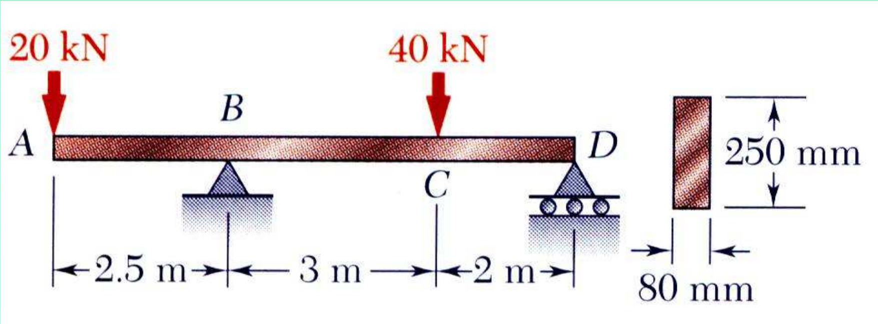
Problem.1

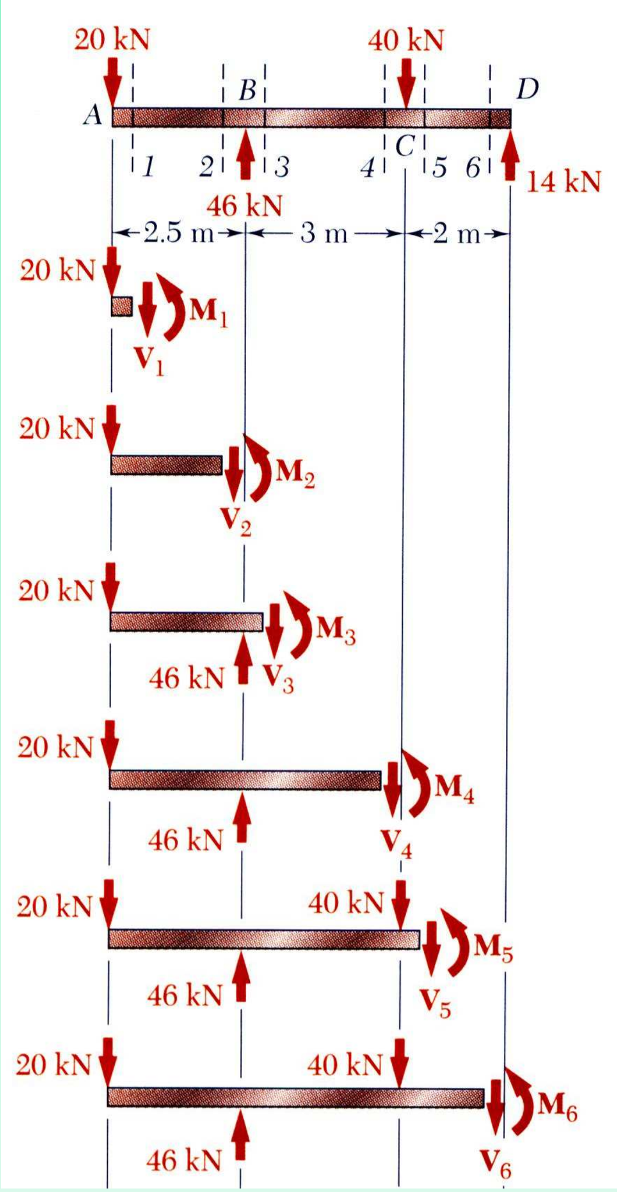


For the timber beam and loading shown, draw the shear and bend-moment diagrams and determine the maximum normal stress due to bending.

SOLUTION:

- Treating the entire beam as a rigid body, determine the reaction forces
- Section the beam at points near supports and load application points. Apply equilibrium analyses on resulting free-bodies to determine internal shear forces and bending couples
- Identify the maximum shear and bending-moment from plots of their distributions.
- Apply the elastic flexure formulas to determine the corresponding maximum normal stress.

Problem 1



SOLUTION:

- Treating the entire beam as a rigid body, determine the reaction forces

  from $\sum F_y = 0 = \sum M_B: \quad R_B = 40\,\text{kN} \quad R_D = 14\,\text{kN}$

- Section the beam and apply equilibrium analyses on resulting free-bodies

$$\sum F_y = 0 \quad -20\,\text{kN} - V_1 = 0 \qquad V_1 = -20\,\text{kN}$$

$$\sum M_1 = 0 \quad (20\,\text{kN})(0\,\text{m}) + M_1 = 0 \quad M_1 = 0$$

$$\sum F_y = 0 \quad -20\,\text{kN} - V_2 = 0 \qquad V_2 = -20\,\text{kN}$$

$$\sum M_2 = 0 \quad (20\,\text{kN})(2.5\,\text{m}) + M_2 = 0 \quad M_2 = -50\,\text{kN}\cdot\text{m}$$

