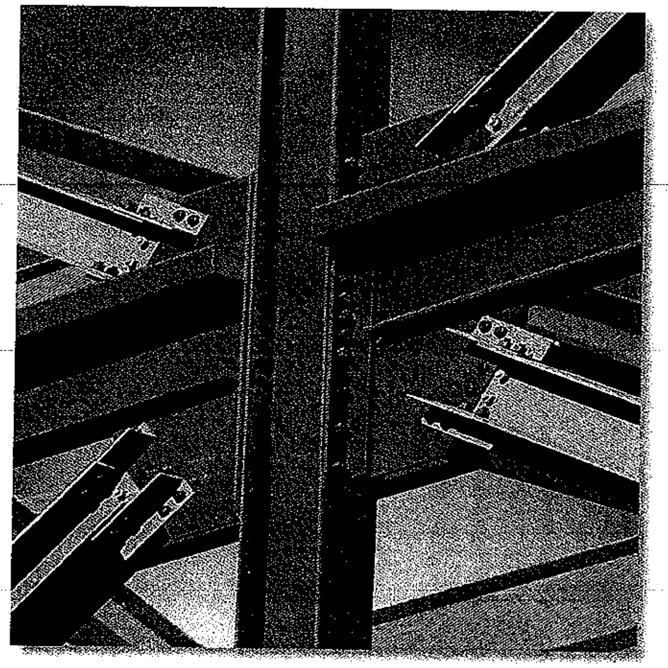
MECH 320-Mechanics of Materials

AMERICAN UNIVERSITY OF BEIRUT



STUDENT NAME:

ID:

Spring semester 2009-2010

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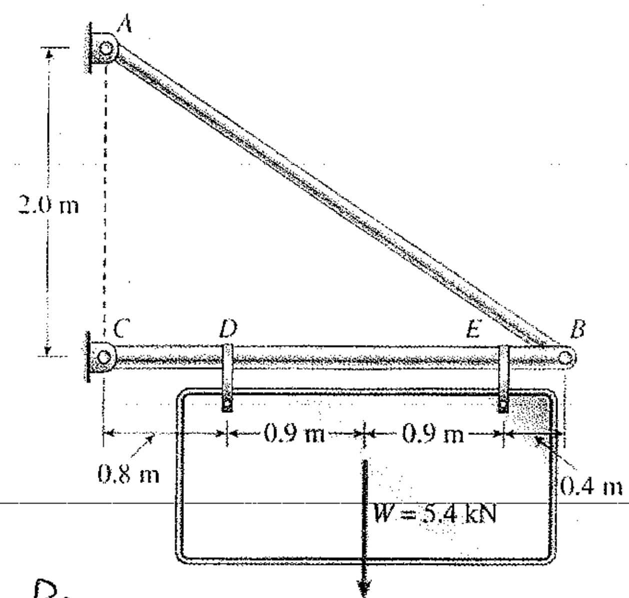
TIME ALLOWED: 2H

Problem 1. (25pts).

The two-bar truss ABC shown in the figure below has pin supports at points A and C, which are 2.0 m apart. Members AB and BC are steel bars, pin connected at joint B. The length of bar BC is 3.0 m. A sign weighing 5.4 kN is suspended from bar BC at points D and E, which are located 0.8 m and 0.4m, respectively, from the ends of the bar.

Determine the required cross sectional area of bar AB and the required diameter of the pin at support C if the allowable stresses in tension and shear are 125 MPa and 45 MPa, respectively. (Note: The pins at the supports are in double shear. Also, disregard the weights of members AB and BC).

