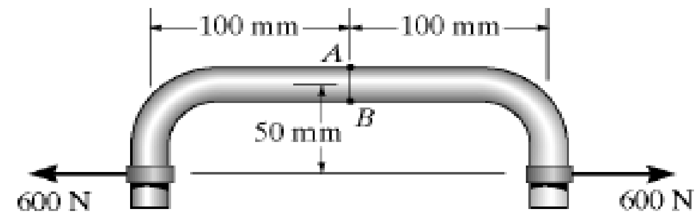
The bent rod has a diameter of 15 mm and is subjected to the force of 600 N. Determine the principal stresses and the maximum in-plane shear stress that are developed at point A and point B. Show the results on elements located at these points.



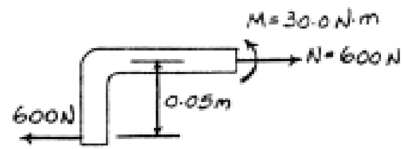
Section Properties:

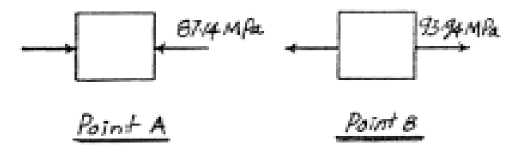
$$A = \pi\left(0.0075^2\right) = 56.25\pi\left(10^{-6}\right)\ \text{m}^2$$

$$I = \frac{\pi}{4}\left(0.0075^4\right) = 2.4850\left(10^{-9}\right)\ \text{m}^4$$

$Q_A = Q_B = 0$

Stress:





$$\sigma = \frac{N}{A} \pm \frac{Mc}{I}$$

$$= \frac{600}{56.25\pi(10^{-6})} \pm \frac{30.0(0.0075)}{2.4850(10^{-9})}$$

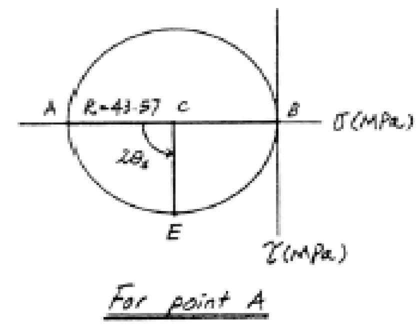
$$\sigma_A = 3.3953 - 90.5414 = -87.14\ \text{MPa}$$

$$\sigma_B = 3.3953 + 90.5414 = 93.94\ \text{MPa}$$

$$\tau_A = \tau_B = 0 \text{ since } Q_A = Q_B = 0$$

Construction of the Circle: In accordance with the sign convention, $\sigma_x = -87.14$ MPa, $\sigma_y = 0$, and $\tau_{xy} = 0$ for point A. Hence,

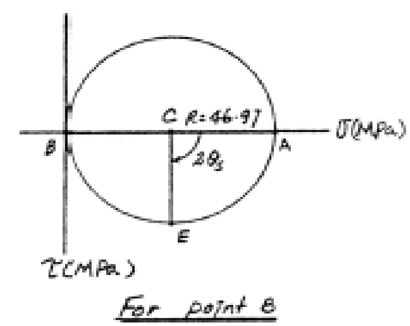
$$\sigma_{avg} = \frac{\sigma_x + \sigma_y}{2} = \frac{-87.14 + 0}{2} = -43.57\ \text{MPa}$$



The coordinates for reference points A and C are $A(-87.14,\ 0)$ $C(-43.57,\ 0)$.

The radius of the circle is $R = 87.14 - 43.57 = 43.57$ MPa

In-Plane Principal Stresses: The coordinates of points B and A represent σ_1 and σ_2, respectively.



