$$\frac{A_c}{A_m} = \frac{1}{V_m}$$

which, when substituted into the previous expression leads to

$$\frac{F_f}{F_m} = \frac{E_c}{E_m V_m} - 1$$

Also, from Equation 16.10a, $E_c = E_m V_m + E_f V_f$, which, when substituted for E_c into the previous expression, yields

$$\frac{F_f}{F_m} = \frac{E_m V_m + E_f V_f}{E_m V_m} - 1$$

$$= \frac{E_m V_m + E_f V_f - E_m V_m}{E_m V_m} = \frac{E_f V_f}{E_m V_m}$$

the desired result.

(b) This portion of the problem asks that we establish an expression for F_f/F_c. We determine this ratio in a similar manner. Now $F_c = F_f + F_m$ (Equation 16.4), or division by F_c leads to

$$1 = \frac{F_f}{F_c} + \frac{F_m}{F_c}$$

which, upon rearrangement, gives

$$\frac{F_f}{F_c} = 1 - \frac{F_m}{F_c}$$

Now, substitution of the expressions in part (a) for F_m and F_c that resulted from combining Equations 6.1 and 6.5 results in

$$\frac{F_f}{F_c} = 1 - \frac{A_m \varepsilon E_m}{A_c \varepsilon E_c} = 1 - \frac{A_m E_m}{A_c E_c}$$

Since the volume fraction of fibers is equal to $V_m = A_m/A_c$, then the above equation may be written in the form