

16.16 In this problem, for an aligned carbon fiber-epoxy matrix composite, we are given the desired longitudinal tensile strength (500 MPa), the average fiber diameter (1.0×10^{-2} mm), the average fiber length (0.5 mm), the fiber fracture strength (4 GPa), the fiber-matrix bond strength (25 MPa), and the matrix stress at composite failure (7.0 MPa); and we are asked to compute the volume fraction of fibers that is required. It is first necessary to compute the value of the critical fiber length using Equation 16.3. If the fiber length is much greater than l_c , then we may determine V_f using Equation 16.17, otherwise, use of either Equation 16.18 or Equation 16.19 is necessary.

Thus,

$$l_c = \frac{\sigma_f^* d}{2\tau_c} = \frac{(4 \times 10^3 \text{ MPa})(1.0 \times 10^{-2} \text{ mm})}{2(25 \text{ MPa})} = 0.80 \text{ mm}$$

Inasmuch as $l < l_c$ ($0.50 \text{ mm} < 0.80 \text{ mm}$), then use of Equation 16.19 is required. Therefore,

$$\sigma_{cd}^* = \frac{l\tau_c}{d}V_f + \sigma_m'(1 - V_f)$$

$$500 \text{ MPa} = \frac{(0.5 \times 10^{-3} \text{ m})(25 \text{ MPa})}{0.01 \times 10^{-3} \text{ m}}(V_f) + (7 \text{ MPa})(1 - V_f)$$

Solving this expression for V_f leads to $V_f = 0.397$.