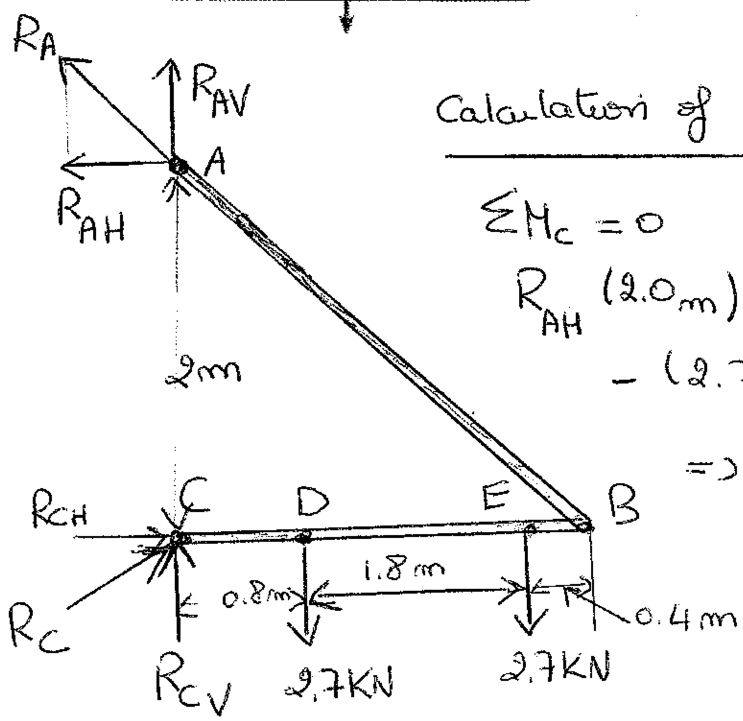


Solution

1) Reactions

FBD

(fig a)

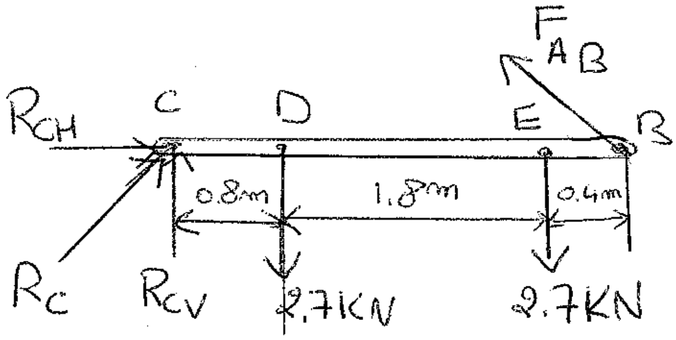


Calculation of R_{AH}

$\Sigma M_C = 0$

$$R_{AH}(2.0\,m) - (2.7\,KN)(0.8\,m) - (2.7\,KN)\cdot(2.6\,m) = 0$$

$$\Rightarrow \boxed{R_{AH} = 4.6\,KN}$$



(fig b)

Calculation of R_{CH}: $\boxed{R_{CH} = R_{AH} = 4.6\ KN}$

Calculation of R_{CV} and R_{AV} (fig b)

$\Sigma M_B = 0 \qquad -R_{CV}(3.0\,m) + (2.7\,KN) \times 2.2\,m + (2.7\,KN)\times(0.4\,m)$

$\Rightarrow R_{CV} = 2.34\ KN$

fig (a): $\Sigma F_{Vert} = 0$

$R_{AV} + R_{CV} - 2.7\,KN - 2.7\,KN = 0$

$\Rightarrow R_{AV} = 3.060\ KN$

calculation of R_A: $R_A = \sqrt{(R_{AH})^2 + (R_{AV})^2} = 5.516\ KN$

calculation of R_C: $R_C = \sqrt{(R_{CH})^2 + (R_{CV})^2} = 5.152\ KN$

Tensile force in bar AB: $F_{AB} = R_A = 5.516\ KN$

shear force acting in pin C: $V_C = R_C = 5.152\ KN$

Required area of bar: $A_{AB} = \frac{F_{AB}}{\sigma_{All}} = \frac{5.516\ KN}{125\ MPa} = 44\ mm$

