AMERICAN UNIVERSITY OF BEIRUT

ENGINEERING MATERIALS-MECH 340

QUIZ I-DEGREE FROM 100

STUDENT NAME:

ID:

DURATION: 1H10

Problem 1.(Parts 1 and 2 are independent)- (20 pts)

Part 1- (10 pts)

(a) Show that the c/a ratio (height of unit cell divided by its edge length) is 1.633 for the ideal hcp structure. (b) Comment on the fact that real hcp metals display c/a ratios varying from 1.58 (for Be) to 1.89 (for Cd). (You can use the following geometric configurations to solve part a)

Answer:

a-

$$x = \frac{a/2}{\cos 30^\circ} = 0.5774a$$

$$h^2 = a^2 - x^2 = a^2 - (0.5774a)^2 = 0.6667a^2$$

$$h = 0.8165a$$

$$c = 2h = 1.633a$$

or $\underline{\underline{c/a = 1.633}}$

b-

(b) Rather than perfect spheres, the atoms are effectively ellipsoids (due to some assymmetry in atomic bonding)

Part 2- (10 pts)

Data taken from a stress–strain test for a ceramic are given in the table. The curve is linear between the origin and the first point. Plot the diagram, and determine the modulus of elasticity and the modulus of resilience.

σ (ksi)	ϵ (in./in.)
0	0
33.2	0.0006
45.5	0.0010
49.4	0.0014
51.5	0.0018
53.4	0.0022

$$U_r \cong \frac{1}{2}\sigma_y \varepsilon_y$$

Answer: (10 pts)

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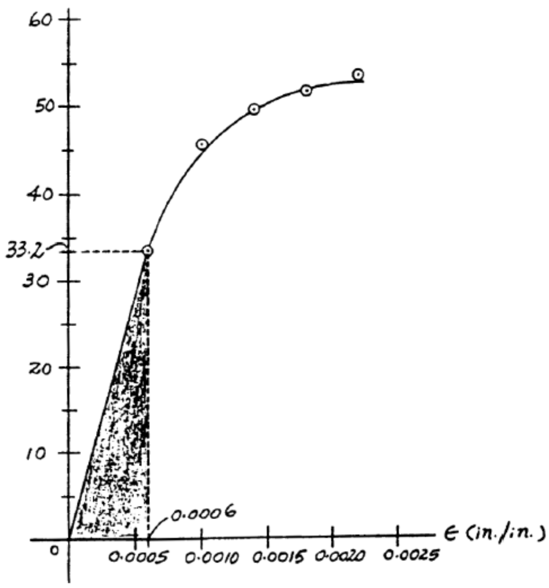


Modulus of Elasticity : From the stress – strain diagram

$$E = \frac{33.2 - 0}{0.0006 - 0} = 55.3\left(10^3\right) \text{ ksi} \qquad \textbf{Ans}$$

Modulus of Resilience : The modulus of resilience is equal to the area under the *linear portion* of the stress – strain diagram (shown shaded).

$$u_r = \frac{1}{2}(33.2)\left(10^3\right)\left(\frac{\text{lb}}{\text{in}^2}\right)\left(0.0006\ \frac{\text{in.}}{\text{in.}}\right) = 9.96\ \frac{\text{in}\cdot\text{lb}}{\text{in}^3} \qquad \textbf{Ans}$$

Problem 2- (15 pts)

We find that 10 h are required to successfully carburize a batch of 500 steel gears at 900°C, where the iron has the FCC structure. We find that it costs $1000 per hour to operate the carburizing furnace at 900°C and $1500 per hour to operate the furnace at 1000°C. Is it economical to increase the carburizing temperature to 1000°C? What other factors must be considered?

Algorithm:

1- **Find the required time to successfully carburize the batch of gears at 1000** °C
2- **Find the cost per part**
3- **Compare between costs at 2 temperatures**

$$\frac{c_s - c_x}{c_s - c_0} = \mathrm{erf}\left(\frac{x}{2\sqrt{Dt}}\right)$$

Answer:

Again assuming, we can use the solution to Ficks's second law given by Equation 5-7

$$\frac{c_s - c_x}{c_s - c_0} = \mathrm{erf}\left(\frac{x}{2\sqrt{Dt}}\right)$$

Note that, since we are dealing with only changes in heat treatment time and temperature, the term Dt must be constant.