$\sigma_1 = 0$ **Ans**

$\sigma_2 = -87.1$ MPa **Ans**

Maximum In-Plane Shear Stress: Represented by the coordinates of point E on the circle.

$$\tau_{\substack{max \\ in\text{-}plane}} = R = 43.6 \text{ MPa} \qquad \textbf{Ans}$$

Orientation of the Plane for Maximum In-Plane Shear Stress: From the circle

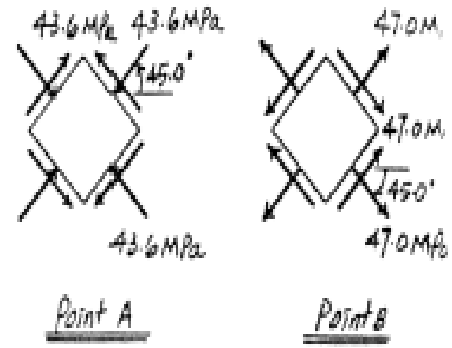
$$2\theta_s = 90^\circ \qquad \theta_s = 45.0^\circ \quad (\textit{Counterclockwise}) \qquad \textbf{Ans}$$

Construction of the Circle: In accordance with the sign convention, $\sigma_x = 93.94$ MPa, $\sigma_y = 0$, and $\tau_{xy} = 0$ for point B. Hence,

$$\sigma_{avg} = \frac{\sigma_x + \sigma_y}{2} = \frac{93.94 + 0}{2} = 46.97 \text{ MPa}$$

The coordinates for reference points A and C are $A(93.94,\ 0)$ $C(46.97,\ 0)$.

The radius of the circle is $R = 93.94 - 46.97 = 46.97$ MPa



In-Plane Principal Stresses: The coordinates of points A and B represent σ_1 and σ_2, respectively.

$\sigma_1 = 93.9$ MPa **Ans**

$\sigma_2 = 0$ MPa **Ans**

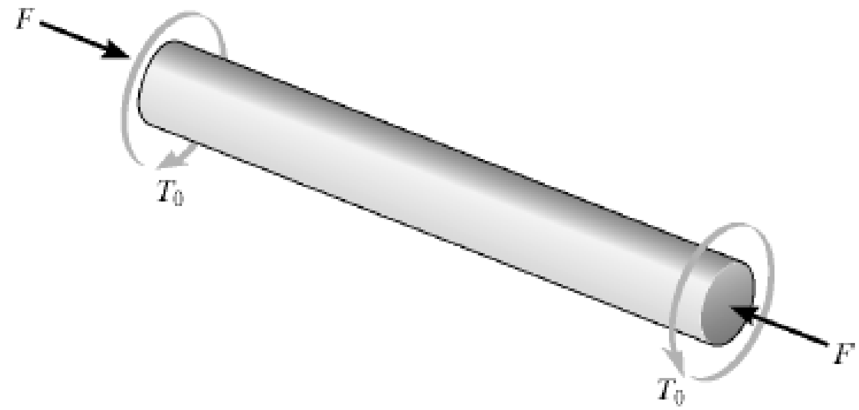
Maximum In-Plane Shear Stress: Represented by point E on the circle.

$$\tau_{\substack{max \\ in\text{-}plane}} = R = 47.0 \text{ MPa} \qquad \textbf{Ans}$$

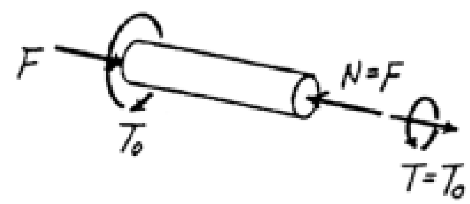
Orientation of the Plane for Maximum In-Plane Shear Stress: From the circle

$$2\theta_s = 90^\circ \qquad \theta_s = 45.0^\circ \quad (\textit{Clockwise}) \qquad \textbf{Ans}$$

The shaft has a diameter d and is subjected to the loadings shown. Determine the principal stresses and the maximum in-plane shear stress that is developed anywhere on the surface of the shaft.



Internal Forces and Torque: As shown on **FBD (a)**.

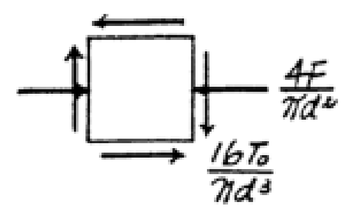


Section Properties:

$$A = \frac{\pi}{4}d^2 \qquad J = \frac{\pi}{2}\left(\frac{d}{2}\right)^4 = \frac{\pi}{32}d^4$$