Required diameter of pin

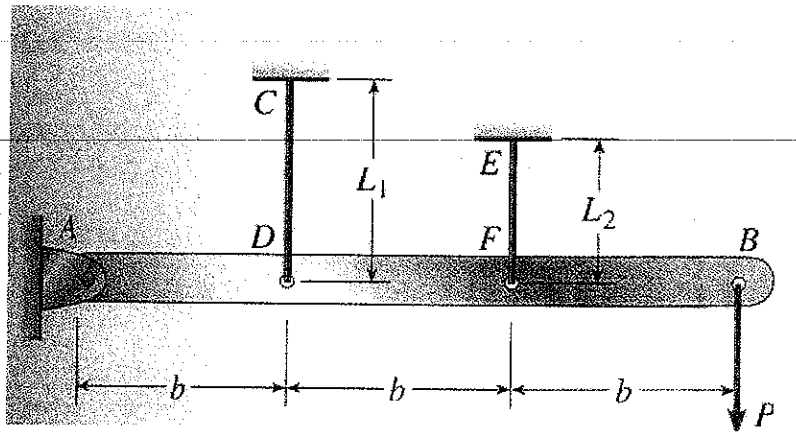
$A_{pin} = \frac{V_C}{2\tau} = \frac{5.152\ KN}{2(45\ MPa)} = 57.2\ mm^2 \Rightarrow d_{pin} = \sqrt{\frac{4A_{pin}}{\pi}} = 8.54\ mm$

Problem 2.(25 pts)

A horizontal rigid bar AB is pinned at end A and supported by two wires CD and EF at points D and F as shown in the figure below. A vertical load P=1.5 kN acts at end B of the bar. The bar has a length =3b. The wires CD and EF have length L_1=0.40m and L_2=0.30 m. Also the wires CD and EF have diameters of 4 mm and 3mm respectively. The wire CD is made of aluminum with E_{Al}=72 GPa while the wire EF is made of Magnesium with E_{Mg}=45 GPa knowing that the allowable stresses in the Aluminum and Magnesium wires are 200 MPa and 175 MPa respectively

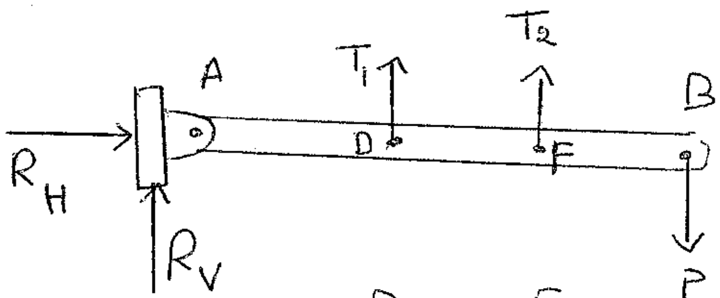
a-Calculate the normal stresses in bars EF and CD.

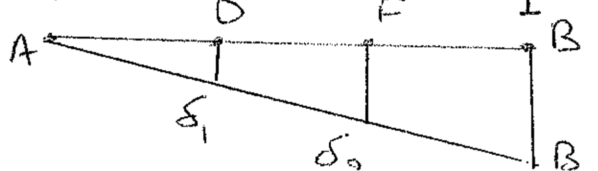
b-By reference to the values found in part a, determine if the materials used in the design are adequate.



Solution

Equilibrium





$\Sigma M_A = 0$

$T_1 b + T_2 (2b) - P(3b) = 0 \Rightarrow T_1 + 2T_2 = 3P \quad (1)$

<u>Compatibility</u> $AD = DF$

$$\delta_2 = 2\delta_1 \quad (2)$$

<u>Force displacement relations</u>

$$\delta_1 = \frac{T_1 L_1}{E_1 A_1} \qquad \delta_2 = \frac{T_2 L_2}{E_2 A_2}$$

$$A_1 = \frac{\pi d_1^2}{4} \qquad A_2 = \frac{\pi d_2^2}{4}$$

For simplicity $f_1 = \frac{L_1}{E_1 A_1} \quad f_2 = \frac{L_2}{E_2 A_2}$

$$\Rightarrow \delta_1 = f_1 T_1 \qquad \delta_2 = f_2 T_2 \qquad (3, 4)$$

<u>solution of equations</u>

Substitute (3, 4) into (2) $\Rightarrow$

$$f_2 T_2 = 2 f_1 T_1 \quad (5)$$

Solving (5) and (1) each containing T_1 & T_2 –

$$T_1 = \frac{3 f_2 P}{4 f_1 + f_2} \qquad T_2 = \frac{6 f_1 P}{4 f_1 + f_2}$$

<u>Normal stresses</u>

$$\sigma_1 = \frac{T_1}{A_1} = \frac{3P}{A_1}\left(\frac{f_2}{4f_1 + f_2}\right)$$