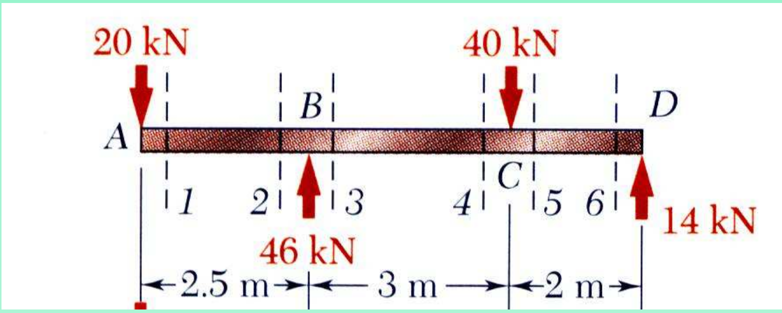
$$V_3 = +26\,\text{kN} \quad M_3 = -50\,\text{kN}\cdot\text{m}$$

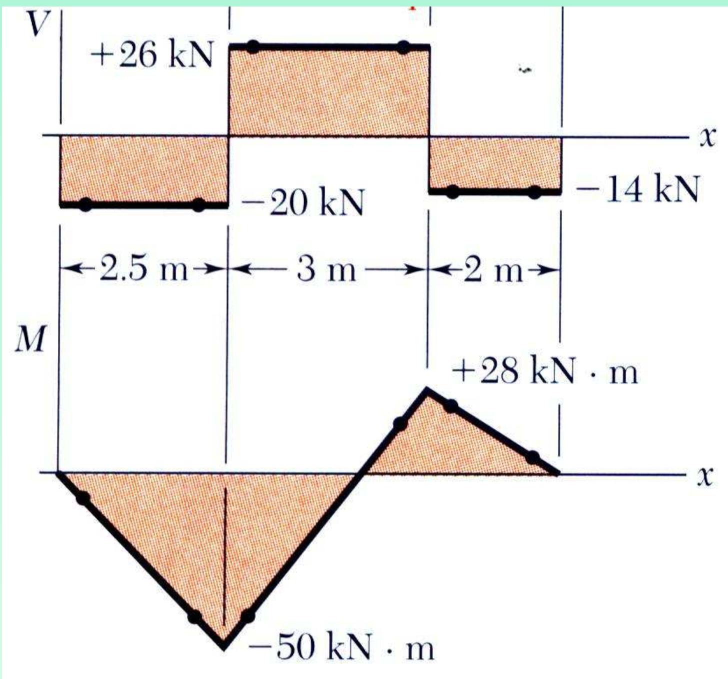
$$V_4 = +26\,\text{kN} \quad M_4 = +28\,\text{kN}\cdot\text{m}$$

$$V_5 = -14\,\text{kN} \quad M_5 = +28\,\text{kN}\cdot\text{m}$$

$$V_6 = -14\,\text{kN} \quad M_6 = 0$$

Problem 1





- Identify the maximum shear and bending-moment from plots of their distributions.

$$V_m = 26\,\mathrm{kN} \quad M_m = |M_B| = 50\,\mathrm{kN \cdot m}$$

- Apply the elastic flexure formulas to determine the corresponding maximum normal stress.

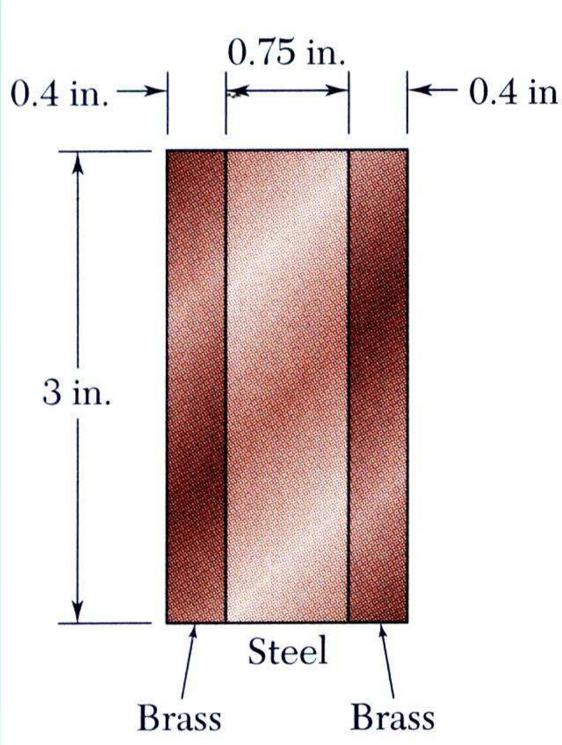
$$S = \tfrac{1}{6} b h^2 = \tfrac{1}{6}(0.080\,\mathrm{m})(0.250\,\mathrm{m})^2$$