Normal Stress:

$$\sigma = \frac{N}{A} = \frac{-F}{\frac{\pi}{4}d^2} = -\frac{4F}{\pi d^2}$$



Shear Stress: Applying the torsion formula,

$$\tau = \frac{Tc}{J} = \frac{T_0\left(\frac{d}{2}\right)}{\frac{\pi}{32}d^4} = \frac{16T_0}{\pi d^3}$$

In-Plane Principal Stresses: $\sigma_x = -\frac{4F}{\pi d^2}$, $\sigma_y = 0$, and $\tau_{xy} = -\frac{16T_0}{\pi d^3}$ for any point on the shaft's surface. Applying Eq. 9 – 5,

$$\sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$= \frac{-\frac{4F}{\pi d^2} + 0}{2} \pm \sqrt{\left(\frac{-\frac{4F}{\pi d^2} - 0}{2}\right)^2 + \left(-\frac{16T_0}{\pi d^3}\right)^2}$$

$$= \frac{2}{\pi d^2}\left(-F \pm \sqrt{F^2 + \frac{64T_0^2}{d^2}}\right)$$

$$\sigma_1 = \frac{2}{\pi d^2}\left(-F + \sqrt{F^2 + \frac{64T_0^2}{d^2}}\right)$$ **Ans**

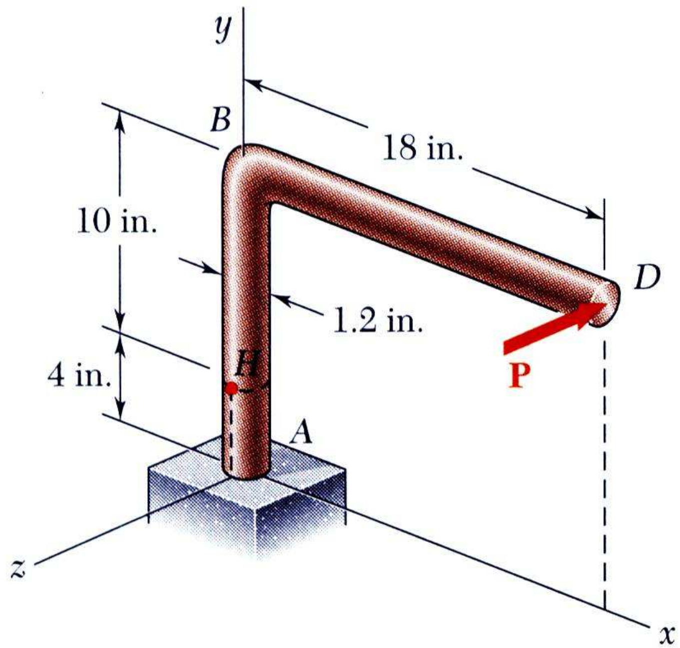
$$\sigma_2 = -\frac{2}{\pi d^2}\left(F + \sqrt{F^2 + \frac{64T_0^2}{d^2}}\right)$$ **Ans**

Maximum In-Plane Shear Stress: Applying Eq. 9 – 7,

$$\tau_{\substack{max \\ in\text{-}plane}} = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$= \sqrt{\left(\frac{-\frac{4F}{\pi d^2} - 0}{2}\right)^2 + \left(-\frac{16T_0}{\pi d^3}\right)^2}$$

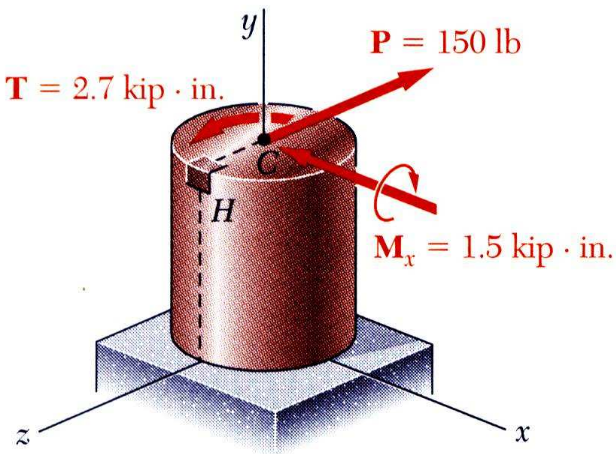
$$= \frac{2}{\pi d^2}\sqrt{F^2 + \frac{64T_0^2}{d^2}}$$ **Ans**

