma_c}{\varepsilon}$ (from Equation 6.5) and that $\sigma_c = \dfrac{F_m + F_f}{A_c}$ (from Equation 6.1). Thus

$$E_c = \frac{\sigma_c}{\varepsilon} = \frac{\dfrac{F_m + F_f}{A_c}}{\varepsilon} = \frac{F_m + F_f}{\varepsilon A_c}$$