To achieve the same carburizing treatment at 1000°C as at 900°C:

$$D_{1273}t_{1273} = D_{1173}t_{1173}$$

At 900°C, the cost per part is ($1000/h) (10 h)/500 parts = $20/part
At 1000°C, the cost per part is ($1500/h) (3.299 h)/500 parts = $9.90/part

How to find D:

$$D_1 t_1 = D_2 t_2$$

$$D = D_0 exp(\frac{-Q_d}{RT})$$

So $t_2 = exp\left(\frac{-Q_d}{R}\left(\frac{1}{T_1} - \frac{1}{T_2}\right)\right) t_1 = 360000 exp(\frac{-148x10^3}{8.314}\left(\frac{1}{1173} - \frac{1}{1273}\right) = 3h2min$

Problem 3 (10 pts)- determine for each one of the listed properties if it belongs to an Edge dislocation or to a screw dislocation (b=burger vector)

- Has an extra half plane of atoms **answer: Edge**
- b is parallel to the dislocation line **answer: Screw**
- Dislocation moves perpendicular to b **answer: Screw**
- Moves on the plane containing b and the line **answer: Edge**
- Moves on any plane containing b and the line **answer: Screw**
- Cannot cross slip **answer: Edge**
- Can Climb **answer: Edge**

Problem 4- 20 pts

a. Calculate the diffusion coefficient of carbon in (i) Austenite (= γ-Fe see table 5.2) and (ii) Ferrite (α-Fe. See table 5.2) at 920°C. User your knowledge to explain the difference between the two values. (10 pts)

b. A mild steel component (carbon content 0.2%) is to be case-hardened by placing it in a furnace in an atmosphere rich in hydrocarbon gas so that the surface concentration is 0.8% carbon. The design of the component requires that at the completion of this process, the carbon concentration at 1 mm below the surface will be 0.55%. The furnace is set at 1050°C. Determine the time required for the heat treatment. (10 pts)

$$D = D_0 \exp\left(\frac{-Q}{RT}\right)$$

Answer: From tables

For γ:

$$D = D_0 e^{\left(-\frac{Q_d}{RT}\right)}$$

(γ Fe)

$Q_d = 148$ kJ/mol
$D_0 = 2.3 \times 10^{-5}$ m²/s

T= 1193 K.

So D1=7.77 x 10-^{12}m^2/s

For α:

α (Fe)

$Q_d = 80 \times 10^3$
$D_0 = 6.2 \times 10^{-7}$ m²/s

So D2=1.95 x 10^{-10} m^2/s

$D_1 < D_2 \Rightarrow$ diffusion occurs at a lower rate in γFe because it has FCC that is more packed that BCC.

b) $C_0 = 0.2$ wt%
$C_s = 0.8$ wt%
$x = 1.0 \times 10^{-3}$ m
$T = 1323$ K
$C_x = 0.55$ wt%

$$\frac{C_x - C_0}{C_s - C_0} = 1 - \text{erf}\left(\frac{x}{2\sqrt{Dt}}\right) \Rightarrow 1 - \left(\frac{0.55 - 0.2}{0.8 - 0.2}\right) = \text{erf}\left(\frac{x}{2\sqrt{Dt}}\right)$$

$$\Rightarrow 0.4166 = \text{erf}\left(\frac{x}{2\sqrt{Dt}}\right)$$

Using tables $\Rightarrow D \times t = 1.661 \times 10^{-6}$

At $T = 1323$ K $\Rightarrow$ (γ) Fe

$$D = 2.3 \times 10^{-5} e^{-\left(\frac{148 \times 10^3}{1323 \times 8.31}\right)} = 3.276 \times 10^{-11}$$

$\Rightarrow t = 14.08$ h

Problem 4- (35 pts)

Part 1- (20 pts)