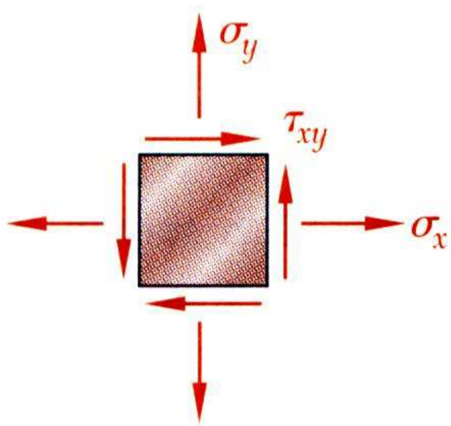


A single horizontal force *P* of 150 lb magnitude is applied to end D of lever *ABD*. Determine (a) the normal and shearing stresses on an element at point *H* having sides parallel to the *x* and *y* axes, (b) the principal planes and principal stresses at the point *H*.

SOLUTION:

- Determine an equivalent force-couple system at the center of the transverse section passing through *H*.
- Evaluate the normal and shearing stresses at *H*.
- Determine the principal planes and calculate the principal stresses.





SOLUTION:

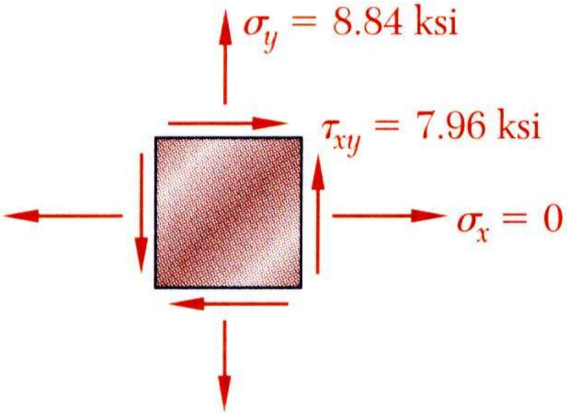
- Determine an equivalent force-couple system at the center of the transverse section passing through *H*.

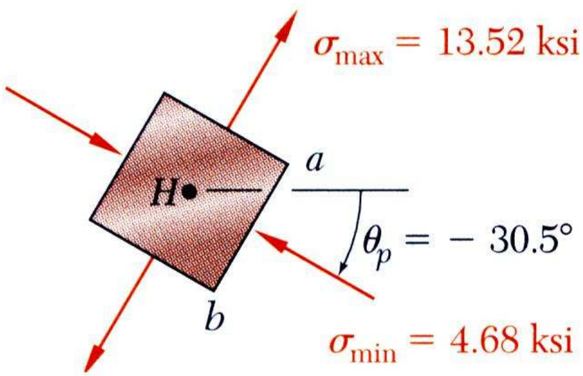
$$P = 150\,\text{lb}$$
$$T = (150\,\text{lb})(18\,\text{in}) = 2.7\,\text{kip}\cdot\text{in}$$
$$M_x = (150\,\text{lb})(10\,\text{in}) = 1.5\,\text{kip}\cdot\text{in}$$

- Evaluate the normal and shearing stresses at *H*.

$$\sigma_y = +\frac{Mc}{I} = +\frac{(1.5\,\text{kip}\cdot\text{in})(0.6\,\text{in})}{\frac{1}{4}\pi(0.6\,\text{in})^4}$$
$$\tau_{xy} = +\frac{Tc}{J} = +\frac{(2.7\,\text{kip}\cdot\text{in})(0.6\,\text{in})}{\frac{1}{2}\pi(0.6\,\text{in})^4}$$

$$\boxed{\sigma_x = 0 \quad \sigma_y = +8.84\,\text{ksi} \quad \tau_y = +7.96\,\text{ksi}}$$





- Determine the principal planes and calculate the principal stresses.