$$f_1 = \frac{L_1}{A_1 E_1} = 0.4420 \times 10^{-6} \text{ m/N};$$

$$f_2 = \frac{L_2}{E_2 A_2} = 0.9431 \times 10^{-6} \text{ m/N};$$

$P = 1.5 \times 10^3$ N ; $A_1 = \frac{\pi d_1^2}{4} = 12.57 \text{ mm}^2$

$$\Rightarrow \sigma_1 = \frac{3 \times 1.5 \times 10^3}{12.57 \times 10^{-6}} \left(\frac{0.9431 \times 10^{-6}}{4 \times 0.442 \times 10^{-6} + 0.9431 \times 10^{-6}} \right)$$

$$= 124.44 \text{ MPa}$$

$$\sigma_2 = \frac{6P}{A_2} \left(\frac{f_1}{4f_1 + f_2} \right) = \frac{6 \times 1.5 \times 10^3}{7.069 \times 10^{-6}} \left(\frac{0.4420}{4 \times 0.442 + 0.9431} \right)$$

$$= \cancel{1.\ldots} \; \cancel{2\ldots} \; 208 \text{ MPa}$$

Material 1 → Acceptable
because $\sigma_1 < \sigma_{All}$

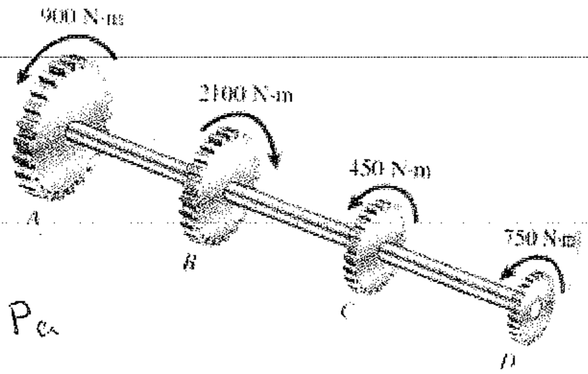
Material 2 → Non acceptable
because $\sigma_2 > \sigma_{Allg}$
change it to a stronger material.

Problem 3. (25 pts)

Four gears are attached to a circular shaft and transmit the torques shown in the figure below. The Allowable shear stress in the shaft is 70 MPa.

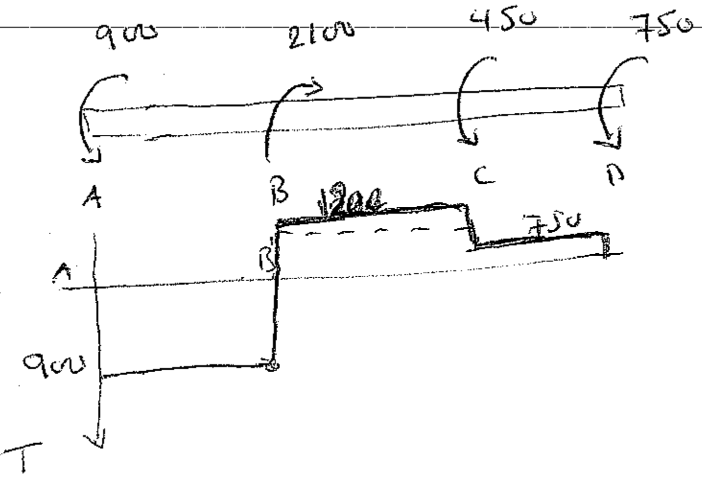
a- What is the required diameter d of the shaft if it has a solid cross-section?