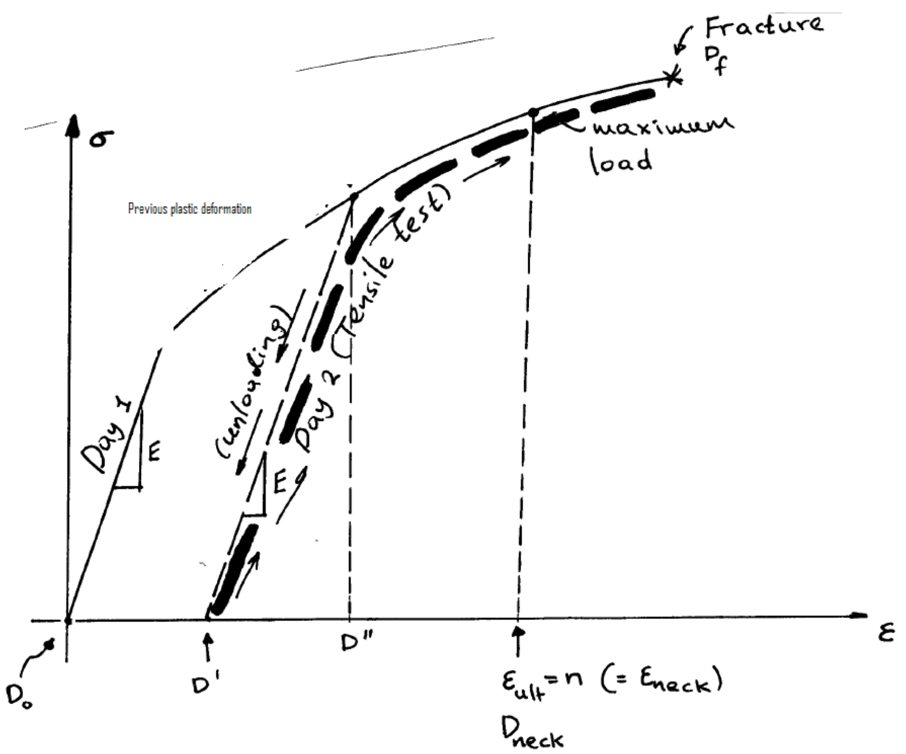
A tensile specimen is machined to a gage diameter of 0.357-in and is marked with a starting gage length of 2-in. When subjected to a test, the following results were found:

- yield load = 2,000 lbf
- fracture diameter = 0.27-in
- diameter at ultimate load = 0.31-in
- elastic modulus = 25×10^6 psi

After completing this test, you are informed that the tensile specimen had been plastically deformed some amount before it was machined and tested and that in this stage:

$$\sigma = K\epsilon^n \text{ with n=0.5.}$$

Refer to data found on the following curve describing the history of actions during this experiment and answer to the following questions:



What is the yield strength Y for this specimen**? (7 pts)**

How much strain was induced by the unknown amount of plastic deformation at day 1**? (7 pts)**

What maximum load (i.e. Fu) was reached during the test? **(6 pts)**

Use:

$$\epsilon = 2\ell n\left(\frac{D_o}{D}\right)$$

Pre-strain: When considering necking, we must consider the curve representing the material and we can write $\epsilon_{\text{neck}} = n, \epsilon$ starting without pre-strain. So

$$\epsilon_{\text{neck}} = n = 0.5 = 2\ell n\frac{D_o}{D_{\text{neck}}} \tag{11}$$

$$\Rightarrow D_o = D_{\text{neck}}\exp\left(\frac{\epsilon_{\text{neck}}}{2}\right) = 0.398 - in \tag{12}$$

Pre-strain:

$$\epsilon' = 2\ell n\frac{D_o}{D'} = 0.217 \tag{13}$$

Maximum load: Let us first determine K from yielding point of Day:2 experiment:

$$\sigma' = Y = K\epsilon'^n \Rightarrow K = \frac{Y}{\epsilon'^n} = 42,892 \text{ psi} \tag{14}$$

Let us use K and n at necking where the maximum load is applied.

$$\epsilon_{\text{neck}} = n, \sigma_{\text{neck}} = K\epsilon_{\text{neck}}^n = Kn^n = 30,329 \text{ psi} \tag{15}$$

And

$$F_u = \frac{\pi D_{\text{neck}}^2 \sigma_{\text{neck}}}{4} = 2289 \text{ lbf} \tag{16}$$

Part 2- (15 pts)

An annealed brass specimen of 0.505-in starting diameter supports a maximum tensile load of 120,000-lbf at which point the initial area is reduced by 40%. If a second identical specimen were loaded until the induced strain was half the magnitude of n, what load would be needed to reach this condition?

r is the reduction of area defined as

$$r = \frac{A_o - A}{A_o}$$

Use:

$$\epsilon_{\text{neck}} = \ell n\left(\frac{1}{1-r}\right)$$

$$\epsilon_{\text{neck}} = n, \sigma_{\text{neck}} = K\epsilon_{\text{neck}}^n$$

Answer:

From the first tensile specimen, we can infer K and n. Area reduction $= 40\% \Rightarrow \epsilon_{\text{neck}} = \ell n\left(\frac{1}{1-r}\right) = \ell n\left(\frac{1}{1-0.4}\right) = 0.51$ at necking. So, $n = 0.51$.