$$\tan 2\theta_p = \frac{2\tau_{xy}}{\sigma_x - \sigma_y} = \frac{2(7.96)}{0 - 8.84} = -1.8$$

$$2\theta_p = -61.0°, 119°$$

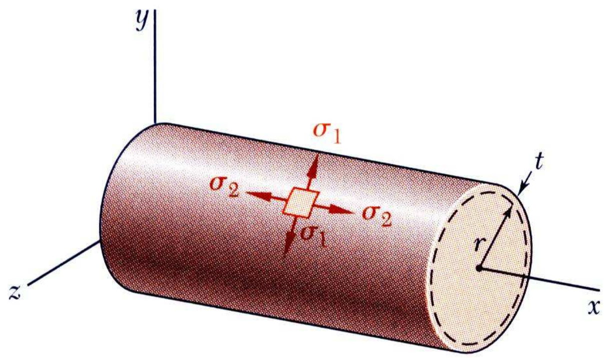
$$\theta_p = -30.5°,\ 59.5°$$

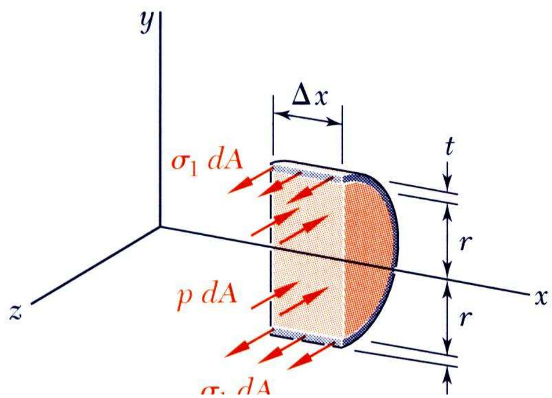
$$\sigma_{max,min} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

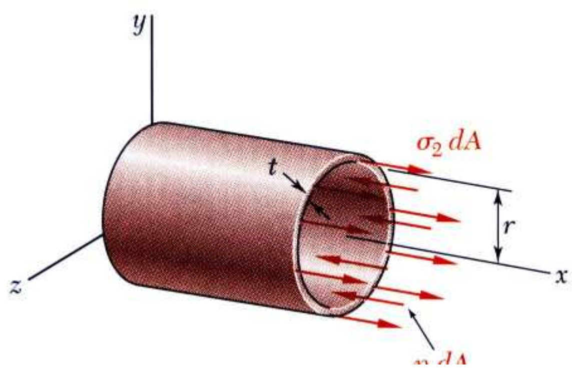
$$= \frac{0 + 8.84}{2} \pm \sqrt{\left(\frac{0 - 8.84}{2}\right)^2 + (7.96)^2}$$

$$\sigma_{max} = +13.52\,\text{ksi}$$
$$\sigma_{min} = -4.68\,\text{ksi}$$

PB 1







- Cylindrical vessel with principal stresses

  σ_1 = hoop stress

  σ_2 = longitudinal stress

- Hoop stress:

$$\sum F_z = 0 = \sigma_1(2t\,\Delta x) - p(2r\,\Delta x)$$

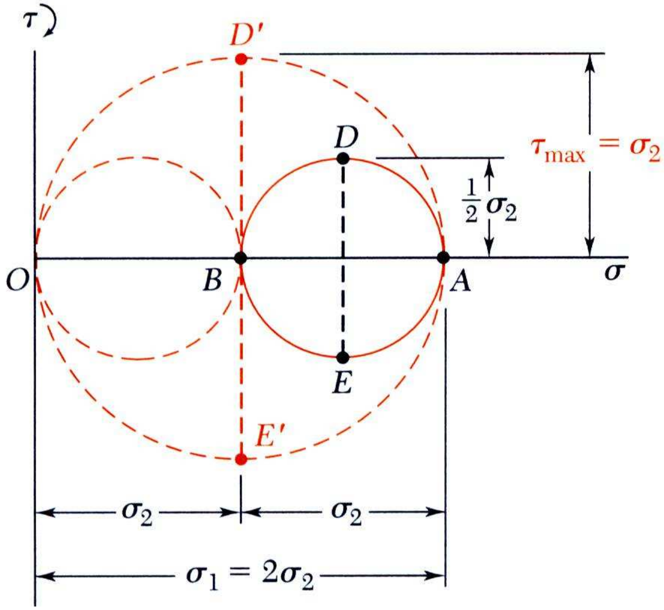
$$\sigma_1 = \frac{pr}{t}$$

- Longitudinal stress:

$$\sum F_x = 0 = \sigma_2(2\pi rt) - p(\pi r^2)$$

$$\sigma_2 = \frac{pr}{2t}$$

$$\sigma_1 = 2\sigma_2$$



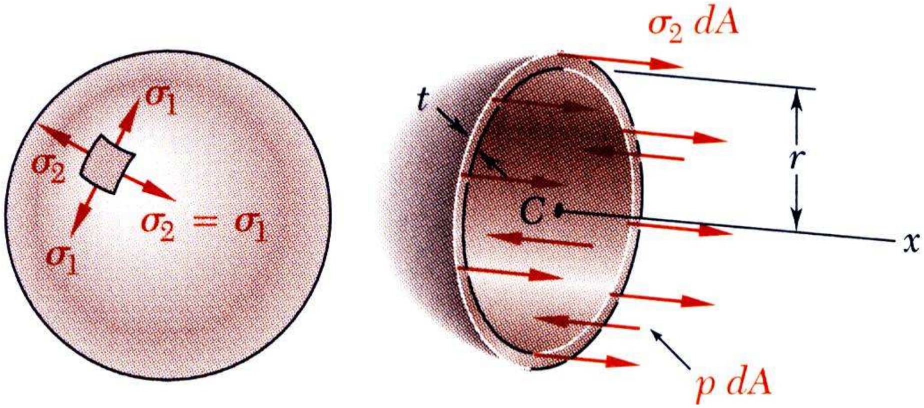
- Points *A* and *B* correspond to hoop stress, σ_1, and longitudinal stress, σ_2

- Maximum in-plane shearing stress:

$$\tau_{\max(\text{in-plane})} = \frac{1}{2}\sigma_2 = \frac{pr}{4t}$$

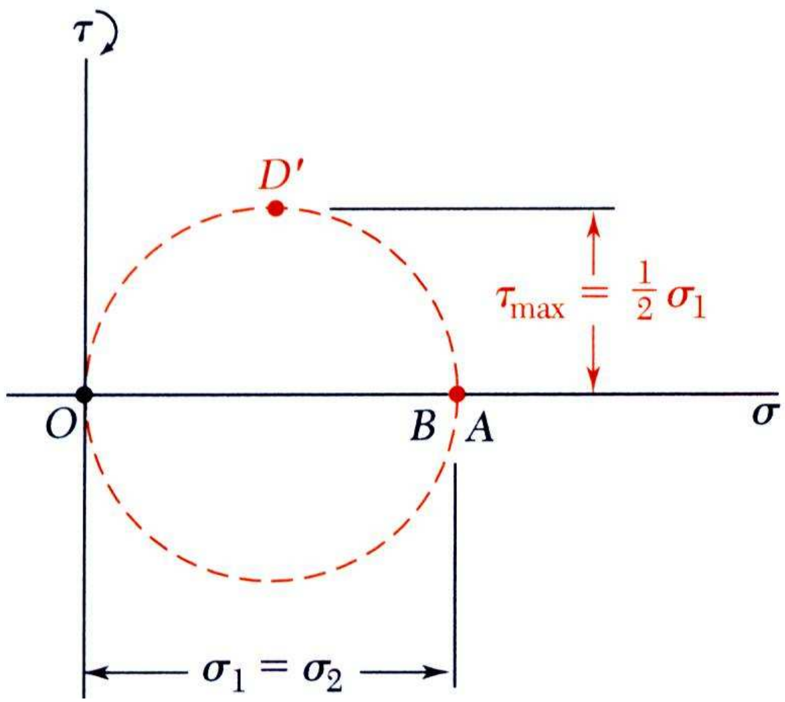
- Maximum out-of-plane shearing stress corresponds to a 45° rotation of the plane stress element around a longitudinal axis

$$\tau_{\max} = \sigma_2 = \frac{pr}{2t}$$



- Spherical pressure vessel:

$$\sigma_1 = \sigma_2 = \frac{pr}{2t}$$



- Mohr's circle for in-plane transformations reduces to a point

$$\sigma = \sigma_1 = \sigma_2 = \text{constant}$$

$$\tau_{\text{max(in-plane)}} = 0$$

- Maximum out-of-plane shearing stress

$$\tau_{\max} = \tfrac{1}{2}\sigma_1 = \frac{pr}{4t}$$