$$= 833.33 \times 10^{-6}\,\mathrm{m}^3$$

$$\sigma_m = \frac{|M_B|}{S} = \frac{50 \times 10^3\,\mathrm{N \cdot m}}{833.33 \times 10^{-6}\,\mathrm{m}^3}$$

$$\boxed{\sigma_m = 60.0 \times 10^6\,\mathrm{Pa}}$$

Problem 2

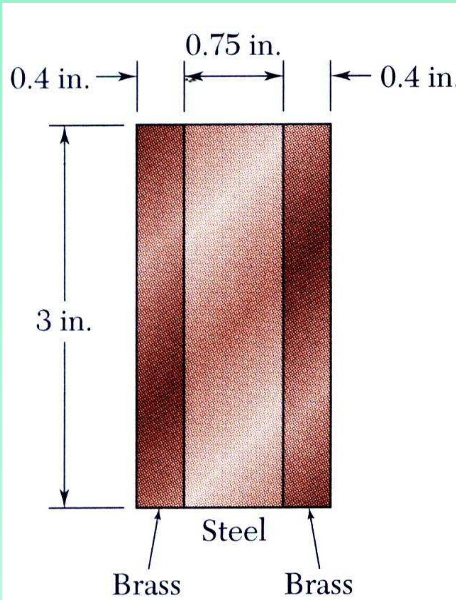


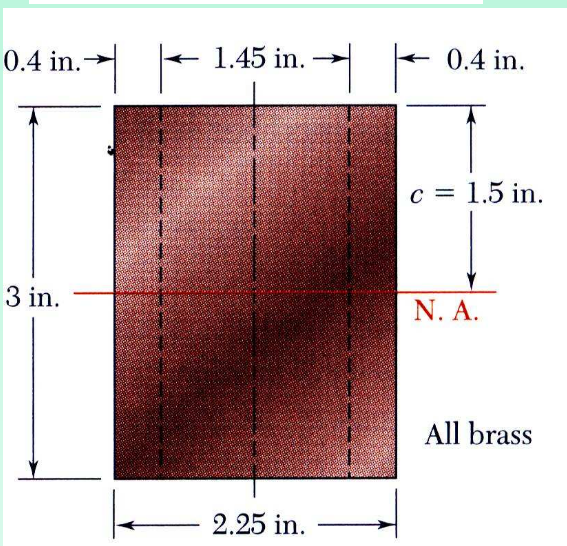
Bar is made from bonded pieces of steel (E_s = 29x10^6 psi) and brass (E_b = 15x10^6 psi). Determine the maximum stress in the steel and brass when a moment of 40 kip*in is applied.

SOLUTION:

- Transform the bar to an equivalent cross section made entirely of brass
- Evaluate the cross sectional properties of the transformed section
- Calculate the maximum stress in the transformed section. This is the correct maximum stress for the brass pieces of the bar.
- Determine the maximum stress in the steel portion of the bar by multiplying the maximum stress for the transformed section by the ratio of the moduli of elasticity.

Problem 2





SOLUTION:

- Transform the bar to an equivalent cross section made entirely of brass.

$$n = \frac{E_s}{E_b} = \frac{29\times10^6\,\text{psi}}{15\times10^6\,\text{psi}} = 1.933$$

$$b_T = 0.4\,\text{in} + 1.933\times0.75\,\text{in} + 0.4\,\text{in} = 2.25\,\text{in}$$

- Evaluate the transformed cross sectional properties

$$I = \tfrac{1}{12} b_T h^3 = \tfrac{1}{12}(2.25\,\text{in.})(3\,\text{in})^3$$

$$= 5.063\,\text{in}^4$$

- Calculate the maximum stresses

$$\sigma_m = \frac{Mc}{I} = \frac{(40\,\text{kip}\cdot\text{in})(1.5\,\text{in})}{5.063\,\text{in}^4} = 11.85\,\text{ksi}$$

$$(\sigma_b)_{max} = \sigma_m$$

$$(\sigma_s)_{max} = n\sigma_m = 1.933\times11.85\,\text{ksi}$$

$$\boxed{\begin{aligned}(\sigma_b)_{max} &= 11.85\,\text{ksi}\\ (\sigma_s)_{max} &= 22.9\,\text{ksi}\end{aligned}}$$