Starting diameter: $0.505'' \to D_{\text{neck}} = D_o \exp\left(-\frac{\epsilon_{\text{neck}}}{2}\right) = 0.391''$.

$$\text{UTS} = 120,000 \text{ lbf} \Rightarrow \sigma_{\text{neck}} = \frac{4 \times \text{UTS}}{\pi D_{\text{neck}}^2} = 999 \times 10^3 \text{ psi} \tag{17}$$

$$K = \frac{\sigma_{\text{neck}}}{n^n} = 1409 \times 10^3 \text{ psi} \tag{18}$$

Second specimen: $\epsilon = \frac{n}{2} = 0.255$.

So $\sigma = K\epsilon^n = 702 \times 10^3$ psi, $D = D_o \exp\left(-\frac{\epsilon}{2}\right) = 0.445''$.

And load $= \frac{\pi D^2 \sigma}{4} = 110,000$ lbf.

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Avogadro's number: $6.023x10^{23}$ /mol

Gas Constant: 8.31 J/mol•K, 1.987 cal/mol•K

Boltzmann's constant: $1.38x10^{-23}$ J/atom•K, $8.62x10^{-5}$ eV/atom•K

Table 5.1 Tabulation of Error Function Values

z	$erf(z)$	z	$erf(z)$	z	$erf(z)$
0	0	0.55	0.5633	1.3	0.9340
0.025	0.0282	0.60	0.6039	1.4	0.9523
0.05	0.0564	0.65	0.6420	1.5	0.9661
0.10	0.1125	0.70	0.6778	1.6	0.9763
0.15	0.1680	0.75	0.7112	1.7	0.9838
0.20	0.2227	0.80	0.7421	1.8	0.9891
0.25	0.2763	0.85	0.7707	1.9	0.9928
0.30	0.3286	0.90	0.7970	2.0	0.9953
0.35	0.3794	0.95	0.8209	2.2	0.9981
0.40	0.4284	1.0	0.8427	2.4	0.9993
0.45	0.4755	1.1	0.8802	2.6	0.9998
0.50	0.5205	1.2	0.9103	2.8	0.9999

Table 5.2 A Tabulation of Diffusion Data

Diffusing Species	*Host Metal*	$D_0(m^2/s)$	*Activation Energy* Q_d		*Calculated Values*	
			kJ/mol	*eV/atom*	$T(°C)$	$D(m^2/s)$
Fe	α-Fe (BCC)	2.8×10^{-4}	251	2.60	500	3.0×10^{-21}
					900	1.8×10^{-15}
Fe	γ-Fe (FCC)	5.0×10^{-5}	284	2.94	900	1.1×10^{-17}
					1100	7.8×10^{-16}
C	α−Fe	6.2×10^{-7}	80	0.83	500	2.4×10^{-12}
					900	1.7×10^{-10}
C	γ-Fe	2.3×10^{-5}	148	1.53	900	5.9×10^{-12}
					1100	5.3×10^{-11}
Cu	Cu	7.8×10^{-5}	211	2.19	500	4.2×10^{-19}
Zn	Cu	2.4×10^{-5}	189	1.96	500	4.0×10^{-18}
Al	Al	2.3×10^{-4}	144	1.49	500	4.2×10^{-14}
Cu	Al	6.5×10^{-5}	136	1.41	500	4.1×10^{-14}
Mg	Al	1.2×10^{-4}	131	1.35	500	1.9×10^{-13}
Cu	Ni	2.7×10^{-5}	256	2.65	500	1.3×10^{-22}

Source: E. A. Brandes and G. B. Brook (Editors), *Smithells Metals Reference Book,* 7th edition, Butterworth-Heinemann, Oxford, 1992.