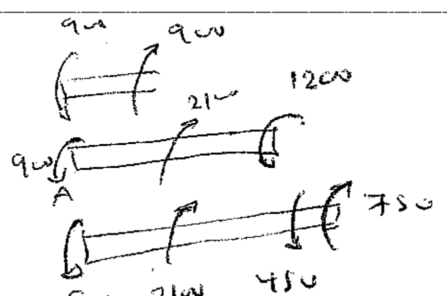
b- Another design is suggested such that a hollow shaft of an inside diameter of 40 mm and an outside diameter of 60 mm is used, would the shaft sustain the applied torque ? Justify your answer.



$\tau_{all} = 70$ MPa

Draw the torque diagram





From the torque diagram, the maximum torque is applied in the section BC and that is where the maximum stress will be induced.

(a)

$$\tau = \frac{Tr}{J}$$

$$\cancel{\tau_{max} = \frac{16T}{\pi d^3} = \frac{16 \times (1200)}{\pi (\ldots)} = 70 \times 10^6}$$

$$\tau_{max} = \frac{16\,T}{\pi d^3} \quad \cancel{\Rightarrow d = \tau_{max}}$$

$$d = \left(\frac{16\,T}{\pi\,\tau_{max}}\right)^{1/3} \quad \cancel{\Rightarrow \frac{16 \times 70 \times 10^6}{\pi}}$$

$$d = \left(\frac{16 \times 1200}{\pi \times 70 \times 10^6}\right)^{1/3} = 0.0443\ m$$

$$\boxed{\therefore d = 44.3\ mm}$$

(b) $d_i = 40\ mm \qquad d_o = 60\ mm$.

$$\tau = \frac{Tr}{J}, \qquad J = \frac{\pi}{32}(d_o^4 - d_i^4) = \frac{\pi}{32}(60^4 - 40^4) \times 10^{-12}\ m^4$$

$$\therefore J = 1.021 \times 10^{-6}\ m^4$$

$$r = 30 \times 10^{-3}$$

$$\therefore \tau = \frac{1200 \times 30 \times 10^{-3}}{1.021 \times 10^{-6}} = 35.26\ MPa$$

The shaft would sustain the applied torque because $\tau_{max} < \tau_{all}$.

$$\delta_D = \frac{20 \times 0.7}{E_{st} A_{st}} + \frac{20 \times 0.5}{E_{al} A_{al}} + \frac{12 \times 0.3}{E_{al} A_{al}}$$

$$A_{st} = \frac{\pi}{4}(10\times10^{-3})^2 = 78.54 \times 10^{-6} \ m^2$$

$$A_{al} = \frac{\pi}{4}(30\times10^{-3})^2 = 706.86 \times 10^{-6} \ m^2$$

$$\therefore \delta_D = \frac{20 \times 0.7 \times \cancel{1000}}{200\times\cancel{10^9}\times 78.54\times10^{-6}} + \frac{20\times0.5\times\cancel{1000}}{70\times\cancel{10^9}\times706.86\times10^{-6}} + \frac{12\times0.3\times\cancel{1000}}{70\times\cancel{10^9}\times706.86\times10^{-6}}$$

$$\therefore \delta_D = 0.000891 + 0.000202 + 0.000072762$$

$$\boxed{\delta_D = 0.001165 \ m = 1.165 \ mm}$$

b- Maximum stress. $\Rightarrow$ since the steel rod has smaller cross section and the maximum load is applied on it then the max stress will be induced there.

$$\sigma_{max} = \frac{P}{A}$$

$$\sigma_{max} = \frac{20\times10^3}{\frac{\pi}{4}(10\times10^{-3})^2} = 254.65 \ MPa.$$

c- $\sigma_{st} = E\,\varepsilon_{st} \Rightarrow \varepsilon = \frac{\sigma}{E} = \frac{254.65}{200\times10^3} = 1.273\times10^{-3}$

$\nu_{st} = -\frac{\varepsilon_{lat}}{\varepsilon_{ax}} \Rightarrow \varepsilon_{al} = -\nu_{st}\,\varepsilon_{max} = -(0.3)(1.273)\times10^{-3}$

$\therefore \varepsilon_{lat} = -0.382\times10^{-3} = \frac{\delta D}{D}$

$\therefore \delta D = (-0.382\times10^{-3})(10 \ mm) = -3.82\times10^{-3} \ mm$